

book

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CANADIAN BUSINESS MATHEMATICS

***CATHERINE
LUND***

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CANADIAN BUSINESS MATHEMATICS

book 1

CATHERINE LUND

B. A., M. COM., C. G. A.

G R E G G D I V I S I O N

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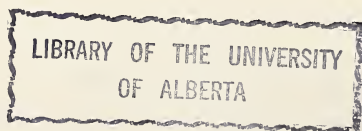
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P R E F A C E

CANADIAN BUSINESS MATHEMATICS is a modern simplified approach to the fundamentals of basic mathematics and their application to the business world. The book is designed primarily for use in commercial courses in the Canadian high schools and business colleges. It also provides a background for more advanced work in mathematics of finance.

CANADIAN BUSINESS MATHEMATICS has a typically Canadian approach with a business problem introducing the subject matter of each chapter. A liberal use is also made of illustrations from the Canadian business world. Each chapter is divided into small self-contained units with extensive Workout Exercises after each section.

Graphs form the motif for the book, are introduced early, and are stressed throughout. Each chapter is headed with a graph pertinent to the chapter material, and some problems in all chapters call for graphical presentation. Knowledge through pictures is an important aspect of our modern life; this is particularly true in the business and financial world where pictures and graphs are used extensively to tell the story. It is important that business men and women learn to read graphs and, still more important, learn to present their own material in a pictorial manner.

Part I deals with basic skills; the concept of sets is introduced in the opening chapters but is kept in self-contained units which can be included or excluded without interfering with the continuity of the material.

Algebra is presented not as mathematical mystery but as a tool to be used to solve problems and in equations and formulas. This section also provides the necessary mathematical background for students who may later wish to transfer to academic courses.

Part II deals with specific business applications. This section will be continued and enlarged upon in Book 2.

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In the subject of mathematics, too, there is much scope for fun and entertainment; and it is hoped that readers may yield to the fascination of playing some of the games of the mathematician. The Figuring for Fun section at the end of each chapter has been included to whet the appetite of readers for such mathematical play. To have fun with mathematics will dispel any lingering fears or dislikes a person may have; and if this can be accomplished, the effort will be worth while.

CATHERINE LUND

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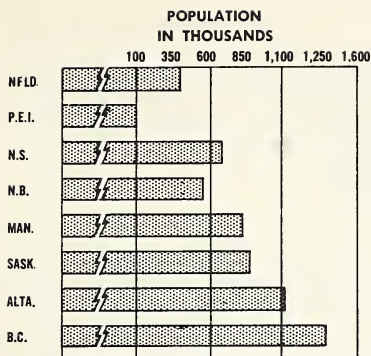
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Addition and Subtraction

1



* SOURCE: CANADA YEAR BOOK

A Business Problem

Happy Daze Ltd., pack their product Wimpey in a cardboard box. They purchase these cardboard boxes at \$20 per thousand and use ten million in a year. Mr. Ima Wowe, the production manager, has suggested that a box-making machine be installed and that Happy Daze Ltd., manufacture their own cardboard boxes. The estimated costs per thousand boxes would be: paper and cardboard, \$9; labels, \$1; glue, \$1; labour, \$3; payment on machine, \$2; other costs, \$2. Do you think that Happy Daze Ltd., should install this machine? Why? What would be the difference in cost for a thousand boxes? For a year's supply? What problems, other than costs, must be considered?

Mathematics

Mathematics deals with numbers, and the numbers are used in counting, comparing, and computing. For example, business organizations make use of mathematical calculations in counting, weighing, and measuring merchandise. Business organizations also use ratios, proportions, and percentages for the comparison of one set of numbers with one or more other sets in analyzing financial information.

In our numbering system we use ten symbols: 0,1,2,3,4,5,6,7,8,9. The Hindus perfected and used a similar system about 500 A.D. The Arabs adopted this about 800 A.D., and as they traded throughout Southern Europe and Africa, they transmitted it to the countries bordering the Mediterranean Sea. The Moorish Arabs also took the system to Spain during their domination of that country from about 900 A.D. until 1200 A.D. From there it gradually filtered throughout the civilized world. We call it the Hindu-Arabic System.

There are two great advantages this system has over the more ancient ones, such as the Greek and Roman notations:

- The Hindus invented the 0 to be used in any column not having a numerical value. It was thought of as a place-holder.
- When denoting a number, digits are so arranged that they have a place value, e.g. 6,941.

* For graphical presentation of data see Chapter 4.

$$\begin{array}{rcl}
 6 \text{ is the thousands digit} & = & 6,000 \text{ the three zeros hold its place.} \\
 9 \text{ is the hundreds digit} & = & 900 \text{ the two zeros hold its place.} \\
 4 \text{ is the tens digit} & = & 40 \text{ the one zero holds its place.} \\
 1 \text{ is the units digit} & = & \underline{\underline{1}} \\
 & & \underline{\underline{6,941}}
 \end{array}$$

The Concept of Sets

A *set* is simply a collection of objects or things. We speak of a set of cutlery, meaning a collection of knives, forks, and spoons, a set of chess pieces, and so forth. In mathematics, the term *set* has the same meaning, but, as with anything in mathematics, we must be sure to describe the set very carefully. Just as in addition, subtraction, multiplication, and division we use special symbols, so also there are special symbols for sets.

To describe sets more briefly, we can choose some letter of the alphabet to represent the set. Using the same examples given above:

$$C = \{\text{cutlery}\}$$

$$M = \{\text{chess men}\}$$

The braces { } signify the words 'the set of'. The objects in the set are called *elements* of the set. Thus 'knife' is an element of the set C.

To indicate that an object belongs to, or is an element of, a set, we use the symbol $\in$ which is the Greek letter Epsilon. To say that 'knife belongs to the set of cutlery', we would write:

$$k \in C$$

where k has been chosen to represent the object 'knife'.

In the same way, we may describe sets of numbers. The set of all the natural positive integers might be denoted as

$$N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \dots\}$$

(The three dots here are read as "and so on indefinitely.") Any other group of numbers could also constitute a set:

$$P = \{1, 3, 5, 7, 9\}$$

would be read 'P is the set of all the odd numbers between 0 and 10.' 7 therefore belongs to, or is an element of, P:

$$7 \in P$$

The ten digits of our numbering system could be denoted as

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

It is also possible to describe a set which would have no members, for example, the set containing all the two-digit numbers that are less than 10. A set such as this is called an "empty set" and has a special symbol ϕ (the Greek letter Phi).

WORKOUT EXERCISE I

1. If $T = \{0,1,2,3,4,5,6,7,8,9\}$, list the elements of each of the following sets:
 - (a) The set M whose numbers are between 2 and 9 in the set T ,
 - (b) The set N whose numbers are the odd numbers of set T between 2 and 8,
 - (c) The set O whose numbers are the even numbers of set T between 0 and 9,
 - (d) The set P containing the numbers belonging to both set M and set N ,
 - (e) The set R containing the numbers belonging to both set N and O ,
 - (f) The set S which contains the numbers belonging to neither set P nor set R .
2. List the following sets:
 - (a) The provinces of Canada,
 - (b) The months of the year which contain 31 days,
 - (c) A set containing 7 elements,
 - (d) The set of the four gospels of the Bible,
 - (e) The natural numbers greater than 6 and less than 7,
 - (f) The set of all the two-digit natural numbers,
 - (g) The set of all the odd numbers that are even numbers,
 - (h) The set of all the natural numbers less than 6.
3. Write a description of all the members of a set listed between braces in the following examples:
 - (a) {January, June, July}.
 - (b) {Newfoundland, Prince Edward Island, New Brunswick, Nova Scotia.}
 - (c) {A, B, C . . . Z}.
4. Indicate whether the following pairs of sets are equal ($=$) to each other or are unequal ($\neq$) to each other:
 - (a) $x, y, z =$ or $\neq z, x, y$. (b) $b, c, d =$ or $\neq c, d$.
 - (c) all odd one-digit natural numbers $=$ or $\neq 0,1,2,3,4,5,7,9$.
 - (d) $0 =$ or $\neq \phi$.
 - (e) $97, 98, 99, 100 =$ or $\neq 98, 100, 97, 99$.

<u>1</u> <u>1</u> <u>XX</u>	<u>2</u> <u>1</u> <u>XX</u>	<u>3</u> <u>1</u> <u>XX</u>	<u>4</u> <u>1</u> <u>XX</u>	<u>5</u> <u>1</u> <u>XX</u>	<u>6</u> <u>1</u> <u>XX</u>	<u>7</u> <u>1</u> <u>XX</u>	<u>8</u> <u>1</u> <u>XX</u>	<u>9</u> <u>1</u> <u>XX</u>
	<u>2</u> <u>2</u> <u>XX</u>	<u>3</u> <u>2</u> <u>XX</u>	<u>4</u> <u>2</u> <u>XX</u>	<u>5</u> <u>2</u> <u>XX</u>	<u>6</u> <u>2</u> <u>XX</u>	<u>7</u> <u>2</u> <u>XX</u>	<u>8</u> <u>2</u> <u>XX</u>	<u>9</u> <u>2</u> <u>XX</u>
		<u>3</u> <u>3</u> <u>XX</u>	<u>4</u> <u>3</u> <u>XX</u>	<u>5</u> <u>3</u> <u>XX</u>	<u>6</u> <u>3</u> <u>XX</u>	<u>7</u> <u>3</u> <u>XX</u>	<u>8</u> <u>3</u> <u>XX</u>	<u>9</u> <u>3</u> <u>XX</u>
			<u>4</u> <u>4</u> <u>XX</u>	<u>5</u> <u>4</u> <u>XX</u>	<u>6</u> <u>4</u> <u>XX</u>	<u>7</u> <u>4</u> <u>XX</u>	<u>8</u> <u>4</u> <u>XX</u>	<u>9</u> <u>4</u> <u>XX</u>
				<u>5</u> <u>5</u> <u>XX</u>	<u>6</u> <u>5</u> <u>XX</u>	<u>7</u> <u>5</u> <u>XX</u>	<u>8</u> <u>5</u> <u>XX</u>	<u>9</u> <u>5</u> <u>XX</u>
					<u>6</u> <u>6</u> <u>XX</u>	<u>7</u> <u>6</u> <u>XX</u>	<u>8</u> <u>6</u> <u>XX</u>	<u>9</u> <u>6</u> <u>XX</u>
						<u>7</u> <u>7</u> <u>XX</u>	<u>8</u> <u>7</u> <u>XX</u>	<u>9</u> <u>7</u> <u>XX</u>
							<u>8</u> <u>8</u> <u>XX</u>	<u>9</u> <u>8</u> <u>XX</u>
								<u>9</u> <u>9</u> <u>XX</u>

Figure 1-1

45 Basic Groupings
In Addition of Two Digits

Addition

Addition is the process of finding the sum of two or more numbers without counting. The terms used in addition are:

The *addends*, the figures to be added.

The *sum*, the result of the addition of the addends.

SECTION 1 The Principle of Grouping

When we add two numbers together, we do not increase anything; we regroup the numbers under the Hindu-Arabic number system which is based on groups of ten. If we add 8 and 5 (the addends), we obtain 13 (the sum); we have the same number (of objects) as we had before, but we have changed the arrangement into the standard arrangement of 13, that is, 1 ten and 3 units.

The ten digits of our numbering system, 0,1,2,3,4,5,6,7,8,9, if added together *two at a time*, make 100 combinations. The nine digits from 1 to 9 added together two at a time make 81 combinations. If we now eliminate 36 duplicate combinations (such as either $3 + 4$ or $4 + 3$), we are left with 45 basic combinations of the nine digits added together two at a time. These combinations are given in figure 1-1. They should be practised until they can be recognized automatically, either when added down from the top or up from the bottom.

WORKOUT EXERCISE II

The groupings of figure 1-1 should be read aloud in not more than 45 seconds. They should then be practised until the answers can be given in 30 seconds or less.

A two-digit number should be combined directly with a one-digit number.

Example: Add 45 and 8.

This should be combined as "45 and 8 are 53". We should not have to think "5 and 8 are 13, carry 1; 4 and 1 are 5: 53".

WORKOUT EXERCISE III

1. $79 + 3 =$

2. $87 + 4 =$

3. $5 + 36 =$

4. $9 + 98 =$

5. $7 + 38 =$

6. $1 + 52 =$

7. $66 + 6 =$

8. $2 + 59 =$

9. $39 + 8 =$

10. $3 + 18 =$ 11. Add 6 and 19 12. Add 33 and 9
 13. Add 3 and 23 14. Add 17 and 8 15. Add 7 and 76
 16. 1 and 99 are 17. 87 and 4 are 18. 7 and 52 are
 19. 73 and 5 are 20. 9 and 47 are

SECTION 2 Adding in Columns

Single Columns

Examples:		Grouping the Digits Mentally into Tens	
Adding the Digits Singly			
Addends	(6 ...	6	9
	(1 ... 7	1	3
	(3 ... 10	3	8 ... 20
	(5 ... 15	5	3
	(2 ... 17	2	2 ... 30
	(3 ... 20	3	20 (7
	(6 ... 26	6	8 ... 40
	(4 ... 30	4	30 5 ... 45
Sum <u>30</u>		<u>30</u>	<u>45</u>

Referring back to figure 1-1, the combinations in bold print are the basic combinations of two digits whose sum is equal to ten. In figure 1-2 are the basic combinations of three digits whose sum is equal to ten.

WORKOUT EXERCISE IV

- Practise reading the answers to the combinations in figure 1-2 until they can all be recognized automatically in 35 seconds.
- Add the following columns:

1	8	7	7	1	9	3	4
	3	9	1	6	3	3	8
7	8	1	2	3	4	6	3
	1	3	2	4	4	4	4
9	4	1	4	9	2	4	2
	5	2	5	7	7	9	2
3	2	7	1	5	1	8	7
	7	3	2	4	3	1	8
8	2	5	8	4	4	2	7
2	4	2	3	1			

1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8
<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>
<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>
2	2	2	2	2	2	2	
1	2	3	4	5	6	7	
<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>	
<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	
3	3	3	3	3	3		
1	2	3	4	5	6		
<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>		
<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>		
4	4	4	4	4			
1	2	3	4	5			
<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>			
<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>			
5	5	5	5				
1	2	3	4				
<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>				
<u>XX</u>	<u>XX</u>	<u>XX</u>	<u>XX</u>				
6	6	6					
1	2	3					
<u>3</u>	<u>2</u>	<u>1</u>					
<u>XX</u>	<u>XX</u>	<u>XX</u>					
7	7						
1	2						
<u>2</u>	<u>1</u>						
<u>XX</u>	<u>XX</u>						
8							
1							
<u>1</u>							
<u>XX</u>							

Figure 1-2

36 Basic Groupings
In Addition of Three Digits

Multiple Columns

Example: Add 19,427 and 629 and 498.

Ten Thousands Column	Thousands Column	Hundreds Column	Tens Column	Units Column
(1) 1	(1) 9	(1) 4	(2) 2	7
		6	2	9
		4	9	8
	(1)	(1)	(1)	(2)
2	0	5	5	4

Procedure: Add units column: 7 plus 9 plus 8 = 24 units.

This is 2 tens and 4 units.

Place the 4 under the units column.

Carry the 2 tens to the tens column.

Repeat for each column remembering to 'add in' the figure carried from the previous column.

WORKOUT EXERCISE V

Add the following columns:

1.	2.	3.	4.	5.	6.
7,862	3,478	56,798	79,183	317,824	4,814,792
4,193	15,762	91,437	94,376	972,392	3,639,812
6,382	9,416	75,191	68,139	219,786	7,348,171
5,791	7,321	84,424	176,172	678,324	4,388,778
8,157	12,724	71,382	38,197	196,391	7,176,834
		53,792	14,396	1,624,819	9,141,728
			116,182	3,712,623	3,176,482
			526,645		7,132,841

7. According to the last census, the population of each province and territory of Canada was: Newfoundland, 415,074; Prince Edward Island, 99,285; Nova Scotia, 694,717; New Brunswick, 554,616; Quebec, 4,628,378; Ontario, 5,404,933; Manitoba, 850,040; Saskatchewan, 880,665; Alberta, 1,123,116; British Columbia, 1,398,464; Yukon, 12,190; Northwest Territories, 19,313. See chapter heading for graphical presentation. (a) What was the total population of Canada? (b) What

was the population in the four Atlantic Provinces? (c) What was the total population in the three Prairie Provinces?

8. The area in square miles of each province and territory of Canada is as follows: Newfoundland, 155,364; Prince Edward Island, 2,184; Nova Scotia, 21,068; New Brunswick, 27,985; Quebec, 594,860; Ontario, 412,582; Manitoba, 246,512; Saskatchewan, 251,700; Alberta, 255,285; British Columbia, 366,255; Yukon, 207,076; Northwest Territories, 1,304,903. (a) What is the total area of Canada? (b) What is the total area of the Atlantic Provinces? (c) What is the total area of the Prairie Provinces? (d) Present this information in graphical form, using the bar graph in the chapter heading as a guide.

9. How many calories would you consume if you ate the following dinner: cream of tomato soup, 125 calories; tossed salad with mayonnaise, 96 calories; roast chicken, 175 calories; stuffing, 125 calories; jelly, 85 calories; lima beans, 100 calories; corn-on-the-cob (one), 110 calories; french-fried potatoes, 225 calories; roll, 125 calories; pat butter, 86 calories; banana-split, 558 calories; glass of chocolate milk, 300 calories?

10. (a) Find the total area of the Great Lakes on the Canada-United States border. The area of each lake in square miles is: Lake Superior, 31,820; Lake Huron, 23,010; Lake Michigan, 22,400; Lake Erie, 9,940; Lake Ontario, 7,540; Lake St. Clair, 460. (b) Illustrate this information by means of a bar graph. (See chapter heading.)

SECTION 3 Checking Vertical Addition

Reverse-Order Method

The most effective method of checking addition is to add the columns once from the top downwards and repeat adding from the bottom upwards. Different combinations will be chosen as the column is added from the bottom to the top than will be selected when adding from the top to the bottom. Counter-balancing errors will seldom occur if this method is followed.

WORKOUT EXERCISE VI

Check your answers to Workout Exercise V by the reverse-order method.

SECTION 4 Horizontal Addition

Many problems in the business world, particularly in accounting, bookkeeping, and statistical recording, involve horizontal addition; therefore, proficiency should be cultivated in this area.

Example: Hours of overtime for the last 12 months:

132; 216; 333; 310; 296; 175; 273; 155; 198; 153; 321; 310.

Total 2,872.

Procedure:

Add the unit digits across

$$2 + 6 + 3 + 0 + 6 + 5 + 3 + 5 + 8 + 3 + 1 + 0 = \underline{42}$$

Insert 2 in the unit column and carry (4)

Add the tens digits

$$(4) + 3 + 1 + 3 + 1 + 9 + 7 + 7 + 5 + 9 + 5 + 2 + 1 = \underline{57}$$

Insert 7 in the tens column and carry (5)

Add the hundreds

$$(5) + 1 + 2 + 3 + 3 + 2 + 1 + 2 + 1 + 1 + 1 + 3 + 3 = \underline{28}$$

Insert the 8 in the hundreds column and the 2 in the thousands column.

WORKOUT EXERCISE VII

1.

TIME SHEET Week ending June 30							
	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
Adams, A	8	10	9	8	8	4	XX
Barler, J	7	8	6	9	12	3	XX
Dodds, P	8	8	9	10	9	4	XX
Fraser, L	6	0	0	12	10	4	XX
Graham, S	8	12	10	12	10	0	XX
Martin, O	9	8	7	12	9	3	XX
Paul, B	8	10	9	8	7	4	XX
Right, W	8	8	8	8	8	4	XX
Salmon, R	10	11	12	10	9	0	XX
Walker, L	8	8	9	8	7	3	XX
Totals	XX	XX	XX	XX	XX	XX	XX

Find:

- The total number of hours worked by each employee.
- The total hours worked per day.
- The total hours worked in the week.

2.

[illegible]

Find:

- The total purchases of all departments for each month.
- The total purchases of each department for the six months.
- The total purchases of all departments for the six months.
- Compare the total purchases for each month by means of a bar graph. (See chapter heading.)

3.

ROBEMWERKES DISCOUNT HOUSE CASH RECEIPTS						
DATE	RECEIVED FROM	ACCOUNTS PAID	CASH SALES			TOTAL
			READY WEAR	FURNITURE	OTHER	
19-1						
Jan.3	Cash sales		\$ 3,172	\$ 7,062	\$10,012	\$ XX
4	A. Jones	\$ 310				
	P. Osborne	76				
	T. Taylor	91				XX
5	Cash Sales		5,398	6,892	9,143	XX
6	Cash Sales		4,937	5,828	8,762	XX
7	W. Williams	154				
	O. Murphy	37				
	P. Appleton	85				XX
8	Cash Sales		4,979	6,384	8,762	XX
		\$ XX	\$ XX	\$ XX	\$ XX	\$ XX

- Find the six daily totals to be deposited in the bank.
- Total the columns and find the total cash receipts for the week.

4.

[illegible]

- Add the columns vertically to find the total monthly sales.
- Add horizontally to find the total department sales for six months.
- Add the total column to find the total of all sales and check by horizontally adding the totals of each month's sales.
- Compare the monthly sales by means of a bar graph. (See chapter heading.)

SECTION 5 Addition of Decimals

Our monetary system is a typical example of the use of the decimal system in the business world, and calculations involving the decimal system should be thoroughly mastered.

Vertical Addition of Decimals

When adding vertical columns, arrange the figures so that the decimal place always occupies the same relative position.

Example: Add \$16.65 and \$8.49 and \$11.20.

$$\begin{array}{r}
 \$16.65 \\
 8.49 \\
 \underline{11.20} \\
 \underline{\underline{\$36.34}}
 \end{array}$$

Horizontal Addition of Decimals

When adding numbers arranged in horizontal rows, keep them "in place" by inserting zeros.

Example: Add \$107 and \$4,263.64 and \$371.

$$\$107.00 + \$4,263.64 + \$1,371.00 = \$5,741.64$$

WORKOUT EXERCISE VIII

Add the following numbers horizontally; check by arranging in vertical columns and adding, first upwards, and then downwards:

- $\$138.67 + \$0.67 + \$1,408 + \$43.40 + \$567 + \$807.06$
- $\$603.05 + \$37 + \$0.60 + \$793.71 + \$815.71 + \$972.14 + \$78.19 + \$0.57$
- $\$718.19 + \$301.10 + \$718 + \$32 + \$813.31 + \$702.29 + \$19 + \$17.50$

4. $\$171.17 + \$321.18 + \$77.77 + \$302.29 + \$762.72 + \$492.92 + \$381.71$
5. $\$721.19 + \$832.16 + \$912.73 + \$892.04 + \$734.77 + \$32 + \$719 + \$1,802.21$
6. $\$321.62 + \$712.06 + \$3,102 + \$11,061 + \$1,111.11 + \$18,818.81 + \$1,716.98 + \$13,719.58$
7. Complete the following petty cash record:

PETTY CASH RECORD						
Date		General Expense	Office Expense	Store Expense	Delivery Expense	TOTAL
19 --						
Mar 1.	Taxi fare	\$ 1.50				\$ XX
2.	Window cleaning	4.50	\$ 3.00	\$ 1.50		XX
3.	Postage stamps	5.00	1.00	.50		XX
4.	Freight-out				\$ 0.75	XX
	Stationary		2.59	1.75		XX
5.	Extra help			4.00	5.00	XX
8.	Cleaning supplies	2.66	1.99	1.59		XX
9.	Stencils	1.60				XX
	Wrapping Paper			3.54	2.79	XX
10.	Telegrams	3.71				XX
11.	Pencils, ink		2.19			XX
	Freight-out				1.53	XX
12.	Entertainment	5.59				XX
Total		\$ XX	\$ XX	\$ XX	\$ XX	\$ XX

8. Using the same form as in question 7, prepare a petty cash record for the following transactions:

Mar. 15: Window cleaning, \$9.00 (same as above); Freight-out, \$0.89; Pencils, \$0.60.

Mar. 16: Extra help: in store, \$5.50; for delivery, \$3.50; Postage stamps: for office, \$2.54; for store, \$1.65.

Mar. 17: Telegram, \$1.74; Taxi, \$2.10; Entertainment, \$2.40.

Mar. 18: Stationery: for office, \$1.69; for store, \$1.19; Wrapping paper: for store, \$2.06; for delivery, \$0.79.

Mar. 19: General expenses, \$8.05; Washroom supplies (general), \$3.79; Stapling machine for office, \$5.89.

9. Referring to the record of purchases on the next page:

Find: (a) The horizontal total owing to creditors. (b) The total purchases for each department. (c) The total of all purchases. (d) Compare the total department purchases by means of a vertical bar graph. (See Chapter 4.)

HELGARD SUPPLY HOUSE RECORD OF PURCHASES					
Date	Bought From	Hardware	Electrical	Gardening	Total
19 —					
Sept 1	Wright Bros.	\$1,796.17	\$ 932.39	\$7,162.17	\$ XX
2	Campbell Cowan		1,062.76	63.98	XX
	Simpson Styles	384.76	842.19		XX
3	Hardtack Harry Ltd.	317.84	142.17	384.19	XX
	'Onest' Erb	401.18	714.32	162.91	XX
4	Farmer Tomes	17.16		89.93	XX
	Tally Cormes		17.98	15.17	XX
5	Fry & Broil Co.	173.18	419.15	73.18	XX
	Tell & Dotell	78.13	49.10	17.19	XX
Totals		\$ XX	\$ XX	\$ XX	\$ XX

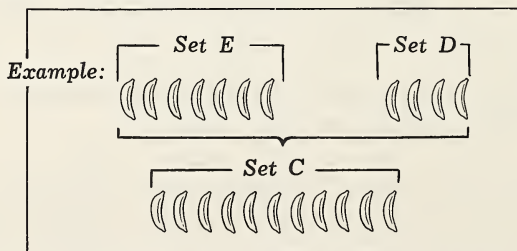
10. Using the same form as in problem 9 above, prepare a purchase record, in chronological order, for the Pilladay Drug Company departmentalized for: drugs, cosmetics, gifts, and magazines. Purchases are made from:

Homey Cosmetics: on October 15: cosmetics, \$374.15; gifts, \$87.59; on October 17: cosmetics, \$79.85; gifts, \$38.14; magazines, \$14.18; on October 20: gifts, \$17.19; cosmetics, \$33.17.

Nitenday Pharmacy: on October 14: drugs, \$1,176.18; gifts, \$318.17; cosmetics, \$17.18; on October 17: drugs, \$378.79; gifts, \$18.19; cosmetics \$318.76; on October 19: drugs, \$164.48; gifts, \$13.17; magazines \$14.60.

Alphonse Drugs: on October 15: drugs, \$329.69; gifts, \$79.81; magazines, \$8.70; on October 18 drugs \$176.79; gifts, \$38.79; cosmetics, \$17.81; on October 19: drugs, \$38.76; cosmetics, \$57.77; gifts \$18.19; on October 20: drugs, \$41.77; cosmetics, \$16.66; magazines, \$21.12.

SECTION 6 Adding with Sets



If we have two sets of the same kind of things, one with 7 members (say 7 bananas) and one with 4 members (say 4 bananas) and no

member (banana) is common to both, we can combine these two sets to form a single set with 11 bananas.

This illustrates one of the fundamental properties of addition; namely, that only those things which are alike, or for our particular purpose are considered to be alike, can be added. We cannot, for instance, add 10 feet and 6 yards together. However, if we can change the addends to numbers which represent sets of the same kind of objects, we can then add the sets.

Example: Add 10 feet and 6 yards together.

$$\begin{aligned} \text{(a) } 10 \text{ feet} + 6 \text{ yards} &= 3\frac{1}{3} \text{ yards} + 6 \text{ yards} \\ &= \underline{9\frac{1}{3} \text{ yards}} \end{aligned}$$

Change the numbering of the set containing
10 feet to a set containing $3\frac{1}{3}$ yards.

$$\begin{aligned} \text{(b) } 10 \text{ feet} + 6 \text{ yards} &= 10 \text{ feet} + 18 \text{ feet} \\ &= \underline{28 \text{ feet}} \end{aligned}$$

Change the numbering of the set containing
6 yards to a set containing 18 feet.

WORKOUT EXERCISE IX

1. Set A = 100 feet; Set B = 51 yards; Set C = Set A + Set B. Express Set C in: (a) feet, (b) yards.
2. Set S = 16 horses; Set T = 21 cows; Set V = 13 pigs. Set W = Set S + Set T + Set V. How many "animals" are there in Set W?
3. Set E = 15 dollars; Set F = 145 cents; Set G = 13 dimes; Set H = 21 nickels. If all the sets are added to give Set J, express J in terms of: (a) dollars, (b) cents, (c) nickels, (d) dimes.
4. Set M = 10 one-dollar bills; Set N = 15 two-dollar bills; Set O = 18 five-dollar bills. Express Set P, the total of Set M, N, and O, in: (a) one-dollar bills, (b) two-dollar bills, and (c) five-dollar bills.
5. Set K = 51 feet; Set L = 10 yards; Set M = 73 inches. Set N = Set K + Set L + Set M. Express Set N: (a) in feet, (b) in yards, (c) in inches.
6. List all possible numbers of the following sets. If it is not possible to list, write "impossible". If the set is empty, write ϕ . (a) The set of all odd numbers. (b) The set of all even prime numbers. (c) The set of all natural numbers greater than 5 and less than 6.
7. What do three dots indicate?
8. What does the equality of sets mean?

9. What are the objects of a set called?
10. Describe the Set $P = \{3, 6, 9, 12, 15\}$.

Subtraction

Subtraction is the process of finding what remains when one number is taken from another and is the reverse of adding two numbers.

The terms used in subtraction are:

The *minuend*, the number from which another is subtracted.

The *subtrahend*, the number which is subtracted from the minuend.

The *difference*, the number which remains after the above operation.

SECTION 7 Types of Subtraction Problems

Three basic types of problems in the business world involve the process of subtracting.

A. How much is left?

Example: We buy office supplies during the month worth \$98. During the month we use \$75 of these supplies. What is the value of the supplies left?

$$\$98 - \$75 = \$23$$

B. By how much do two figures differ?

Example: Last year in January, sales in a ladies' hat shop amounted to \$310; this year in January sales amounted to \$345. What is the difference?

$$\$345 - \$310 = \$35 \text{ more}$$

C. How much more do we need?

Example: A firm wishes to build an addition to their plant costing \$25,000. They have available funds of \$13,500. How much more money do they need to raise?

$$\$25,000 - \$13,500 = \$11,500$$

SECTION 8 Methods of Subtracting Whole Numbers**Take-Away-Borrow Method**

Example: Subtract 389 from 523.

$$\begin{array}{r} 5 \quad 2 \quad 3 \quad \text{Minuend} \\ 3 \quad 8 \quad 9 \quad \text{Subtrahend} \\ \hline \end{array}$$

We cannot subtract 9 units from 3 units, so we borrow 1 ten (or 10 units) from the 2 tens; we then have:

$$\begin{array}{r} 5 \quad 1 \quad 13 \quad \text{Minuend} \\ 3 \quad 8 \quad 9 \quad \text{Subtrahend} \\ \hline 4 \end{array}$$

We cannot subtract 8 tens from 1 ten, so we borrow 1 (hundred) from the 5 hundred; we then have:

$$\begin{array}{r} 4 \quad 11 \quad 13 \quad \text{Minuend} \\ 3 \quad 8 \quad 9 \quad \text{Subtrahend} \\ \hline 1 \quad 3 \quad 4 \quad \text{Difference} \\ \hline \hline \end{array}$$

Addition-Borrow Method

In this method we think of adding a figure to the subtrahend to get the minuend.

Example: Subtract 10,726 from 37,614.

$$\begin{array}{r} 3 \quad 7 \quad 6 \quad 1 \quad 4 \\ 1 \quad 0 \quad 7 \quad 2 \quad 6 \\ \hline \end{array}$$

6 and what give 4?

We cannot add a positive number to 6 to give us 4, so first we borrow 1 (ten) from the tens column:

$$\begin{array}{r} 3 \quad 7 \quad 6 \quad 0 \quad 14 \\ 1 \quad 0 \quad 7 \quad 2 \quad 6 \\ \hline 8 \end{array}$$

Now we think, "What number added to 6 gives us 14?" The answer is 8.

2 and what give 0?

We cannot add a positive figure to 2 to give us 0, so we borrow 1 (hundred) from the 6 hundred:

$$\begin{array}{r}
 3 \quad 7 \quad 5 \quad 10 \quad 14 \\
 1 \quad 0 \quad 7 \quad 2 \quad 6 \\
 \hline
 8 \quad 8
 \end{array}$$

Now we think, "What number added to 2 gives us 10?"

And we have the answer 8.

Proceeding as above we have:

$$\begin{array}{r}
 3 \quad 6 \quad 15 \quad 10 \quad 14 \quad \text{Minuend} \\
 1 \quad 0 \quad 7 \quad 2 \quad 6 \quad \text{Subtrahend} \\
 \hline
 2 \quad 6 \quad 8 \quad 8 \quad 8 \quad \text{Difference}
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{add to} \\ \text{check} \end{array}$$

= 37614, the original minuend.

WORKOUT EXERCISE X

Perform the following subtractions by the take-away-borrow method. Check by adding the difference and the subtrahend to find the minuend: (Difference + Subtrahend = Minuend)

$$\begin{array}{lllllll}
 1. \quad 317 & 2. \quad 596 & 3. \quad 792 & 4. \quad 1,702 & 5. \quad 17,029 & 6. \quad 59,000 & 7. \quad 1,029,010 \\
 \hline 208 & \hline 321 & \hline 569 & \hline 979 & \hline 13,927 & \hline 47,989 & \hline 999,999
 \end{array}$$

8. 8,129 from 9,018

9. 7,299 from 10,027

10. 8,927 from 9,129

11. 1,079,238 from 3,239,019

12. 7,029,011 from 8,018,009

13. 989,898 from 1,000,000

Find the difference between:

14. 7,832 and 7,082

15. 1,000,000 and 989,989

16. 7,382 and 1,989

17. 17,092 and 14,329

18. 919,000 and 721,998

19. 3,021,111 and 2,989,787

20. 28,029 is how much more than 25,971?

21. 3,019,019 is how much larger than 987,654?

22. 709,119 is how much greater than 708,929?

23. 79 is how much less than 95?

24. 3,029,000 is how much smaller than 5,000,000?

25. $7,777 - 6,669$

26. $33,333 - 9,999$

27. $5,079,219 - 2,581,324$

28. The populations of the provinces of Canada are given in Workout Exercise V, problem 7. By how much does the population of the central provinces, Ontario and Quebec, exceed the population of all the other eight provinces?

29. The areas of the ten provinces of Canada are given in Workout Exercise V, problem 8. How much less is the area of the Northwest Territories and the Yukon than the total area of the ten provinces? How much greater is the area of the Northwest Territories than the area of the Prairie provinces?

30. How much greater is the area of Lake Superior than the area of each of the other Great Lakes? (Workout Exercise V, problem 10.)

31. The Stoney Batter Pudding Mix Company purchased the necessary raw material to make their product from several suppliers. In each case the supplier allowed the company a discount off the purchase price for prompt payment. The cheques are entered in a cheque voucher register illustrated below. Calculate the actual amount of the cheques paid. Total the columns and cross-check.

THE STONEY BATTER PUDDING MIX COMPANY CHEQUE VOUCHER REGISTER					
Date	Voucher No.	Cheque No.	Purchase Price	Discount Allowed	Amount of Cheque
19 - 1					
Nov. 1	311	107	\$ 7,831.16	\$156.62	\$ XX
2	312	108	983.06	19.66	XX
3	313	109	792.96	15.85	XX
	314	110	27.59	.55	XX
6	315	111	1,723.75	34.47	XX
7	316	112	10,731.06	214.62	XX
	317	113	620.61	12.41	XX
8	318	114	377.98	7.56	XX
	319	115	1,384.29	27.69	XX
9	320	116	841.63	16.83	XX
	321	117	9,813.63	196.27	XX
10	322	118	783.59	15.67	XX
			\$ XX	\$ XX	\$ XX

32. Continue the above cheque voucher register for the cheques issued during the following week, November 13-17. Total the columns and cross-check.

Date	Purchase Price	Discount
November 13	\$ 349.68	\$ 6.99
13	1,386.74	27.74
14	734.18	14.68
14	4,790.38	95.81
15	7,801.10	156.02
16	3,498.90	69.98
16	918.76	18.38
17	49.76	.99
17	719.39	14.39
17	1,810.65	36.21

SECTION 9 Closure Property of Sets

Let N = the set of all natural positive integers:

$$N = \{1, 2, 3, 4, \dots\}$$

If we add any two positive integers, the result is also a member of the set N . We say that the set N "*is closed under the operation of adding two positive integers.*"

However, under the operation of subtraction you cannot always obtain a positive integer by subtracting one positive integer from another. The answer to $4 - 9$ can be found by asking "What number added to 9 gives 4?" This answer is not a positive integer; therefore we say that the set N "*is not closed under the operation of subtraction.*"

Example: $\{S = 1, 3, 5, 7, \dots\}$

Is this set closed under the operation of addition? of subtraction?

The sum of two odd numbers is always an even number; therefore the sum would not be a member of S ; therefore the set S is not closed under the operation of addition. The difference between any two numbers would always be an even number, but not always positive; and therefore the set S is not closed under the operation of subtraction.

WORKOUT EXERCISE XI

1. Is the set $T = \{2, 4, 6, 8, \dots\}$ (a) closed under addition? (b) closed under subtraction?
2. Is the set $S = \{1, 4, 7, 10, 13, 16, \dots\}$ (a) closed under addition? (b) closed under subtraction?
3. Write the set $P = \{\text{all prime numbers less than } 100\}$. Is this set closed under addition? under subtraction?
4. What is meant by a set being closed under addition and under subtraction? Give illustrations of: a set closed under both addition and subtraction; a set closed under addition, but not subtraction; a set not closed under either addition or subtraction.
5. $N = \{1, 2, 3, 4, 5, \dots\}$ (a) If 21 is the least of four consecutive numbers in set N , what are the other three? (b) If 1,301 is the highest of five consecutive numbers in set N , what are the other four? (c) Write a sentence to describe set N .

6. Given the set $S = \{24, 34, 1, 57, 63, 39, 18, 17, 101, 104, 51, 321, 2, 49\}$: (a) List the numbers of S which are odd. (b) List the numbers of S which are even. (c) List the numbers of S which are prime. (d) List the numbers which are multiples of 7, 13, 17.
7. Describe the sets: (a) $R = \{2, 3, 4, 5, 6, 7\}$; (b) $S = \{1, 3, 5, 7\}$; (c) $T = \{\text{Tuesday, Thursday}\}$; (d) $V = \{\text{January, June, July}\}$.
8. What does the equality of sets mean?
9. $S = \{1, 3, 5\}$. Write five equal sets.
10. List the numbers of the set of all the even numbers that are odd.

SECTION 10 Figuring for Fun

1. Arrange 23 twos so that they add up to 1,000.
2. Arrange 34 fives so that they add up to 24,000.
3. Find your two lucky numbers.

To find your first lucky number proceed as follows: Write down your birthday in figures. Suppose your birthday is March 20, 1948; write: 20.3.48 and add the digits, $2 + 0 + 3 + 4 + 8 = 17 =$

$$148 + 3 + 48 = 199 \quad 1 + 7 = \underline{\underline{8}}$$

To find your second lucky number proceed as follows: Write down your full name. Assign a number to each letter starting with $A = 1$, $B = 2$, $C = 3$, ... $Z = 26$. Suppose your name is John Charles Doeman; write:

J O H N C H A R L E S D O E M A N
10 15 8 14 3 8 1 18 12 5 19 4 15 5 13 1 14

Add the digits:

$$\text{John} = 1 + 0 + 1 + 5 + 8 + 1 + 4 = 20 = 2 + 0 = \quad 2$$

$$\text{Charles} = 3 + 8 + 1 + 1 + 8 + 1 + 2 + 5 + 1 + 9 = 39 = \quad 3 + 9 = 12 = 3$$

$$\text{Doeman} = 4 + 1 + 5 + 5 + 1 + 3 + 1 + 1 + 4 = 25 = \quad 7$$

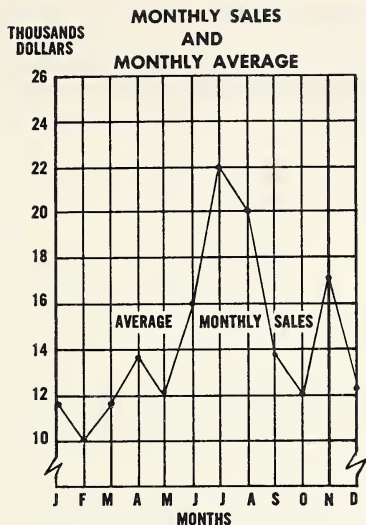
$$12 = \underline{\underline{12}} \\ 1 + 2 = \underline{\underline{3}}$$

Your two lucky numbers are therefore 8 and 3. Your lucky numbers are supposed to "vibrate" favourably for you! What is the mathematical term for these lucky numbers?

4. This is a magic square:

5	1	2	4	3
4	3	5	1	2
1	2	4	3	5
3	5	1	2	4
2	4	3	5	1

The rows across add to 15. The columns add to 15. The two diagonals add to 15. See if you can make a magic square, either three-square, four-square, or five-square, where the rows, the columns, and the diagonals all add to the same figure.



Multiplication and Division 2

A Business Problem

Mr. Thomson, principal of Redfern High, has invited Mr. I.A.M. Smart Scientist to talk to the students. The talk is to take place in the school auditorium, and Mr. Thomson wishes all students to attend. On the ground floor there are 11 rows of 27 seats, and 7 rows of 23 seats. In the balcony are 9 rows of 37 seats.

An honorarium of \$150 is to be paid to Mr. Smart, and expenses will amount to \$30.25. Assuming the hall will be filled, it must be decided how much to charge. Should all students be charged the same amount, and how much should this be? Should the 11 rows, the 7 rows, and the 9 balcony rows be charged different prices, and, if so, what should these prices be?

Multiplication

Multiplication is a method of finding the sum of two or more addends of the same value without adding. The terms used in multiplication are:

The *multiplicand*, the number that you multiply,

The *multiplier*, the number by which you multiply,

The *product*, the result of the process of multiplying,

The *factors*, the numbers that were used, or could be used, to make the product (from the Latin "facere", meaning "to make"). Both the multiplicand and the multiplier are factors.

MULTIPLICATION TABLES

X 1	X 2	X 3	X 4	X 5	X 6	X 7	X 8	X 9	X 10	X 11	X 12
<u>1</u> <u>1</u> <u>1</u>	<u>2</u> <u>1</u> <u>2</u>	<u>3</u> <u>1</u> <u>3</u>	<u>4</u> <u>1</u> <u>4</u>	<u>5</u> <u>1</u> <u>5</u>	<u>6</u> <u>1</u> <u>6</u>	<u>7</u> <u>1</u> <u>7</u>	<u>8</u> <u>1</u> <u>8</u>	<u>9</u> <u>1</u> <u>9</u>	<u>10</u> <u>1</u> <u>10</u>	<u>11</u> <u>1</u> <u>11</u>	<u>12</u> <u>1</u> <u>12</u>
	<u>2</u> <u>2</u> <u>4</u>	<u>3</u> <u>2</u> <u>6</u>	<u>4</u> <u>2</u> <u>8</u>	<u>5</u> <u>2</u> <u>10</u>	<u>6</u> <u>2</u> <u>12</u>	<u>7</u> <u>2</u> <u>14</u>	<u>8</u> <u>2</u> <u>16</u>	<u>9</u> <u>2</u> <u>18</u>	<u>10</u> <u>2</u> <u>20</u>	<u>11</u> <u>2</u> <u>22</u>	<u>12</u> <u>2</u> <u>24</u>
		<u>3</u> <u>3</u> <u>9</u>	<u>4</u> <u>3</u> <u>12</u>	<u>5</u> <u>3</u> <u>15</u>	<u>6</u> <u>3</u> <u>18</u>	<u>7</u> <u>3</u> <u>21</u>	<u>8</u> <u>3</u> <u>24</u>	<u>9</u> <u>3</u> <u>27</u>	<u>10</u> <u>3</u> <u>30</u>	<u>11</u> <u>3</u> <u>33</u>	<u>12</u> <u>3</u> <u>36</u>
			<u>4</u> <u>4</u> <u>16</u>	<u>5</u> <u>4</u> <u>20</u>	<u>6</u> <u>4</u> <u>24</u>	<u>7</u> <u>4</u> <u>28</u>	<u>8</u> <u>4</u> <u>32</u>	<u>9</u> <u>4</u> <u>36</u>	<u>10</u> <u>4</u> <u>40</u>	<u>11</u> <u>4</u> <u>44</u>	<u>12</u> <u>4</u> <u>48</u>
				<u>5</u> <u>5</u> <u>25</u>	<u>6</u> <u>5</u> <u>30</u>	<u>7</u> <u>5</u> <u>35</u>	<u>8</u> <u>5</u> <u>40</u>	<u>9</u> <u>5</u> <u>45</u>	<u>10</u> <u>5</u> <u>50</u>	<u>11</u> <u>5</u> <u>55</u>	<u>12</u> <u>5</u> <u>60</u>
					<u>6</u> <u>6</u> <u>36</u>	<u>7</u> <u>6</u> <u>42</u>	<u>8</u> <u>6</u> <u>48</u>	<u>9</u> <u>6</u> <u>54</u>	<u>10</u> <u>6</u> <u>60</u>	<u>11</u> <u>6</u> <u>66</u>	<u>12</u> <u>6</u> <u>72</u>
						<u>7</u> <u>7</u> <u>49</u>	<u>8</u> <u>7</u> <u>56</u>	<u>9</u> <u>7</u> <u>63</u>	<u>10</u> <u>7</u> <u>70</u>	<u>11</u> <u>7</u> <u>77</u>	<u>12</u> <u>7</u> <u>84</u>
							<u>8</u> <u>8</u> <u>64</u>	<u>9</u> <u>8</u> <u>72</u>	<u>10</u> <u>8</u> <u>80</u>	<u>11</u> <u>8</u> <u>88</u>	<u>12</u> <u>8</u> <u>96</u>
								<u>9</u> <u>9</u> <u>81</u>	<u>10</u> <u>9</u> <u>90</u>	<u>11</u> <u>9</u> <u>99</u>	<u>12</u> <u>9</u> <u>108</u>
									<u>10</u> <u>10</u> <u>100</u>	<u>11</u> <u>10</u> <u>110</u>	<u>12</u> <u>10</u> <u>120</u>
										<u>11</u> <u>11</u> <u>121</u>	<u>12</u> <u>11</u> <u>132</u>
											<u>12</u> <u>12</u> <u>144</u>

Figure 2-1

78 Basic Combinations
and their Products

SECTION 1 Multiplication of Whole Numbers

If we eat three meals a day each day of the week, how many meals do we consume in a week? To find an answer, we might set up the problem thus:

Monday	3 meals
Tuesday	3 meals
Wednesday	3 meals
Thursday	3 meals
Friday	3 meals
Saturday	3 meals
Sunday	3 meals
Total	<u>21 meals</u>

Or we might say we eat three meals on each of seven days and set up the problem thus:

$$3 \times 7 = 21 \text{ meals.}$$

Multiplication, therefore, is a short method of addition and constitutes a more efficient method of finding a sum when the addends are of equal value. Consider the addition that would be involved if we were asked to find the number of meals we eat in a month or in a year!

Figure 2-1 gives 78 basic combinations of the digits 1 to 12 and their products. Only one combination of any two numbers has been included; i.e., only one of 7×9 and 9×7 . Master this thoroughly so that the multiplication of any two numbers from 1 to 12 becomes automatic.

Example: Read the 8×8 multiplication table.

Find the column headed $\times 8$.

Proceed down this column to the last combination which is

$$8 \times 8 = 64.$$

Then proceed from left to right along this row.

WORKOUT EXERCISE I

1. Count by 2's to 20

by 3's to 30

by 4's to 40

by 5's to 50

by 6's to 60

by 7's to 70

2. Count by 8's to 80

by 9's to 90

by 10's to 100

by 11's to 121

by 12's to 144

2. Practise the combinations in figure 2-1:

- Across the rows from left to right,
- Across the rows from right to left,
- Down the columns from top to bottom,
- Up the columns from bottom to top.

When these combinations are mastered, use this acquired facility as an aid to multiply all types of number combinations.

To Multiply Several Digits By One Digit

The positional notation or place value principle of our numbering system facilitates multiplication.

As in addition, when multiplying, we have the problem of “carry-
ing” a figure from one column to another.

Example: What is the product of $3,279 \times 7$?

	Ten Thousands Column	Thousands Column	Hundreds Column	Tens Column	Units Column
Multiplicand		3	2	7	9
Multiplier					7
7×9				(6)	3
$+ 7 \times 70$			(4)	9	0
$+ 7 \times 200$		(1)	4	0	0
$+ 7 \times 3000$	(2)	1	0	0	0
Product	2	2	9	5	3

The figures in brackets have been carried from one column to the next highest. In practice, these are carried mentally, and the place zeros to the right are omitted, thus making it easy to write the answer in one line:

$$\begin{array}{r}
 3279 \\
 7 \\
 \hline
 22,953
 \end{array}$$

To Multiply Several Digits By Several Digits

Example: What is the product of 853×794 ?

	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	
Multiplicand				8	5	3	
Multiplier				7	9	4	
4×3					(1)	2	
$+ 4 \times 50$				(2)	0	0	
$+ 4 \times 800$			(3)	2	0	0	
$+ 90 \times 3$				(2)	7	0	
$+ 90 \times 50$			(4)	5	0	0	
$+ 90 \times 800$		(7)	2	0	0	0	
$+ 700 \times 3$			(2)	1	0	0	
$+ 700 \times 50$		(3)	5	0	0	0	
$+ 700 \times 800$	(5)	6	0	0	0	0	
Product: $853 \times 794 =$	6	7	7	2	8	2	
Again, in practice, the figures in brackets are carried mentally and place zeros omitted, making it possible to obtain the answer in four lines; thus: $\begin{array}{r} 853 \\ 794 \\ \hline 3,412 \\ 76,77 \\ 597,1 \\ \hline 677,282 \end{array}$ 3,412 First Partial Product 76,77 Second Partial Product 597,1 Third Partial Product <u>677,282</u> Final Product							

WORKOUT EXERCISE II

Find the product of:

1. 137×7
2. 669×9
3. $8,793 \times 731$
4. $7,717 \times 373$
5. Multiply 1,798 by 5,136.
6. Multiply $7,982 \times 9,281$.
7. Find the product of 3,798 and 87.
8. $7,693 \times 1,592 \times 8,329$.
9. $6,685 \times 3,712 \times 8,973$.
10. Multiply 813 by 167 by 93.
11. Find the product of 7,963, 7,639 and 17.
12. Find the product of 7,982, 315 and 222.
13. Multiply 7,645 by 8,391 by 42,659.
14. Multiply 3,978,649 by 9,874,567.
15. Practice on figure 2-1.

SECTION 2 Multiplication of Decimal Fractions

Decimal fractions are multiplied in exactly the same way as are whole numbers. However, we have the problem of placing the decimal point in the product. The following rule, which should be memorized, shows us how to determine the position of the decimal point.

The number of decimal places in the product of two numbers is equal to the number of decimal places in the multiplicand, plus the number of decimal places in the multiplier.

Example: Find the product of 7.1×0.23 .

Ignore the decimal point and multiply.

71	7.1 has 1 decimal place.
23	0.23 has 2 decimal places.
213	
142	
1633	Product has 3 decimal places.

Figure 2-2

SALES SLIP			
N. O. GOOD CLOTHING			
463 TURNBERRY STREET, TRENDON			
Date, <u>Jan. 31</u>		19 <u>63</u>	
Name <u>Mrs. Spendit</u>			
Address <u>14 Spicer Lane, Trendon</u>			
QTY.	ITEM		
14	pairs of hosiery @ 1.65 pr.	23	10
3	blouses @ 4.98	14	94
2	shirts @ 11.99	23	98
		62	02
176		SOLD BY <u>N. Good</u>	

A familiar illustration of the multiplication of decimals in business practice is the sales slip. (Figure 2-2.)

When we multiply the price of an item by the quantity to find the amount, we are finding the *extension*.

The symbol @ stands for 'at'.

WORKOUT EXERCISE III

- Multiply: (a) 3.682 by 63.91 (b) 845.13 by 732.62
(c) 21,349.1 by 873.2 by 121.8 (d) 673.671 by 143.9 by 7,193.98
- An invoice is a bill sent to a customer for goods purchased. Complete the extensions and total the following invoice:

INVOICE		BIMEBY DRUGS LTD. BIMBOTON	
SOLD TO: Mr. Sonny Patch Druggist Bimboton		SHIP TO: Same	
YOUR ORDER NO. 63		VIA Truck	
TERMS NET 30 DAYS	DATE July 6, 1963	ACCOUNT NUMBER A. 4393	F.O.B. TORONTO
		INVOICE NUMBER 43213	
QUANTITY	ITEM	LIST PRICE	AMOUNT
6 doz.	bottles of shampoo	.69	
36	tubes of toothpaste	1.23	
27	tins of baby powder	.77	
45	tubes of deodorant	.58	
			\$

- Make out the following invoices dated on the last business day of last month to be sent out by the Bimeby Drugs Ltd.

(a) To Samuel Stone, Bimboton, for 4 picnic bags @ \$1.88 each; 6 picnic jugs @ \$5.95 each; 6 dozen picnic-table covers @ 27¢ each; 5 cameras @ \$14.95; 4 dozen bottles of Tan-in-the-Sun cream @ \$1.79 a bottle; 27 pairs of sun-glasses @ \$1.53 a pair.

(b) To Karl Kramer, Springbow, for 9 bags of epsom salts @ 33¢ a bag; 18 bottles of saccharin tablets @ 69¢ a bottle; 27 toothbrushes @ 49¢ each; 35 toothbrushes @ 99¢ each; 10 dozen cakes of soap @ 11¢ a cake.

(c) To Terry Turner, Snowdin, for 6 bottles of oriental bubble-bath oil @ 89¢ a bottle; 7 sets of barbecue utensils @ \$1.49 a set; 11 bottles of Gloriana shampoo @ \$1.17 a bottle; 13 magazine racks @ \$2.47 each; 8 record racks @ \$1.78 each.

SECTION 3 The Zero Digit in Multiplication

Zero as the Multiplicand or the Multiplier

Examples: $3 \times 0 = 0$ $17 \times 216 \times 0 = 0$
 $0 \times 233 = 0$ $6.325 \times 0 = 0$

Zero Between the First and Last Digits of the Multiplicand

Example: Find the product of $4,008 \times 26$.

4,008	Multiplicand
26	Multiplier
24,048	First Partial Product
80,16	Second Partial Product
<u>104,208</u>	Final Product

Zero Between the First and Last Digit of the Multiplier

Example: Find the product of 534×403 .

534	Multiplicand
403	Multiplier
1,602	First Partial Product
000	Second Partial Product
213,6	Third Partial Product
<u>215,202</u>	Final Product

As the second partial product consists of zeros, the final product is unaffected by its inclusion or omission. We can therefore shorten our problem by omitting the second partial product and exercising care to give the partial products their correct value as follows:

534
403
1,602
213,6
<u>215,202</u>

Zero at the End of the Multiplicand and/or the Multiplier

Multiply the two factors ignoring the zeros to the right of the last figure in both the multiplier and the multiplicand. When the product of these two numbers is found, attach after the right-hand digit as

many zeros as were ignored in both the multiplicand and the multiplier.

Example: Find the product of 17200×620 .

$$\begin{array}{r}
 172 \text{ Drop 2 zeros.} \\
 \underline{62 \text{ Drop 1 zero.}} \\
 344 \\
 1032 \\
 \underline{10664 \text{ Attach 3 zeros.}} \\
 \text{Final product is } 10,664,000.
 \end{array}$$

Zero or Zeros Following the Digit One (i.e., 10 or a multiple of 10)

(i) A Whole Number

Examples: Find the product of 632×100 .

$$\begin{array}{r}
 632 \\
 \underline{1 \text{ Drop 2 zeros.}} \\
 632 \text{ Attach 2 zeros.} \\
 = 63,200
 \end{array}$$

This indicates that we could have added the two zeros of the multiplier 100 to the multiplicand to obtain the product.

Find the product of $1,643 \times 1,000$.

$$\begin{array}{r}
 1643 \\
 \underline{1 \text{ Drop 3 zeros.}} \\
 1643 \text{ Attach 3 zeros.} \\
 = 1,643,000
 \end{array}$$

(ii) A Decimal Fraction

Example: Find the product of $987.321 \times 1,000$.

$$\begin{array}{r}
 987321 \quad \text{Ignore three decimal places in multiplicand.} \\
 \underline{1 \quad \text{Drop 3 zeros in multiplier.}} \\
 987321 \\
 987321000 \quad \text{Attach 3 zeros in product.} \\
 987321.000 \quad \text{Place the decimal point so that the number} \\
 \quad \quad \quad \text{of places is equal to the number of places in} \\
 \quad \quad \quad \text{the multiplicand} = 3.
 \end{array}$$

Therefore $987.321 \times 1,000 = 987,321$.

The three zeros in the multiplier have the effect of moving the decimal place in the multiplicand three places to the right; or as many places to the right as there are zeros in the multiplier.

Examples: $11.23 \times 10 = 112.3$
 $1.643 \times 100 = 164.3$
 $.0932 \times 10,000 = 932$

WORKOUT EXERCISE IV

1. Practise the combinations of figure 2-1.
2. Multiply: (a) 1.682×310 (b) $693.04 \times 27,005$
(c) $31,006.006 \times 698.2$ (d) $760.60 \times 371.23 \times 51.65$
3. Find the product of: (a) 0.376 and 60.207 and 3,708.2
(b) 3,798.4 and 709,601 and 11.09 (c) 1,372.4 and 819.19 and 3,475.69
4. Many business firms record the stock they have on hand at the end of the month on inventory sheets. Extend and total the following inventory sheet.

Inventory Sheet			
Date: Aug. 20		Sheet No. 11	
Priced by: Abe Cede		Extended by L. S.	
Description	Quantity	Unit Price	Extended Total
1/4" valves	105	\$.27	\$ XX
1/2" valves	372	.41	XX
7/8" valves	91	.49	XX
Casters	107 sets	.58	XX
Insulation	620 lbs.	.11	XX
Insulated wire	417 ft.	.03	XX
			\$ XX

5. Make out inventory sheets dated the last day of the month for:
(a) 30 Radios @ \$33.50; 14 Television sets @ \$125; 13 Television sets @ \$149.99; 15 Clocks @ \$7.15; 12 Clocks @ \$8.50; 24 Toasters

@ \$13.13; 43 Irons @ \$7.50; 9 Ironers @ \$94.95.

(b) 31 Electric blankets @ \$12.98; 6 Air-conditioners @ \$179.89; 13 Electric stoves @ \$159.78; 69 Home freezers @ \$167.69; 63 Food mixers @ \$27.18; 27 Refrigerators @ \$137.74; 19 Electric door-chimes @ \$13.75; 29 Floor lamps @ \$32.15; 17 Table lamps @ \$11.19.

SECTION 4 Checking Multiplication

A straightforward method of checking your multiplication is by interchanging multiplicand and multiplier.

Example: Find the product of 721×302 and check the answer.

Multiplicand	721	↔	302	Multiplicand
Multiplier	302	↔	721	Multiplier
	1,442		302	
	<u>216,3</u>		6,04	
	<u>217,742</u>		<u>211,4</u>	
			<u>217,742</u>	

This example also illustrates the fact that if there is a zero digit in either of the factors it is more expedient to use the number containing that digit as the multiplier.

WORKOUT EXERCISE V

Prove the answers to the practice exercises in Sections 1, 2, and 3 by interchanging the multiplicand and the multiplier.

Division

Division is a method of finding the remainder of two or more subtrahends of the same value without subtracting. It can also be thought of as the inverse of multiplication. The terms used in division are:

The *dividend*, the number being divided.

The *divisor*, the number by which we divide.

The *quotient*, the result of the process of division.

The *remainder*, the number that is left if the division is not exact.

to become proficient you should practise carrying the remainder in your head.

To Divide Several Digits By Several Digits (Long Division)

Example: Divide 95,278 by 623.

Divisor	623	152 Quotient 95278 Dividend
		623
		3297
		3115
		1828
		1246
		582 Remainder

Procedure: Move along the dividend from left to right until a higher number than the divisor, 623, is obtained; this is the first partial dividend. The largest number of times 623 is contained in this partial dividend of 925 (hundreds) is 1 (hundred). 1 is therefore the first digit of the quotient and is placed in the hundreds column or above the last digit of the partial product. Subtract 623 (hundreds) from 952 (hundreds) to obtain the remainder 329 (hundreds). Bring down the 7 (tens) and proceed with the division.

Example: Divide 329,982 by 47.

Divisor	47	7020 Quotient 329981 Dividend
		329
		98
		94
		41 Remainder

The dividend is not always exactly divisible by the divisor, and after the process of division has taken place, there will be a remainder. In other words, 47 is contained in 329,981: 7,020 times with 41 remaining.

Note carefully the procedure followed in the last example when there is no remainder from a partial product. Care must also be taken

duct (i.e., 82). Fill any columns between the decimal point and the first figure with place-holding zeros. Proceed as for whole numbers.

B. The divisor is a decimal fraction.

Consider the problem, $3.84 \div 2.4$.

If we could eliminate the decimal point from the divisor, we could proceed as in (A). This can be done if we multiply the divisor by 10. If we also multiply the dividend by 10, we shall not have changed the value of the quotient. For instance:

$$\begin{array}{r} 3 \overline{) 9} \\ 30 \overline{) 90} \\ 300 \overline{) 900} \\ 3,000 \overline{) 9,000} \end{array}$$

In each instance the quotient has remained the same although the original divisor and dividend have each been multiplied in turn by 10, 100, and 1,000.

Example: Divide 3.84 by 2.4.

$$\begin{array}{r} 1.6 \\ 2.4 \overline{) 3.84} \\ \quad \wedge \quad \wedge \\ \quad 24 \\ \quad \underline{24} \\ \quad 144 \\ \quad \underline{144} \\ \quad 0 \end{array}$$

Procedure: Move the decimal points in both the divisor and the dividend as many places to the right as is necessary to make the divisor a whole number. Mark the new position of the decimal point with the sign $\wedge$, called a *caret*. Place the decimal point in the quotient immediately above the caret and proceed as in (A).

Example: Divide 624 by 0.39.

$$\begin{array}{r} 1600. \\ 0.39 \overline{) 624.00} \\ \quad \wedge \quad \wedge \\ \quad 39 \\ \quad \underline{39} \\ \quad 234 \\ \quad \underline{234} \\ \quad 0 \end{array}$$

Note particularly the two zeros added to the dividend and the two zeros necessary as place-holders in the quotient.

To Place the Decimal Point in the Remainder

Example: Divide 0.7471 by 0.47.

$$\begin{array}{r}
 \text{Divisor } 0.47 \overline{) 0.7471} \\
 \underline{47} \\
 277 \\
 \underline{235} \\
 421 \\
 \underline{376} \\
 .045 \text{ Remainder}
 \end{array}$$

The decimal point in the remainder is placed beneath the decimal point in the *dividend*, and any vacant columns are filled with place-holding zeros. In the last step of this problem we have subtracted 376 (ten thousandths) from 421 (ten thousandths) leaving a remainder of 45 (ten thousandths) or 0.0045.

WORKOUT EXERCISE VIII

The Cut & Cumagen Nursery advertise in their Spring Catalogue the following prices for unit purchases and for quantity purchases. Determine the saving per unit if these items are purchased in quantity. Answer to the nearest cent (for rounding off numbers, see Appendix A).

Description	Price Per Unit Purchase		Price Per Quantity Purchase		Cost Per Unit At Quantity Price	Saving Per Unit
	Unit	Price	# Units	Price		
1. Begonia, seeds	Pkt.	\$.50	3	\$1.35	\$ xx	\$ xx
2. Petunia, seeds	100	.50	500	2.00	xx	xx
3. Coleus, seeds	150	.50	450	1.02	xx	xx
4. Pansies, seeds	75	.25	375	1.00	xx	xx
5. Gaillardia, seeds	Pkt.	.25	3	.66	xx	xx
6. Sweet peas, seeds	30	.50	180	2.50	xx	xx
7. Zinnias, seeds	Pkt.	.40	3	1.05	xx	xx
8. Miniature roses, plant	1	1.25	10	9.75	xx	xx
9. Gloxinia, plant	1	2.00	3	5.10	xx	xx
10. Shasta daisies, plant	2	.95	26	8.75	xx	xx
11. Pyrethrum, plant	2	.95	12	4.50	xx	xx
12. Gladiolus, bulb	1	.50	25	8.00	xx	xx
13. Dahlia, tuber	3	1.00	12	2.96	xx	xx
14. Orchids	1	.85	3	2.25	xx	xx
15. Begonia, plant	1	.50	12	5.04	xx	xx

16. The space ship Luna III leaves the space ship station on the moon on January 1 and travels, at a speed of 24,000 miles an hour, to the space station circling the planet Venus. Venus at this time of the year is approximately 50,000,000 miles from the moon. On what day does Luna III arrive at the Venus space station?

SECTION 7 Checking Division

Check your division by the inverse process of multiplication.

Example: When there is no remainder. Divide 31,261 by 43 and check the answer.

		$ \begin{array}{r} 727 \text{ Quotient} \\ \text{Divisor } 43 \overline{) 31261} \text{ Dividend} \\ \underline{301} \\ 116 \\ \underline{86} \\ 301 \\ \underline{301} \end{array} $		$ \begin{array}{r} 727 \text{ Quotient} \\ \times 43 \text{ Divisor} \\ \hline 2181 \\ 2908 \\ \hline 31261 \text{ Dividend} \end{array} $
--	--	---	--	--

$$\text{Divisor} \times \text{Quotient} = \text{Dividend}$$

Example: When there is a remainder. Divide 67,281 by 79 and check the answer.

		$ \begin{array}{r} 851 \text{ Quotient} \\ \text{Divisor } 79 \overline{) 67281} \text{ Dividend} \\ \underline{632} \\ 408 \\ \underline{395} \\ 131 \\ \underline{79} \\ 52 \text{ Remainder} \end{array} $		$ \begin{array}{r} 851 \text{ Quotient} \\ 79 \text{ Divisor} \\ \hline 7659 \\ 5957 \\ \hline 67229 \\ 52 \text{ Add} \\ \text{remainder} \\ \hline 67281 \text{ Dividend} \end{array} $
--	--	--	--	---

$$\text{Divisor} \times \text{Quotient} + \text{Remainder} = \text{Dividend}$$

WORKOUT EXERCISE IX

Check the answers to Workout Exercise VII.

SECTION 8 Factors and Factoring**Factors**

At the beginning of the chapter, 'factors' were listed as the numbers that were used or could be used to make the product. The multiplicand and the multiplier are two special 'factors' of the product, but it is possible for the product to have other factors.

$$12 \times 5 = 60$$

The multiplicand (12) and the multiplier (5) are two factors of 60.

But there are several other sets of 'makers' or factors for 60, e.g.,

$$2 \times 30$$

$$4 \times 15$$

$$6 \times 10$$

$$3 \times 20$$

With this knowledge we can take 60 apart and find its various factors, or the numbers by which it can be divided, e.g., 2,3,4,5,6,10, 15,20,30, and of course by 1 and by itself, 60. So 60 has 11 factors.

Prime Factors

If a whole number can be divided only by itself and 1, it is called a *prime number*. The prime numbers between 1 and 30 are:

1,2,3,5,7,11,13,17,19,23, and 29.

If a whole number has at least one other set of factors in addition to itself and 1, it is called a *composite number*.

Factoring

In multiplication we put factors together to make other numbers, and in division we take the numbers apart to find a factor. So, if we are looking for all factors of a number, we use division and keep dividing by prime factors until we have a remainder which is a prime number and cannot be divided.

Examples: Find the prime factors of 380.

$$2 \overline{) 380}$$

$$2 \overline{) 190}$$

$$5 \overline{) 95}$$

$$19$$

The prime factors of 380 are $2 \times 2 \times 5 \times 19 = 380$.

Find the prime factors of 636.

$$2 \overline{) 636}$$

$$2 \overline{) 318}$$

$$3 \overline{) 159}$$

$$53$$

The prime factors of 636 are $2 \times 2 \times 3 \times 53 = 636$

WORKOUT EXERCISE X

1. Find all the prime numbers between 30 and 200.

Find the prime factors of:

2. 712, 3. 32,691, 4. 12,346, 5. 116, 6. 378,611,
7. 38,216, 8. 782,317 9. 153,972, 10. 1,001,762.

Highest Common Factor (H.C.F.)

To find the highest factor that is common to two or more numbers, list the prime factors of each number, choose the ones common to both numbers, and find the product.

Example: Find the H.C.F. of 84, 2,310 and 6,006.

$$\begin{aligned} 84 &= 2 \times 2 \times 3 \times 7 \\ 2,310 &= 2 \times 3 \times 5 \times 7 \times 11 \\ 6,006 &= 2 \times 3 \times 7 \times 11 \times 13 \\ \text{H.C.F.} &= 2 \times 3 \times 7 = 42 \end{aligned}$$

Lowest Common Multiple (L.C.M.)

A multiple is a number which contains another number some number of times without a remainder; for example, 6 is a multiple of 2 and of 3. 6, 9, 12, and 15 are all multiples of 3.

The L.C.M. of two or more numbers is the lowest number that will be a multiple of the two or more numbers. To find the L.C.M., list the prime factors of each number, then list every factor that is contained in each of the numbers, omitting any common factors that are already listed, and find the product.

Example: Find the L.C.M. of 84, 2310 and 6,006.

$$\begin{aligned} 84 &= 2 \times 2 \times 3 \times 7 \\ 2,310 &= 2 \times 3 \times 5 \times 7 \times 11 \\ 6,006 &= 2 \times 3 \times 7 \times 11 \times 13 \\ \text{L.C.M.} &= 2 \times 2 \times 3 \times 7 \times 5 \times 11 \times 13 \\ &= \underline{60,060} \end{aligned}$$

Alternative solution by continuous division.

$$\begin{array}{r} 2 \) \ 84, \ 2,310, \ 6,006 \\ 3 \) \ 42, \ 1,155, \ 3,003 \\ 7 \) \ 14, \ 385, \ 1,001 \\ 11 \) \ 2, \ 55, \ 143 \\ \quad 2, \quad 5, \quad 13 \end{array}$$

$$\begin{aligned} \text{L.C.M.} &= 2 \times 3 \times 7 \times 11 \times 2 \times 5 \times 13 \\ &= \underline{60,060} \end{aligned}$$

Procedure for Continuous Division:

Write the denominators in a row and divide by the *lowest* number which will divide into two or more of the numbers. The other numbers are brought down intact. Continue until no two numbers have any divisor, other than 1, in common. The factors of the L.C.M. are all the divisors and the numbers in the last row.

WORKOUT EXERCISE XI

Find the highest common factor of:

- | | | |
|---------------------------------|----------------------------------|-----------------------|
| 1. 18, 76, 94. | 2. 24, 36, 56. | 3. 72, 84, 44. |
| 4. 121, 33, 76, 82. | 5. 64, 92, 128, 184. | 6. 84, 108, 156, 192. |
| 7. 42, 378, 462, 882. | 8. 1,326, 3,315, 7,293, 5,967. | |
| 9. 392, 448, 728, 1,176, 1,456. | 10. 144, 738, 792, 1,944, 5,112. | |

Find, by inspection, the lowest common multiple of:

- | | | |
|---------------|----------------|----------------|
| 11. 4, 6, 10. | 12. 5, 6, 9. | 13. 3, 15, 18. |
| 14. 2, 7, 11. | 15. 4, 8, 16. | 16. 3, 9, 15. |
| 17. 6, 9, 10. | 18. 8, 10, 14. | |

Find, by continuous division, the lowest common multiple of:

- | | |
|--------------------------|-------------------------------|
| 19. 14, 18, 22, 26. | 20. 36, 50, 64, 76, 96. |
| 21. 38, 49, 63, 75, 96. | 22. 26, 42, 51, 68. |
| 23. 18, 45, 54, 72, 81. | 24. 33, 48, 55, 120, 264. |
| 25. 48, 84, 108, 180. | 26. 49, 52, 91, 130. |
| 27. 33, 66, 88, 99, 132. | 28. 13, 39, 66, 88, 96. |
| 29. 18, 21, 51, 63, 9. | 30. 19, 34, 51, 76, 152, 323. |

SECTION 9 Estimating Answers

Cultivate the habit of estimating the answer to a problem before actually working it out. In this way, careless errors and unreasonable answers can often be avoided — especially where decimals are involved.

Example: You are planning a motor trip and estimate your gasoline consumption will be 118 gallons and that you will pay 39.9 cents a gallon. How much money will you need for gasoline on your trip?

First, round off the numbers to obtain an approximate answer.

You estimate the money needed by finding how much 120 gallons at 40 cents would cost.

$$120 \times 40 = 4,800 \text{ cents or } \$48.00.$$

By actual multiplication $118 \times 39.9 = \$47.082 = \$47.08$.
 By estimating the answer in advance we avoid giving an unreasonable answer such as \$470.82 or \$4.71.

Example: Divide 8,983.8 by 31.

Estimate the answer by rounding off: $9,000 \div 30$, which is approximately 300.

By actual division we get 289.8.

WORKOUT EXERCISE XII

Estimate the answers to Workout Exercises II, III, IV, VII, in this chapter.

SECTION 10 Averages

If you have had several examinations or tests up to this point in your course, you may be interested to know how you stand. You can determine this by adding up all your marks and then dividing this total mark by the number of tests or examinations you have had. This single figure will represent your standing in the whole course, and is called the *average* of the marks obtained.

Simple Average

Example: Mary Meyers obtained the following grades in her Christmas examinations: Mathematics 79%, English 64%, Social Studies 73%, Bookkeeping 76%, Business Law 61%, Penmanship 85%, Stenography 68%. What was her average mark for the seven examinations?

Total marks for examinations:

$$79 + 64 + 73 + 76 + 61 + 85 + 68 = 506$$

$$\text{Average mark} = \frac{\text{Total marks}}{\text{Number of examinations}}$$

$$= \frac{506}{7}$$

$$= 72.29$$

$\therefore$ Mary's average mark was 72.29.

Example: During the week Sam Snow purchased gasoline at three different locations. At the first station he paid 37¢ a gallon, at the second he paid 41¢ a gallon, and at the third he paid 38¢ a gallon. What was the average price he paid assuming he bought the same amount of gas at all three stations?

Cost per gallon	1st	37¢
	2nd	41
	3rd	38
	Total for 3 gallons	<u>116¢</u>

$$\therefore \text{Average cost per gallon } \frac{116}{3} = 38.67¢$$

An *average* is therefore the quotient when a series of numbers is added together and their sum divided by the number of items in the series. It should be noted that the average does not necessarily have the value of any particular item. The more technical name for the average is the *arithmetical mean*.

Weighted Average

If, in the above example, Sam Snow had purchased 5 gallons of gas at 37¢ a gallon; 6 gallons at 38¢ a gallon; and 10 gallons at 41¢ a gallon, we could not say he had paid an average of 38.67¢ a gallon. 38.67¢ is only the average price if he buys one gallon, or an equal number of gallons, from each station.

If the gallonage purchased differs at each price, then we must *weight* each price according to its importance, or to the number of times it occurs, or in this case, the number of gallons bought at each price.

Example: On a recent trip by car you purchased 5 gallons of gasoline at 37¢ a gallon, 6 gallons at 38¢ a gallon, and 10 gallons at 41¢ a gallon. What was the average price you paid for a gallon of gasoline?

Number of Gallons	Price per Gallon	Total Cost
5	37¢	\$ 1.85
6	38¢	2.28
10	41¢	4.10
21	—	\$ 8.23

You therefore purchased 21 gallons of gasoline for a total cost of \$8.23.

$$\therefore \text{The average price paid per gallon} = \frac{8.23}{21} = 39.19\phi.$$

In the first example we found the average price of gasoline relative to the three stations to be 38.67¢ a gallon. In the last example we found the average price an individual paid to be 39.19¢ a gallon relative to the number of gallons he purchased.

Averages can therefore give us very useful information, but they can sometimes be misleading. For instance, if a store has sales of \$300,000 in December and \$12,000 in each of the other eleven months, the average monthly sales would be:

$$\frac{\$432,000}{12} = \$36,000 \text{ per month}$$

which is not a very representative figure.

Consider too Mrs. Murphy's two sons; John is knock-kneed; Tom is bow-legged.

Figure 2-3



Could Mrs. Murphy claim that 'on the average' her sons have the straightest legs in the country?

WORKOUT EXERCISE XIII

1. In a Grade X class of 40 students, 1 student was 12 years old, 2 students were 13 years old, 10 students were 14 years old, 15 students were 15 years old, 7 students were 16 years old, and 5 students were 17 years old. What was the average age of the students in this Grade X class?
2. 15 students received the following grades on an examination in arithmetic: 95, 93, 92, 90, 86, 86, 83, 80, 75, 75, 75, 74, 72, 64, 60. What was the average grade?
3. Electricity is paid for by the number of kilowatt-hours the consumer uses. In one month the following 80 consumers used electricity as follows:

3 consumers used	15 kilowatt-hours
7 " "	32 " "
13 " "	56 " "
23 " "	71 " "
15 " "	90 " "
6 " "	110 " "
8 " "	125 " "
5 " "	136 " "

- Find the total kilowatt-hours used.
- Find the average kilowatt-hours per consumer.
- If a kilowatt-hour costs 18¢, find the average cost per consumer.
- Find the amount of the largest individual bill to be paid and the smallest individual bill to be paid.

4. Last winter Smiley Swatch drove down to Florida for a holiday. He took four days to drive south driving 403 miles, 280 miles, 310 miles, and 106 miles on consecutive days. On the homeward journey he took the same route, and he drove back in three days, driving 379 miles on the first day and 321 miles on the second day. How far did he have to drive to reach home on the third day? What was the average daily mileage going south? Coming home? The round trip? What was the difference in the daily average going south and coming home?

5.	Sales	For month ended	Sales	For month ended
	\$10,376	January 31	18,921	June 30
	9,716	February 28	20,174	July 31
	11,028	March 31	16,123	August 31
	12,712	April 30	11,269	September 30
	15,976	May 31		

From the above summary of sales:

- Find the average monthly sales, to the nearest dollar, for the six months ended June 30, July 31, August 31, September 30, and for the nine months ended September 30.

- Draw a line graph showing monthly sales and the monthly average (see chapter heading).

6. The number of farms and the total area of farmland in each Canadian Province is given in the table below.

- Fill in the last column giving the average acreage per farm. Give the answer to the nearest acre (for rounding off numbers see Appendix A).

- Draw a vertical bar graph showing the average acreage per farm in each province.

Province	Number Of Farms	Area In Acres	Average Acres Per Farm
Newfoundland	3, 387	71, 814	xx
P. E. I.	9, 432	1, 065, 463	xx
Nova Scotia	21, 075	2, 775, 642	xx
New Brunswick	22, 116	2, 981, 449	xx
Quebec	122, 617	15, 910, 128	xx
Ontario	140, 602	19, 879, 646	xx
Manitoba	49, 201	17, 931, 817	xx
Saskatchewan	103, 391	62, 793, 979	xx
Alberta	79, 424	45, 970, 395	xx
British Columbia	24, 748	4, 538, 881	xx
Territories	22	4, 477	xx
Canada	576, 015	173, 923, 691	xx

7. A table of distances between important cities is given below. The times taken to travel these distances by train and by aeroplane are given. Find the average miles per hour, by train, and by aeroplane, for each trip.

Trip	Miles	Hours By Train	Average Miles Per Hr. By Train	Hours By Aeroplane	Average Miles Per Hr. By Aeroplane
Montreal - Toronto	335	6. 15	xx	1. 35	xx
Montreal - Winnipeg	1, 358	31. 05	xx	5. 45	xx
Montreal - Vancouver	2, 881	70. 50	xx	10. 20	xx
Toronto - Winnipeg	1, 217	28. 50	xx	3. 50	xx
Toronto - Vancouver	2, 704	67. 45	xx	8. 10	xx
Halifax - Montreal	840	21. 10	xx	3. 05	xx
Winnipeg - Calgary	832	13. 30	xx	3. 35	xx
Toronto - New York	538	12. 30	xx	1. 40	xx
Montreal - New York	384	9. 15	xx	1. 35	xx
Saint John - Montreal	482	13. 00	xx	2. 05	xx
St. John's - Montreal	1, 553	66. 45	xx	6. 50	xx

Give answers to two decimal places. Do not forget to estimate your answers. What reasons can you give for the difference in the average miles per hour for the various trips?

8. A space rocket is fired into orbit and attains a speed of 5,000 miles an hour during the first 25 miles. It then loses its rear section and attains 14,000 miles an hour. At about 40 miles out from earth the second section is released and the nose is projected at 18,000 miles per hour. How many miles per minute is the space ship travelling at each phase?

9. Receipts for motion picture theatres in five Provinces amounted in one year to: New Brunswick, \$1,827,544; Quebec, \$19,069,521; Ontario, \$32,436,912; Saskatchewan, \$4,089,415; Alberta, \$7,392,084.

If the population of these provinces was: 554,616; 4,628,378; 5,404,933; 880,665; 1,123,116 respectively, what was the average amount, to the nearest dollar, spent by each person (per capita) in the above mentioned provinces in one year? Draw a horizontal bar graph to illustrate your answer.

10. The following table shows the shirts manufactured by Glamur-tab Limited in one week:

6	men	each	completed	24	shirts
8	"	"	"	31	"
13	"	"	"	38	"
21	"	"	"	45	"
23	"	"	"	49	"
19	"	"	"	56	"
14	"	"	"	59	"
11	"	"	"	63	"
2	"	"	"	67	"

(a) What is the average number of shirts finished per man, per week?

(b) If each man is paid at the rate of 95¢ per shirt, what is the amount of the weekly payroll? What is the average pay, per man? What is the highest pay received? What is the lowest pay received?

(c) If the men are all paid \$40 a week, and then given a bonus of 35¢ per shirt for every shirt they complete above a minimum of 50 shirts, how much will the weekly payroll amount to? By how much does it differ from (b)? What is the average pay per man? What is the highest pay received?

SECTION 11 Sets Again

Factor Sets

In Section 8 we found the factors of 60 to be 1, 2, 3, 4, 5, 6, 10, 15, 20, and 30. If A is the factor set of 60,

then $A = \{1, 2, 3, 4, 5, 6, 15, 20, 30\}$.

The Prime Set: If P is the set of all the prime factors,

then $P = \{2, 3, 5, 7, 11 \dots\}$.

Closure

This is the property of a set which, when a mathematical operation with members of the set is performed, results in a number which is also contained in that set. The set of natural numbers is closed under addition and multiplication. In division, we sometimes find the answer among the set of natural numbers, but not at other times, e.g., $41 \div 5$. Therefore, the set of natural numbers is not closed under division.

Multiplication of Sets

It is possible, and often desirable, to find and combine the total number of objects in several sets where each set contains the same number and the same type of object. This is done by a process of multiplication. For example, if there are nine sets, each containing eleven cows, the sets could be combined as one set:

$$C = \{99 \text{ cows}\}$$

WORKOUT EXERCISE XIV

Construct the prime factor set of:

- | | | | | |
|-------|--------|--------|--------|-----------|
| 1. 16 | 2. 18 | 3. 36 | 4. 49 | 5. 64 |
| 6. 84 | 7. 112 | 8. 124 | 9. 156 | 10. 1,077 |

Construct the set of:

11. Prime numbers between 50 and 75.
12. Prime numbers between 75 and 100.
13. Prime numbers below 50.
14. Prime numbers below 25.
15. Prime numbers between 25 and 50.
16. Prime numbers between 100 and 200.

Given two sets of numbers:

$$P = \{1, 3, 5, 7, 9, 11 \dots\}$$

$$Q = \{2, 4, 6, 8, 10, 12 \dots\},$$

determine whether the new set is closed, or not closed, when the following operations are performed:

- | | | |
|---------------------|---------------------|-----------------|
| 17. $\{P + Q\}$ | 18. $\{P + P + P\}$ | 19. $\{Q + Q\}$ |
| 20. $\{Q + Q + Q\}$ | 21. $\{P + P\}$ | 22. $\{Q + Q\}$ |
| 23. $\{P + P + Q\}$ | 24. $\{P + Q + Q\}$ | |

Combine the following sets into one set:

25. 7 sets of 8 oranges, 5 sets of 11 oranges, and 9 sets of 10 oranges.
26. 19 sets of 11 bananas, 27 sets of 17 bananas, and 47 sets of 13 bananas.
27. 99 sets of 8 boys, 66 sets of 15 boys, and 55 sets of 13 boys.
28. 211 sets of 81 girls, 719 sets of 93 girls, 529 sets of 117 girls.
29. 79 sets of 80 kites, 103 sets of 25 kites, 727 sets of 19 kites.

SECTION 12 Figuring for Fun

1. Russian Multiplication

This is an old method of multiplying two numbers together which involves only multiplying and dividing by two. And it always works!

Proceed as follows:

(a) Form two columns headed by the multiplicand and the multiplier — the order is not important (say $537 \times 1,623$).

(b) Divide the left-hand number by 2 and ignore any remainder. Multiply the right-hand column by two.

(c) Continue step (b) until the number 1 appears in the left-hand column. Rule off the right-hand column.

(d) Cross out all the even numbers in the left-hand column and, at the same time, cross out the number appearing in the right-hand column opposite the even number crossed out.

(e) Add the numbers in the right-hand column that have not been crossed out.

(a) 537	1,623
(b) ~~268~~	~~3,246~~
~~134~~	~~6,492~~
67	12,984
33	25,968
~~16~~	~~51,936~~
~~8~~	~~103,872~~
~~4~~	~~207,744~~
~~2~~	~~415,488~~
1	830,976
	<u>871,551</u>

2. Austrian Division

This is a very fast method of division, and the one you would learn if you went to school in Europe. As each partial product is formed, it is subtracted term by term from the digits in the partial dividend using the additive method of subtraction and writing down only the remainder.

Divide 378,568 by 61:

$$\begin{array}{r} 6 \\ 61 \overline{) 378,568} \\ \underline{12} \end{array}$$

(a) Find and place the first digit in the quotient in the usual way.

(b) Multiply $6 \times 1 = 6$; then $6 + 2 = 8$. Write the 2 under the 8.

Multiply $6 \times 6 = 36$; then $36 + 1 = 37$.

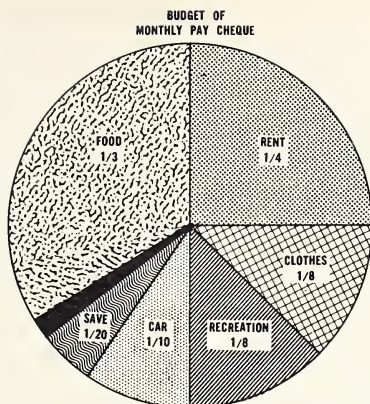
Write the 1 under 7.

This figure, 12, is the remainder.

(c) Bring down the 5, and proceed as in (b).

Quotient 6,206; remainder 2.

$$\begin{array}{r} 6,206 \\ 61 \overline{) 378,568} \\ \underline{125} \\ 368 \\ \underline{2} \\ \hline \end{array}$$



Common Fractions

3

A Business Problem

Whip Cord earns a net salary, after deductions such as income tax, of \$524 a month. He wishes to spend his income as indicated in the above pie graph (see Chapter 4). Whip is transferred by his company to another town. He is paid the same salary. Whip decides that he will live in an apartment for which he will need some new furniture. If he adheres to his proposed budget, how much will he be able to pay in rent? How much would he have left to pay monthly instalments on the furniture? If there is insufficient to pay for furniture, should he use some of the money set aside for savings? If so, how much?

Our system of fractions, like our system of whole numbers, was developed by the Hindus. They invented a vertical positioning of the digits to represent the broken numbers, as they had previously invented a horizontal positioning to represent whole numbers. They arbitrarily designated the bottom number to indicate the number of parts into which the whole unit was broken, and the top number to indicate the number of parts with which the problem dealt. The Arabs also made a contribution; they were responsible for placing the bar between the two numbers. The Romans named the fraction and its components, and these names are used by us today.

If we buy a cake, cut it into 8 equal parts, and give away 5 of these parts, we can say we gave away $\frac{5}{8}$ of the cake. The 8 represents the number of equal pieces into which we cut the cake; the 5 represents the number of pieces we gave away.

The terms used in fractions are:

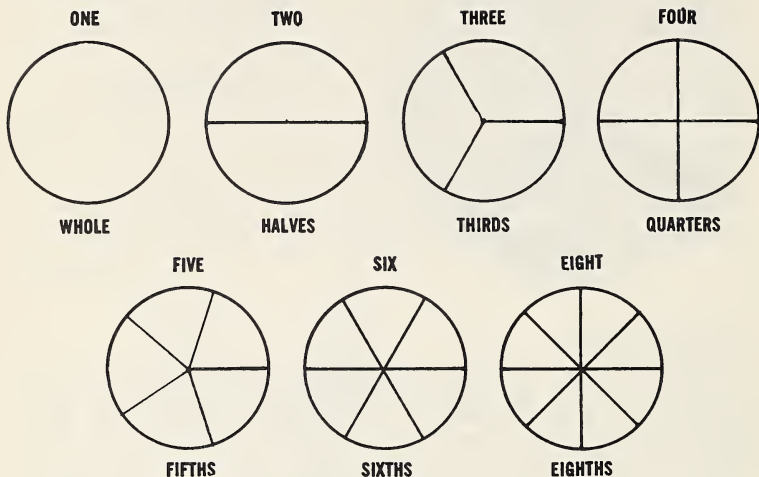
The *denominator*, the lower number indicating the number of equal parts into which the whole has been divided.

The *numerator*, the upper number indicating how many of the equal parts are being used.

The *terms*, the denominator and the numerator together.

SECTION 1 Broken Units

Figure 3-1

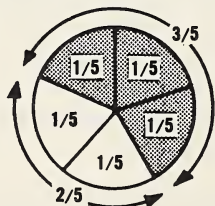


Whole Pie Divided into Parts

If you have a pie, you might wish to divide it up, depending on its size and the number of people desiring a piece, into a number of parts. If it is a very small tart-sized pie, you might eat the whole pie yourself; if you have a friend with whom to share it, you would cut it into two equal parts, and each of you would have one-half; if there were three of you, you would carefully cut the pie into three equal parts, and each of you would take one-third of the pie.

The symbol we use to represent these "parts of a pie," such as, one-third, one-seventh, etc., is the Hindu-Arabic notation $\frac{1}{3}$, $\frac{1}{7}$, etc., and we call these parts *fractions* (from the Latin "frangere" meaning "to break").

Figure 3-2

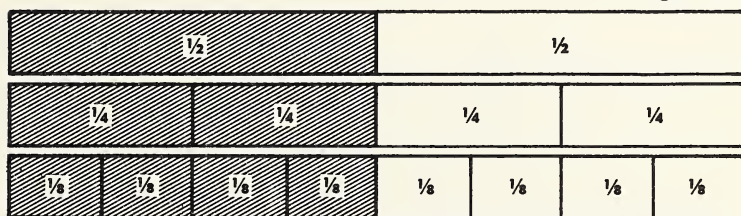


We might cut our pie into 5 equal parts and serve three people each with $\frac{1}{5}$ of the pie. The shaded pieces represent the pieces of pie we have served, i.e., $\frac{3}{5}$ of the pie. The unshaded pieces represent the pieces of pie we have left, i.e., $\frac{2}{5}$ of the pie.

WORKOUT EXERCISE I

Express as a fraction:

1. One-half, one-quarter, one-eighth, one-sixteenth.
2. Three-sixteenths, five-ninths, seven-tenths, three-sevenths.
3. Eleven-twelfths, nine-elevenths, seventeen-nineteenths.
4. If each of the fractions in 1, 2, and 3 are deducted from the whole, what fraction of the whole will remain in each case?

SECTION 2 Equivalent Fractions**Raising a Fraction To Higher Terms****Figure 3-3****Three Chocolate Bars**

You have three Long Tom chocolate bars of equal length and break one into two-halves, one into four-quarters, and the third into eight-eighths. The shaded portions of the three chocolate bars in figure 3-3 show that one-half of the first bar is equal to two-quarters of the second bar and to four-eighths of the third bar, or

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

A whole chocolate bar can be thought of as two-half bars, four-quarter bars, or eight-eighth bars, or

$1 = \frac{1}{1} = \frac{2}{2} = \frac{4}{4} = \frac{8}{8} = \frac{a}{a}$ or any other fraction with identical numerator and denominator.

From this example we can form the following rule:

If the numerator and denominator of a fraction are both multiplied by the same number, the value of the fraction is unchanged.

Examples:

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

Change two-thirds to sixths:

$$\frac{2}{3} = \frac{?}{6}$$

What multiplied by 3 gives 6?

$\therefore$ The new denominator, $6 = 3 \times 2$.

The new numerator therefore should be $2 \times 2 = 4$.

$$\therefore \frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$

Therefore, two-thirds changed to sixths becomes four-sixths.

$$\frac{7}{12} = \frac{?}{60}$$

$$\frac{7}{12} = \frac{7}{12} \times \frac{5}{5} = \frac{35}{60}$$

WORKOUT EXERCISE II

Draw diagrams to show:

1. $\frac{1}{2} = \frac{3}{6}$

2. $\frac{2}{4} = \frac{4}{8}$

3. $1 = \frac{3}{3}$

4. $\frac{2}{3} = \frac{6}{9}$

5. $\frac{3}{4} = \frac{6}{8}$

6. $1 = \frac{4}{4}$

Change to the required equivalent fractions:

7. $\frac{1}{3} = \frac{?}{9}$

8. $\frac{1}{4} = \frac{?}{16}$

9. $\frac{7}{8} = \frac{?}{56}$

10. $\frac{2}{7} = \frac{?}{49}$

11. $\frac{2}{5} = \frac{?}{30}$

12. $\frac{3}{4} = \frac{?}{24}$

13. $\frac{4}{9} = \frac{?}{81}$

14. $\frac{9}{11} = \frac{?}{66}$

15. $\frac{7}{13} = \frac{?}{104}$

16. $\frac{9}{17} = \frac{?}{85}$

17. Change $\frac{5}{12}$ to 48ths.

18. Change $\frac{11}{14}$ to 42nds.

19. Change $\frac{7}{15}$ to 90ths.

20. Change $\frac{13}{16}$ to 64ths.

Reducing a Fraction To Lower Terms

Look again at the shaded portion of our chocolate bars in figure 3-3; we see that:

$$\frac{4}{8} = \frac{1}{2} \text{ or } \frac{2}{4} = \frac{1}{2}$$

$$\frac{4 \div 4}{8 \div 4} = \frac{1}{2} \quad \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

This is the reverse of the process we followed to raise a fraction to higher terms. We are now dividing both the numerator and the denominator by the same number without altering the value of the fraction.

We can now formulate the following rule:

If the numerator and denominator of a fraction are both divided by the same number, the value of the fraction is unchanged.

Examples:

$$\frac{25}{60} = \frac{?}{12}$$

$$\frac{25}{60} = \frac{25 \div 5}{60 \div 5} = \frac{5}{12}$$

By what number can 60 be divided to obtain 12? The number is 5; therefore both numerator and denominator must be divided by 5.

Reducing a Fraction To Its Lowest Terms

If we reduce a fraction so that no whole number, other than 1, will divide evenly into both the numerator and the denominator, then the fraction has been reduced to its lowest terms.

Examples: Reduce to lowest terms:

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Since 2 divides into both 6 and 10 evenly and no whole numbers, other than 1, divide into both 3 and 5, the fraction is in its lowest terms.

$$\frac{9}{21} = \frac{9 \div 3}{21 \div 3} = \frac{3}{7}$$

$$\frac{14}{28} = \frac{14 \div 2}{28 \div 2} = \frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$$

The reduction of these fractions may be performed as follows:

$$\begin{array}{r} 3 \\ \cancel{6} \\ \hline 10 \\ \hline 5 \end{array} \text{ The division by 2 is done mentally.}$$

$$\begin{array}{r} 3 \\ 9 \overline{) 27} \\ \underline{27} \\ 0 \end{array}$$

The division by 3 is done mentally.

$$\begin{array}{r} 1 \\ 7 \overline{) 14} \\ \underline{14} \\ 0 \end{array}$$

The division by 2 and 7 is done mentally.

$$\begin{array}{r} 7 \\ 7 \overline{) 49} \\ \underline{49} \\ 0 \end{array}$$

Note that $7 \div 7 = 1$, and the 1 must always be indicated.

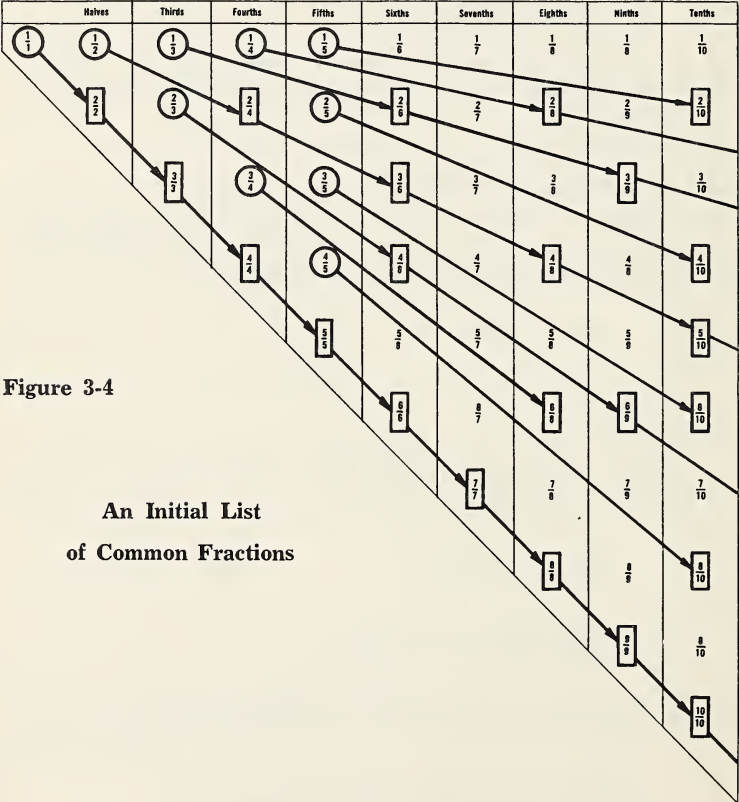


Figure 3-4

An Initial List
of Common Fractions

WORKOUT EXERCISE III

Reduce the following fractions to their lowest terms:

1. $\frac{25}{30}$ 2. $\frac{6}{18}$ 3. $\frac{120}{240}$ 4. $\frac{36}{12}$ 5. $\frac{36}{96}$ 6. $\frac{54}{72}$ 7. $\frac{13}{104}$
8. $\frac{400}{1600}$ 9. $\frac{3600}{900}$ 10. $\frac{270}{90}$ 11. $\frac{44}{121}$ 12. $\frac{27}{102}$ 13. $\frac{45}{117}$ 14. $\frac{73}{73}$
15. $\frac{210}{70}$ 16. $\frac{28}{217}$ 17. $\frac{52}{117}$ 18. $\frac{34}{136}$ 19. $\frac{64}{96}$ 20. $\frac{95}{133}$ 21. $\frac{66}{231}$

22. Extend the table in figure 3-4 to include twelfths. Read explanation of table first.

The ability to change fractions to higher and lower terms will be found very useful as we proceed to deal with mathematical problems in the following sections. The foregoing rules should therefore be thoroughly understood and the procedure mastered before continuing further. The rules for divisibility given in Appendix B will be found helpful when reducing fractions.

Figure 3-4 lists the common fractions up to and including tenths. The table can be extended to the right, in whole or part, as far as required. Each line commences with a fraction reduced to its lowest terms, which is indicated by a circle. The boxed numerals through which the diagonal lines pass are equivalent fractions. These lines can, of course, be extended indefinitely. All other fractions shown in the table are reduced to their lowest terms (that is, the numerator and denominator have no common factor), and would be circled if the table were extended.

SECTION 3 Proper and Improper Fractions

In the sections we have just studied, all the fractions have smaller numerators than denominators. Such fractions are called *proper fractions*.

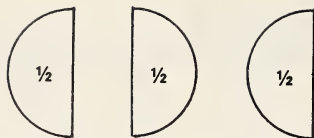
Examples: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{101}{109}$, $\frac{17}{81}$, etc.

When the numerators are larger than, or equal to, the denominators, the fractions are called *improper fractions*.

Examples: $\frac{3}{2}$, $\frac{11}{11}$, $\frac{4}{3}$, $\frac{19}{7}$, $\frac{83}{20}$, $\frac{64}{13}$, etc.

As these fractions are 'improper', it is usually desirable to change them to another form.

Figure 3-5



$\frac{3}{2}$ is the same as three-halves; if we have three half pies, we really have one whole pie and one-half pie, or $1 + \frac{1}{2}$. We can write this as $1\frac{1}{2}$; the addition sign between the 1 and the $\frac{1}{2}$ is understood.

This is called a *mixed number*. A mixed number is therefore the sum of a whole number and a fraction.

Example: Express $\frac{19}{7}$ as a mixed number.

$$\boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} = \frac{7}{7} = 1 \text{ whole}$$

$$\boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} = \frac{7}{7} = 1 \text{ whole}$$

$$\boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\frac{1}{7}} \boxed{\phantom{\frac{1}{7}}} \boxed{\phantom{\frac{1}{7}}} = \frac{5}{7}$$

$$\therefore \frac{19}{7} = 2 + \frac{5}{7} = 2\frac{5}{7}$$

In this problem we ask ourselves how many groups of seven-sevenths are contained in nineteen-sevenths?

We find that there are only two complete groups of seven-sevenths contained in nineteen-sevenths. If we deduct these fourteen-sevenths from the original nineteen-sevenths, we have a remainder of five-sevenths. Our answer is therefore $2\frac{5}{7}$.

Example: Express $\frac{24}{8}$ as a mixed or whole number:

If we divide 24 to see how many groups of eight eighths are contained in 24 eighths, the answer is 3 complete groups with no eighths left over. The answer is therefore 3.

Example: $\frac{5}{5} = 1$

WORKOUT EXERCISE IV

Express the following improper fractions as mixed or whole numbers:

1. $\frac{13}{3}$ 2. $\frac{27}{5}$ 3. $\frac{29}{7}$ 4. $\frac{28}{7}$ 5. $\frac{31}{9}$ 6. $\frac{39}{10}$ 7. $\frac{47}{11}$
8. $\frac{49}{7}$ 9. $\frac{51}{13}$ 10. $\frac{78}{16}$ 11. $\frac{796}{10}$ 12. $\frac{718}{11}$ 13. $\frac{1000}{66}$ 14. $\frac{1000}{75}$
15. $\frac{3600}{90}$ 16. $\frac{310}{44}$ 17. $\frac{1096}{17}$ 18. $\frac{1004}{64}$ 19. $\frac{567}{63}$ 20. $\frac{794}{23}$

In problems dealing with fractions it is sometimes more useful to work with improper fractions than with mixed fractions; so we must be able to convert mixed numbers and whole numbers to improper fractions.

Example: Change $3\frac{4}{5}$ to an improper fraction:

$$3\frac{4}{5} = 3 \text{ wholes} + \frac{4}{5} \text{ of a whole}$$

As we wish to express the improper fraction in terms of fifths, we must convert the 3 wholes to fifths of a whole. As each whole contains 5 fifths, 3 wholes contain 15 fifths, so,

$$3 \text{ wholes} + \text{four-fifths} = 15 \text{ fifths} + 4 \text{ fifths}$$

$$= 19 \text{ fifths}$$

$$\therefore 3\frac{4}{5} = \frac{19}{5}.$$

When we understand the steps, a shorter way to find the numerator is:

Multiply the whole number (3) by the denominator of the fractional part (5) and add the numerator of the fractional part (4).

$$(3 \times 5 = 15; 15 + 4 = 19)$$

Write the result (19) as the numerator of the improper fraction. $\left(\frac{19}{5}\right)$

Examples: Change $5\frac{7}{9}$ to an improper fraction:

$$5\frac{7}{9} = \frac{52}{9} \quad (5 \times 9 = 45; 45 + 7 = 52)$$

Change $9\frac{13}{17}$ to an improper fraction:

$$9\frac{13}{17} = \frac{166}{17} \quad (9 \times 17 = 153; \quad 153 + 13 = 166)$$

With a little practice these steps can be performed mentally.

WORKOUT EXERCISE V

Change the following mixed and whole numbers to improper fractions:

- | | | | |
|-----------------------|-------------------------|------------------------|--------------------------|
| 1. $7\frac{1}{4}$ | 2. $19\frac{7}{8}$ | 3. 11 (to twelfths) | 4. $97\frac{9}{11}$ |
| 5. $17\frac{18}{19}$ | 6. 13 (to thirds) | | 7. $16\frac{4}{5}$ |
| 8. 14 (to ninths) | 9. $16\frac{7}{11}$ | | 10. $14\frac{17}{21}$ |
| 11. 98 (to elevenths) | 12. $19\frac{9}{10}$ | | 13. $16\frac{13}{15}$ |
| 14. 79 (to quarters) | 15. $123\frac{11}{123}$ | | 16. $1,000\frac{2}{3}$ |
| 17. 981 (to sevenths) | 18. $137\frac{23}{27}$ | 19. $371\frac{26}{51}$ | 20. $187\frac{100}{107}$ |

SECTION 4 Addition of Fractions

Like Fractions

Fractions which have a common denominator are called *like* or *similar* fractions. The addition of like fractions is easily performed.

$$\begin{array}{r} \text{If we add together:} \quad 3 \text{ bananas} \\ \quad \quad \quad \quad \quad + 2 \text{ bananas} \\ \quad \quad \quad \quad \quad + 4 \text{ bananas} \\ \hline \text{we have} \quad \quad \quad \underline{\underline{9 \text{ bananas}}} \end{array}$$

$$\begin{array}{r} \text{If we add together:} \quad 3 \text{ tenths} = \frac{3}{10} \\ \quad \quad \quad \quad \quad + 2 \text{ tenths} = \frac{2}{10} \\ \quad \quad \quad \quad \quad + 4 \text{ tenths} = \frac{4}{10} \\ \hline \text{we have} \quad \quad \quad \underline{\underline{9 \text{ tenths} = \frac{9}{10}}} \end{array}$$

or

$$\frac{3}{10} + \frac{2}{10} + \frac{4}{10} = \frac{3 + 2 + 4}{10} = \frac{9}{10}$$

Example: Add $\frac{2}{13} + \frac{4}{13} + \frac{5}{13} + \frac{8}{13}$:

$$\frac{2}{13} + \frac{4}{13} + \frac{5}{13} + \frac{8}{13} = \frac{2+4+5+8}{13} = \frac{19}{13}$$

$\frac{19}{13}$ is an improper fraction and should be changed to a mixed number before stating the answer:

$$\frac{19}{13} = 1\frac{6}{13} \quad \therefore \text{Answer} = 1\frac{6}{13}$$

This can also be added vertically:

$$\begin{array}{r} \frac{2}{13} \\ \frac{4}{13} \\ \frac{5}{13} \\ \frac{8}{13} \\ \hline \frac{19}{13} = 1\frac{6}{13} \end{array}$$

Example: Add $\frac{7}{12} + \frac{5}{12} + \frac{9}{12} + \frac{5}{12}$:

$$\frac{7}{12} + \frac{5}{12} + \frac{9}{12} + \frac{5}{12} = \frac{7+5+9+5}{12} = \frac{26}{12} = 2\frac{\cancel{2}}{\cancel{12}_6}$$

$$\text{Answer} = 2\frac{1}{6}$$

It is left for the student to add this vertically.

The result is usually stated in its lowest terms, and so $\frac{2}{12}$ should be reduced to $\frac{1}{6}$.

The rule can be stated as follows:

To add together fractions which have equal denominators, add the numerators and write the sum over the common denominator.

Unlike Fractions

Fractions which have unlike denominators are called *unlike* fractions.

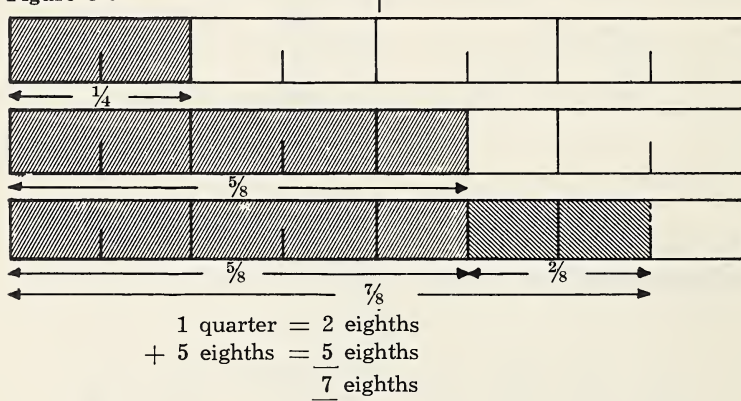
If we add together:

$$\begin{array}{rclcl}
 3 \text{ dimes} & = & 30 \text{ cents} & = & \frac{3}{10} \text{ or } \frac{30}{100} \text{ of a dollar} \\
 + 2 \text{ quarters} & = & 50 \text{ cents} & = & \frac{2}{4} \text{ or } \frac{50}{100} \text{ of a dollar} \\
 \hline
 ? & & 80 \text{ cents} & & \frac{80}{100} \text{ of a dollar} \\
 = & & = & & =
 \end{array}$$

We see that we have to express each of the addends in the same kind of unit before we can add them together.

If we desire to add one-quarter and five-eighths, we must again change to the same kind of units. This is shown in figure 3-6.

Figure 3-6



$$\text{or } \frac{1}{4} + \frac{5}{8} = \frac{2}{8} + \frac{5}{8} = \frac{2+5}{8} = \frac{7}{8}$$

$$\begin{array}{rcl}
 \text{or, vertically } \frac{1}{4} & = & \frac{1}{4} \times \frac{2}{2} = \frac{2}{8} \\
 \frac{5}{8} & & = \frac{5}{8} \\
 \hline
 & & = \frac{7}{8}
 \end{array}$$

Example: Add $\frac{3}{5} + \frac{9}{10} + \frac{7}{15} + \frac{8}{30}$

This can be stated as $\frac{18}{30} + \frac{27}{30} + \frac{14}{30} + \frac{8}{30}$

$$\begin{aligned} \text{or: } \frac{18 + 27 + 14 + 8}{30} &= \frac{67}{30} = 2\frac{7}{30} \\ \text{or vertically: } \frac{3}{5} &= \frac{3}{5} \times \frac{6}{6} = \frac{18}{30} \\ \frac{9}{10} &= \frac{9}{10} \times \frac{3}{3} = \frac{27}{30} \\ \frac{7}{15} &= \frac{7}{15} \times \frac{2}{2} = \frac{14}{30} \\ \frac{8}{30} &= \frac{8}{30} \\ \frac{67}{30} &= 2\frac{7}{30} \end{aligned}$$

The Least Common Denominator

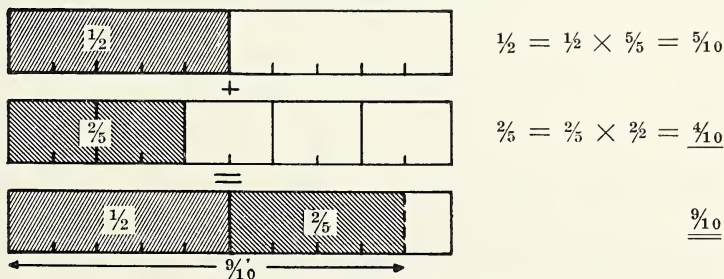
In the previous example the common denominator was a denominator of one of the fractions. This will not always be the case.

A characteristic of the common denominator is that it must be divisible by the denominator of all the fractions which are to be added. One possible common denominator is the product of all the denominators multiplied together.

Example: Add $\frac{1}{2} + \frac{2}{5}$.

$$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{5+4}{10} = \frac{9}{10}$$

Figure 3-7



We could choose any other common denominator such as:

$$\frac{1}{2} + \frac{2}{5} = \frac{10}{20} + \frac{8}{20} = \frac{10+8}{20} = \frac{18}{20} = \frac{9}{10}$$

The result is the same as obtained above; but as we must state the

result in the lowest terms, it involves more steps and is cumbersome; so we usually try and find the *least common denominator*.

Example: Add $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$.

The smallest number for which 2, 3 and 4 are exact divisors is the number 12. The lowest common denominator is therefore 12.

$$\begin{aligned}\frac{1}{2} + \frac{2}{3} + \frac{3}{4} &= \frac{6}{12} + \frac{8}{12} + \frac{9}{12} = \frac{6+8+9}{12} = \frac{23}{12} \\ &= 1\frac{11}{12}\end{aligned}$$

Example: Add $3\frac{1}{3} + 5\frac{2}{5} + 1\frac{7}{10}$.

The smallest number for which 3, 5 and 10 are exact divisors is the number 30; hence 30 is the lowest common denominator. This problem can be done two ways:

(a) Add the whole numbers and the fractions separately.

$$\begin{aligned}3\frac{1}{3} + 5\frac{2}{5} + 1\frac{7}{10} &= 3 + \frac{1}{3} + 5 + \frac{2}{5} + 1 + \frac{7}{10} \\ &= 3 + 5 + 1 + \frac{1}{3} + \frac{2}{5} + \frac{7}{10} \\ &\quad \left(\frac{1}{3} = \frac{10}{30}; \frac{2}{5} = \frac{12}{30}; \frac{7}{10} = \frac{21}{30}\right) \\ &= 9 + \frac{10+12+21}{30} \\ &= 9 + \frac{43}{30} \\ &= 9 + 1\frac{13}{30} \\ &= 10\frac{13}{30}\end{aligned}$$

(b) Change the mixed numbers to improper fractions.

$$3\frac{1}{3} + 5\frac{2}{5} + 1\frac{7}{10} = \frac{10}{3} + \frac{27}{5} + \frac{17}{10}$$

$$\begin{aligned}
 &= \frac{100 + 162 + 51}{30} \\
 &= \frac{313}{30} \\
 &= 10\frac{13}{30}
 \end{aligned}$$

We can now state the following rule:

To add together fractions which have different denominators, change the fractions to fractions with a common denominator, then add the numerators and write the sum over the common denominator.

If the L.C.D. cannot be found by inspection, use the method of continuous division illustrated in Chapter 2, Section 8, to find the L.C.M.

WORKOUT EXERCISE VI

Use both vertical and horizontal methods of addition for problems 1-25. In each case indicate what equivalent fraction must be used to bring the fractions to be added to a common denominator.

$$1. \frac{1}{8} + \frac{5}{8} \qquad 2. \frac{3}{11} + \frac{5}{11} + \frac{7}{11} \qquad 3. 6\frac{2}{3} + 11\frac{1}{3} + 2\frac{2}{3}$$

$$4. 8\frac{1}{4} + 7\frac{3}{4} + 6\frac{3}{4} \qquad 5. 11\frac{1}{12} + 7\frac{5}{12} + 8\frac{7}{12} + 1\frac{11}{12}$$

$$6. 18\frac{13}{17} + 19\frac{11}{17} + 20\frac{9}{17} \qquad 7. 9\frac{7}{9} + 5\frac{5}{9} + 2\frac{2}{9}$$

$$8. 7\frac{11}{21} + 8\frac{19}{21} + 9\frac{17}{21} \qquad 9. 8\frac{29}{32} + 17\frac{17}{32} + 19\frac{11}{32} + 4\frac{7}{32}$$

$$10. 15\frac{4}{15} + 13\frac{7}{15} + 11\frac{4}{15} + 9\frac{14}{15} \qquad 11. \frac{3}{5} + \frac{7}{10}$$

$$12. \frac{7}{24} + \frac{17}{24} + \frac{11}{24} \qquad 13. 18\frac{1}{8} + 9\frac{1}{9} + 2\frac{1}{2}$$

$$14. 11\frac{11}{36} + 9\frac{1}{9} + 12\frac{5}{12} + 2\frac{2}{3} + 4\frac{3}{4}$$

$$15. 3\frac{1}{64} + 17\frac{1}{2} + 8\frac{7}{32} + 7\frac{1}{8} \qquad 16. 3\frac{7}{100} + 8\frac{24}{25} + 7\frac{7}{10} + 8\frac{4}{5}$$

$$17. 8\frac{1}{10} + 7\frac{7}{60} + 4\frac{29}{30} + 8\frac{5}{6} \qquad 18. 4\frac{7}{8} + 8\frac{19}{20} + 17\frac{79}{80} + 31\frac{9}{10}$$

$$19. 7\frac{2}{3} + 82\frac{1}{4} + 31\frac{7}{120} + 78\frac{5}{12} \qquad 20. \frac{2}{5} + \frac{2}{3} \qquad 21. \frac{3}{4} + \frac{2}{3}$$

$$22. \frac{5}{8} + \frac{5}{6} \qquad 23. 3\frac{5}{6} + 5\frac{1}{4} \qquad 24. 17\frac{7}{12} + 21\frac{13}{24} + 18\frac{7}{9}$$

$$25. 7\frac{1}{2} + 3\frac{1}{3} + 8\frac{1}{4} + 9\frac{1}{5}$$

$$26. 6\frac{7}{8} + 4\frac{2}{3} + 7\frac{2}{9}$$

Use the method of continued division to find the L.C.D. when performing the following additions:

$$27. \frac{7}{30} + \frac{11}{64} + \frac{13}{72}$$

$$28. \frac{11}{24} + \frac{7}{18} + \frac{17}{30} + \frac{29}{45}$$

$$29. \frac{11}{24} + \frac{17}{36} + \frac{31}{56} + \frac{19}{64}$$

30. A surveyor measured the sides of a field as $279\frac{7}{32}$ feet, $74\frac{11}{48}$ feet, $281\frac{47}{60}$ feet, $77\frac{29}{54}$ feet. What was the perimeter of the field?

(The perimeter is the sum of the lengths of the sides.)

31. Mr. Green wishes to put a watering system in his garden and measured the following runs: $6\frac{2}{7}$ yards, $5\frac{3}{8}$ yards, $10\frac{3}{11}$ yards, $9\frac{1}{14}$ yards, $7\frac{7}{9}$ yards. How many yards of hose does Mr. Green need? If garden hose costs 24.5 cents a foot and must be purchased by the complete foot, how much will Mr. Green pay?

SECTION 5 Subtraction of Fractions

Fractions Which Have a Common Denominator

If fractions have a common denominator, subtraction is easily performed. If we have 7 apples and give away 5 apples, we have 2 apples left.

$$\begin{array}{r} 7 \text{ apples} \\ - 5 \text{ apples, given away} \\ \hline 2 \text{ apples, remaining} \end{array}$$

If we have a whole pie cut into 5 pieces and give away 3 pieces, we have 2 pieces, or $\frac{2}{5}$ of the pie left.

$$\begin{array}{r} 5 \text{ fifths} \\ - 3 \text{ fifths, given away} \\ \hline 2 \text{ fifths, remaining} \end{array}$$

$$\text{or } \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$$

Examples: $\frac{6}{7} - \frac{3}{7} = \frac{3}{7}$

$$\frac{79}{81} - \frac{17}{81} = \frac{62}{81}$$

$$2\frac{5}{8} - \frac{7}{8} = \frac{21}{8} - \frac{7}{8} = \frac{14}{8} = 1\frac{6}{8} = 1\frac{3}{4}$$

The rule for subtracting fractions with unlike denominators is:

Find a common multiple of the denominators; change the fractions to fractions with this denominator; then proceed as for fractions with like denominators.

It is usually simpler to subtract whole numbers from whole numbers and fractions from fractions.

Examples: Subtract $14\frac{1}{4}$ from $21\frac{1}{3}$

$$21\frac{1}{3} = 21\frac{4}{12}$$

$$\begin{array}{r} 14\frac{1}{4} = 14\frac{3}{12} \\ \underline{\hspace{1cm}} \\ 7\frac{1}{12} \end{array}$$

Subtract $10\frac{5}{8}$ from $13\frac{5}{12}$

$$13\frac{5}{12} = 13\frac{10}{24} = 12\frac{34}{24}$$

$$\begin{array}{r} 10\frac{5}{8} = 10\frac{15}{24} = 10\frac{15}{24} \\ \underline{\hspace{1cm}} \\ 2\frac{19}{24} \end{array}$$

In this example $\frac{15}{24}$ cannot be subtracted from $\frac{10}{24}$; so we 'borrow' one whole unit of $\frac{24}{24}$ from the 13 units to give us $12\frac{10+24}{24}$, and then perform the subtraction as previously directed.

WORKOUT EXERCISE VII

Subtract the following:

1. $\frac{5}{8} - \frac{1}{4}$
2. $\frac{19}{25} - \frac{3}{5}$
3. $16\frac{2}{3} - 11\frac{7}{12}$
4. $53\frac{2}{9} - 48\frac{1}{36}$
5. $17\frac{1}{4} - 13\frac{1}{3}$
6. $51\frac{3}{17} - 40\frac{3}{8}$
7. $65\frac{1}{5} - 61\frac{2}{3}$
8. $108\frac{5}{12} - 99\frac{7}{8}$
9. $77\frac{1}{7} - 55\frac{2}{5}$
10. $333\frac{1}{3} - 299\frac{7}{9}$

Perform the following operations:

11. $16\frac{2}{3} - 11\frac{1}{3} + 2\frac{1}{3} + 5\frac{2}{3} - 7\frac{1}{3}$

$$12. 17\frac{7}{8} - 5\frac{3}{4} + 2\frac{1}{2} + 4\frac{1}{3} - 6\frac{1}{2}$$

$$13. 23\frac{11}{12} - 15\frac{37}{56} + 7\frac{9}{14} + 1\frac{3}{4}$$

$$14. 23\frac{23}{24} - 11\frac{11}{36} + 7\frac{1}{56} - 8\frac{3}{28} + 5\frac{11}{42}$$

$$15. 37\frac{3}{100} - 25\frac{7}{25} + 18\frac{3}{4} - 13\frac{17}{40} - 14\frac{3}{8}$$

16. $1\frac{7}{8}$ inch, $1\frac{5}{12}$ inch and $2\frac{1}{18}$ inch pieces are cut from a metal tube 8 inches long. If $\frac{1}{16}$ of an inch is wasted in each cut, how many inches long is the piece of tube left?

17. From a bolt of terry towelling Mary James, a clerk, sold the following lengths: $6\frac{1}{4}$ yards, $3\frac{7}{8}$ yards, $5\frac{1}{5}$ yards, $4\frac{2}{3}$ yards, $10\frac{5}{9}$ yards, $7\frac{3}{4}$ yards. If the bolt was originally $40\frac{1}{2}$ yards long, how much remained of the bolt?

18. The following pieces were cut from a 20-foot length of lumber: $3\frac{1}{8}$ ft., $\frac{5}{6}$ ft., $17\frac{1}{10}$ ft., $1\frac{1}{5}$ ft., $2\frac{1}{2}$ ft., $1\frac{7}{8}$ ft., and $2\frac{2}{3}$ ft. Arrange these pieces in order of size starting with the smallest first. If $\frac{1}{96}$ of a foot was lost with each cut, how long was the piece left?

19. Sally Thorns, a clerk at the ribbon counter, sold the following ribbon during one week: Red ribbon: $2\frac{1}{3}$ yards, $1\frac{7}{8}$ yards, $15\frac{1}{6}$ yards, $7\frac{2}{3}$ yards, $5\frac{1}{4}$ yards. Blue ribbon: $4\frac{1}{9}$ yards, $2\frac{3}{4}$ yards, $10\frac{1}{3}$ yards, $11\frac{2}{3}$ yards, $7\frac{2}{9}$ yards. Green ribbon: $1\frac{1}{3}$ yards, $\frac{7}{8}$ yard, $1\frac{1}{8}$ yards, $3\frac{1}{9}$ yards, $1\frac{2}{3}$ yards. Pink ribbon: $7\frac{2}{9}$ yards, $4\frac{3}{4}$ yards, 10 yards, $5\frac{1}{3}$ yards, $4\frac{1}{4}$ yards. How much of each colour did she sell? How much altogether did she sell? If each roll of ribbon was originally 50 yards long, how much was left on each roll?

20. Mrs. Dolittle wished to line her shelves with shelf-paper. She measured her shelves as follows: $3\frac{7}{8}$ feet, $4\frac{3}{9}$ feet, $4\frac{1}{9}$ feet, $7\frac{1}{2}$ feet, $6\frac{1}{6}$ feet, $6\frac{1}{4}$ feet, $6\frac{1}{16}$ feet. If the rolls were cut 20 feet long, how many rolls must she buy? Assuming no wastage, how much would she have left on the last roll?

SECTION 6 Multiplication of Fractions

Multiplication of a Fraction By a Whole Number

We now cut our pie into 10 pieces and give away 7 pieces; how much of the pie have we given away? We can set up our problem:

$$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{7}{10}$$

Here we have found the answer by adding the numerator $1 + 1 + 1 + 1 + 1 + 1 + 1$ and using this sum as the numerator over the denominator. We can re-state our problem:

$$\frac{1}{10} \times \frac{7}{1} = \frac{7}{10}$$

The denominator is constant; the numerator is equal to the sum of the seven 1's or 7×1 .

The rule is:

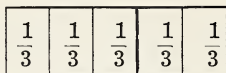
To multiply a fraction and a whole number, multiply the numerator by the whole number and leave the denominator unchanged.

Examples: I have 5 equal pieces of chocolate, each of which is $\frac{1}{3}$ of a bar. How much chocolate have I?

Figure 3-9



$$\frac{1}{3} \times \frac{5}{1} = \frac{5}{3} \text{ bars} = 1\frac{2}{3} \text{ bars}$$



$$\frac{5}{3} = 1\frac{2}{3} \text{ bars}$$

Curley Cutte, the butcher for All Meat & Sons, carved from a carcass five roasts each weighing 4 lbs. $11\frac{1}{4}$ ozs. How much meat did he take from the animal? There are 16 ozs. in 1 lb.

$$\begin{array}{r} 4 \text{ lbs. } 11\frac{1}{4} \text{ ozs.} \\ 5 \end{array}$$

$$\left(5 \times \frac{45}{4} = \frac{225}{4} \right)$$

$$\begin{array}{r} 20 \text{ lbs. } 56\frac{1}{4} \text{ ozs.} \end{array}$$

$$\left(56\frac{1}{4} \text{ ozs.} = 3 \text{ lbs. } 8\frac{1}{4} \text{ ozs.} \right)$$

$$\begin{array}{r} 23 \text{ lbs. } 8\frac{1}{4} \text{ ozs.} \end{array}$$

Multiplication of a Fraction By a Fraction

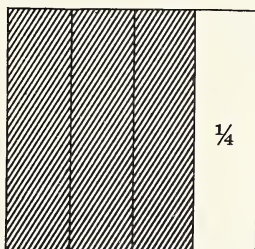
When we multiply a fraction by a whole number, we find a part of a whole, or several wholes; but when we multiply a fraction by a fraction, we find a part of a part.

Example: Tom Jones willed his property as follows: $\frac{1}{4}$ to his daughter, $\frac{3}{5}$ of the remainder to the elder son and the balance to the younger son. What fraction of the property did each of his children receive?

daughter	$= \frac{1}{4}$
first son	$= \frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$
second son	$= \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

We can illustrate our problem as follows:

Figure 3-10 $\longleftrightarrow \frac{3}{4}$

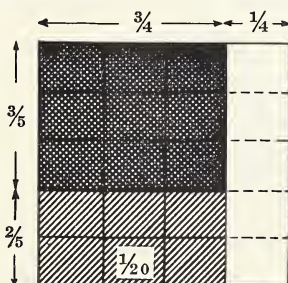


(a)

Divide the area into quarters; measure off three-quarters by cross-hatching.

The $\frac{1}{4}$ section goes to the daughter.

The $\frac{3}{4}$ section goes to the two sons.



(b)

Divide the cross-hatched section into fifths; measure off three-fifths by double cross-hatching.

The elder son receives the double cross-hatched section; that is $\frac{3}{5}$ of the remainder, or $\frac{9}{20}$ of the whole area. The younger son receives the single cross-hatched section; that is $\frac{2}{5}$ of the remainder, or $\frac{6}{20}$, or $\frac{3}{10}$ of the whole area.

The rule is:

When a fraction is multiplied by a fraction, the product has, as its numerator, the product of the two numerators and, as its denominator, the product of the two denominators.

Reduction of Fractions to Lowest Terms Before Multiplication

Work can be saved by the cancellation before multiplication of like factors in the numerators and denominators of the fractions to be multiplied.

Example: Multiply $\frac{22}{35} \times \frac{14}{33}$.

(a) Without cancellation:

$$\frac{22}{35} \times \frac{14}{33} = \frac{\overset{4}{\cancel{28}} \cancel{308}}{\cancel{1155} \underset{15}{\cancel{105}}} = \frac{4}{15}$$

(b) With cancellation: Divide 22 and 33 by 11,
Divide 14 and 35 by 7.

$$\frac{\overset{2}{\cancel{22}}}{\underset{5}{\cancel{35}}} \times \frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{33}}} = \frac{4}{15}$$

The more factors that are involved in the multiplication, the more important it becomes to use the shorter method of cancellation and cross-cancellation illustrated in above example.

Example: Multiply $\frac{1}{3} \times \frac{4}{10} \times \frac{15}{24} \times \frac{3}{4}$.

$$\frac{1}{\cancel{3}} \times \frac{\cancel{4}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{3}} \cancel{15}}{\underset{8}{\cancel{24}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{1}{16}$$

Multiplication of Whole Numbers By Mixed Numbers

If the whole number is large, it will be found simpler to multiply the whole number by each part of the mixed number separately and add the results.

Example: Multiply $15\frac{1}{2} \times 76$.

$$\begin{array}{r} 76 \\ 15\frac{1}{2} \\ \hline 380 \\ 76 \\ \hline 1140 \dots\dots\dots 76 \times 15 \\ 38 \dots\dots\dots 76 \times \frac{1}{2} \\ \hline \underline{\underline{1178}} \end{array}$$

The alternative method is to convert the mixed number to an improper fraction and proceed as for multiplication.

WORKOUT EXERCISE VIII

1. $\frac{5}{6} \times 5$
2. $\frac{11}{15} \times 3$
3. $9 \times \frac{22}{27}$
4. $3\frac{1}{3} \times 9$
5. $7 \times 8 \times \frac{7}{8}$
6. $21 \times 19 \times 23\frac{1}{4}$
7. $\frac{3}{8}$ of 18
8. $9 \times 15 \times 10\frac{11}{15}$
9. $71 \times 81 \times 61\frac{4}{5}$
10. $169 \times 128 \times 37\frac{13}{25}$

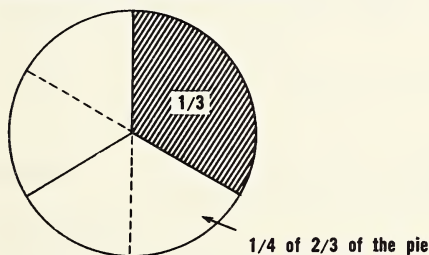
Find the product of:

11. 74, $3\frac{3}{4}$ and $7\frac{5}{6}$
12. 13, $93\frac{1}{2}$ and $101\frac{7}{8}$
13. $7\frac{7}{8}$, $8\frac{2}{5}$ and $9\frac{11}{15}$
14. $312 \times 7\frac{1}{3}$
15. $472 \times 11\frac{2}{11}$
16. $1,078 \times 1,197\frac{7}{8}$
17. $387 \times 792\frac{4}{5}$
18. $1,292 \times 598\frac{7}{9}$
19. $3,333 \times 6,666\frac{2}{3}$

SECTION 7 Division of Fractions

Division of a Fraction By a Whole Number

Figure 3-11



We have $\frac{2}{3}$ of a pie left from supper and wish to divide it equally among 4 people for a late snack. What fraction of the whole pie will each person receive?

Our problem is: $\frac{2}{3} \div 4$

If we eliminate the fraction in the numerator, we would have a simple common fraction. Our old friend 'the equivalent fraction' will come to our aid again. If we multiply the numerator by 3, we shall eliminate the fraction. If we multiply the numerator by 3, we must also multiply the denominator by 3.

$$\text{or, } \frac{2}{3} \div 4 = \frac{2}{3} \times \frac{3}{3} = \frac{2}{12} = \frac{1}{6}$$

Example: Divide $\frac{7}{12}$ by 5.

$$\frac{\frac{7}{12}}{5} = \frac{\frac{7}{12}}{5} \times \frac{12}{12} = \frac{7}{60}$$

Division of a Whole Number By a Fraction

Example: A farmer has a 6 acre field and wishes to use it for market gardening. If he needs $\frac{2}{3}$ of an acre for each different produce, how many different kinds of produce can he grow?

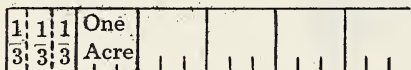
Our problem can be stated: $\frac{6}{\frac{2}{3}}$

This time we must change the denominator into a whole number. This can be done if we multiply it by 3. Therefore the numerator must also be multiplied by 3.

$$\frac{6}{\frac{2}{3}} = \frac{6}{\frac{2}{3}} \times \frac{3}{3} = \frac{18}{2} = 9$$

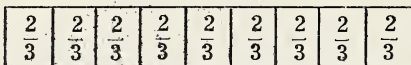
We can illustrate our problem as follows:

Figure 3-12



6 whole acres

18 one-third acre lots



=

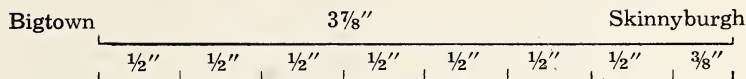
9 two-third acre lots

Example: Divide 11 by $\frac{4}{5}$.

$$\begin{aligned} \frac{11}{\frac{4}{5}} &= \frac{11}{\frac{4}{5}} \times \frac{5}{5} \\ &= \frac{55}{4} \\ &= 13\frac{3}{4} \end{aligned}$$

Division of a Fraction By a Fraction

Figure 3-13



Example: How far is Skinnyburgh from Bigtown? The distance on the map in inches = $3\frac{7}{8}$ where each $\frac{1}{2}$ inch = 1 mile. Our problem is: how many times is $\frac{1}{2}$ inch contained in $3\frac{7}{8}$ inches?

The problem can be stated:

$$\frac{3\frac{7}{8}}{\frac{1}{2}} = \frac{\frac{31}{8}}{\frac{1}{2}}$$

In this case we must change both the numerator and the denominator to whole numbers. Multiplying the numerator by 8 will eliminate the fraction. We must therefore multiply the denominator by 8. Our reciprocal fraction is then $\frac{8}{1}$:

$$\frac{31}{8} \times \frac{8}{1} = \frac{31}{1} = 31$$

Examples: Divide $\frac{6}{7}$ by $\frac{3}{8}$.

$$\frac{\frac{6}{7}}{\frac{3}{8}} = \frac{6}{7} \times \frac{8}{3} = \frac{56}{56} = \frac{2}{1} = 2$$

In this case we will have to multiply numerator and denominator by 7 to clear the top fraction and by 8 to clear the bottom fraction. That is, we must multiply top and bottom by 56.

Divide $3\frac{4}{5}$ by $1\frac{3}{10}$.

Change mixed fractions to improper fractions before dividing.

$$\frac{\frac{19}{5}}{\frac{13}{10}} = \frac{\frac{19}{5}}{\frac{13}{10}} \times \frac{10}{10} = \frac{2 \times 19}{13} = \frac{38}{13}$$

$$= 2\frac{12}{13}$$

We can now state the rule for division of fractions:

To divide fractions or whole numbers by fractions or whole numbers, reduce both numerator and denominator to a whole number using the principle of equivalent fractions.

WORKOUT EXERCISE IX

1. $\frac{7}{8} \div 5$
2. $\frac{11}{13} \div 7$
3. $\frac{115}{131} \div 35$
4. Divide $\frac{21}{25}$ by 8
5. $\frac{48}{71} \div 12$
6. Find the quotient of 24 and $\frac{2}{3}$
7. $105 \div \frac{3}{5}$
8. Divide 36 by $\frac{7}{11}$
9. $428 \div \frac{14}{15}$
10. Divide 104 by $\frac{24}{25}$
11. $\frac{6}{11} \div \frac{5}{7}$
12. $\frac{\frac{7}{12}}{\frac{2}{5}}$
13. $\frac{\frac{11}{15}}{\frac{45}{67}}$
14. $\frac{101}{103} \div \frac{7}{8}$
15. Divide $\frac{31}{37}$ by $\frac{93}{101}$
16. $7\frac{2}{3} \div 6\frac{1}{7}$
17. $4\frac{1}{6} \div 13\frac{13}{18}$
18. Find the quotient of $13\frac{2}{3} \div 7\frac{11}{12}$
19. $\frac{17\frac{1}{3}}{8\frac{1}{4}}$
20. $7\frac{1}{2} \times 3\frac{2}{3} \div 8\frac{2}{5}$

SECTION 8 Reductions

Common fractions and decimal fractions are different methods of expressing the same things, and it is possible to pass from one form to the other. These transfers are known as *reductions*.

To Change the Decimal Fraction Form to the Common Fraction Form

See figure 3-14.

The number of zeros in the denominator of the common fraction is equal to the number of digits to the right of the decimal point. The common fraction should then be reduced to its lowest terms.

Figure 3-14

Decimal Fractions	Place Notation						Common Fraction
	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	
0.6				6			six-tenths = $\frac{6}{10} = \frac{3}{5}$
0.18				1	8		eighteen-hundredths = $\frac{18}{100} = \frac{9}{50}$
0.326				3	2	6	three hundred and twenty-six thousandths = $\frac{326}{1000} = \frac{163}{500}$
0.0921					9	2	nine hundred and twenty-one ten-thousandths = $\frac{921}{10,000}$

Example: Convert the following decimal fractions to common fractions:

$$\begin{aligned}
 0.41 &= \frac{41}{100} = \frac{41}{100} \\
 0.524 &= \frac{524}{1,000} = \frac{131}{250} \\
 0.001 &= \frac{1}{1,000} = \frac{1}{1,000} \\
 3.068 &= 3\frac{68}{1,000} = 3\frac{17}{250} \\
 37.0203 &= 37\frac{203}{10,000} = 37\frac{203}{10,000}
 \end{aligned}$$

Note: When reducing a decimal fraction to its lowest terms, note that the denominator of the initial common fraction is some power of ten. As ten has only the factors 2 and 5, it is only necessary to test the numerator as to its divisibility by 2 or 5.

If the numerator cannot be divided by either 2 or 5, the common fraction is already stated in its lowest terms.

To Change the Common Fraction Form to the Decimal Fraction Form

If we have a common fraction such as $\frac{3}{16}$, we can state it as $3 \div 16$ and perform the division.

$$\begin{array}{r}
 0.1875 \\
 16 \overline{) 3.0000} \\
 \underline{16} \\
 140 \\
 \underline{128} \\
 120 \\
 \underline{112} \\
 80 \\
 \underline{80} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3 \\
 \underline{-75} \\
 1875 \\
 \underline{-10,000} \\
 400 \\
 \underline{-16} \\
 \hline
 \end{array}$$

Proof: $0.1875 = \frac{3}{16}$

Example: Express $\frac{3}{7}$ as a decimal fraction:

$$\begin{array}{r}
 7 \overline{) 3.0206040501} \\
 \underline{0.42857}
 \end{array}$$

We have carried the division to the fifth decimal place. The quotient may be carried to as many decimal places as is required for accuracy and rounded off at that point (rules for rounding off are given in Appendix A). If absolute accuracy is required, express the remainder as the numerator of a fractional part of which the denominator is the divisor; this is called a *complex decimal fraction*. In the above example:

	Round off	With fractional part
To one decimal place	0.4	$0.4\frac{2}{7}$
To two decimal places	0.43	$0.42\frac{6}{7}$
To three decimal places	0.429	$0.428\frac{4}{7}$
To four decimal places	0.4286	$0.4285\frac{5}{7}$
To five decimal places	0.42857	$0.42857\frac{1}{7}$

Example: Find the decimal equivalent of $\frac{1}{3}$:

$$\begin{array}{r} 3 \overline{) 1.0000000} \\ \underline{0.333333} \dots \end{array}$$

The quotient in this example is a decimal without end and is called a repeating or recurring decimal fraction as it will keep repeating itself indefinitely. It is usually written as $0.\overline{3}$. The dot, or dots, above the digit, or digits, indicate the digits that repeat. In some instances the divisor has to be carried out to a number of places before the pattern of repetition occurs.

Example: Find the decimal equivalent of $\frac{1}{7}$:

$$\begin{array}{r} 7 \overline{) 1.0000000000000} \\ \underline{0.14285714285714} \end{array}$$

$= 0.\overline{142857}$ or $0.(142857)$

WORKOUT EXERCISE X

Change to common fractions or mixed numbers:

- | | | | | |
|------------------------|-----------------------|------------------------|-------------------------|-------------------------|
| 1. 0.3 | 2. 0.78 | 3. 0.125 | 4. 0.0325 | 5. 1.054 |
| 6. 9.875 | 7. 13.025 | 8. 0.500 | 9. $0.01\frac{1}{2}$ | 10. 9.375 |
| 11. $0.000\frac{7}{8}$ | 12. $9.29\frac{1}{2}$ | 13. 4.4725 | 14. $0.3333\frac{1}{2}$ | 15. 17.001 |
| 16. $18.18\frac{1}{8}$ | 17. $6.02\frac{1}{2}$ | 18. $0.001\frac{1}{4}$ | 19. $39.89\frac{7}{10}$ | 20. $47.47\frac{1}{40}$ |

Change to decimal fractions and complex decimal fractions where required number of decimal places is indicated in brackets.

- | | | | | |
|-----------------------|-----------------------|------------------------|-------------------------|-------------------------|
| 21. $\frac{1}{4}$ | 22. $\frac{1}{8}$ | 23. $\frac{7}{8}$ | 24. $\frac{1}{2}$ | 25. $\frac{3}{8}$ |
| 26. $\frac{5}{8}$ | 27. $\frac{1}{5}$ | 28. $\frac{3}{5}$ | 29. $\frac{1}{3}$ | 30. $\frac{1}{3}$ (5) |
| 31. $\frac{7}{8}$ (2) | 32. $\frac{2}{5}$ | 33. $\frac{1}{6}$ | 34. $\frac{1}{6}$ (5) | 35. $\frac{1}{7}$ (3) |
| 36. $\frac{5}{7}$ (5) | 37. $\frac{5}{9}$ (4) | 38. $\frac{9}{11}$ (7) | 39. $\frac{11}{12}$ (6) | 40. $\frac{17}{18}$ (5) |

Change to decimal fractions and round off to decimal place indicated in brackets:

- | | | | | |
|-------------------------|------------------------|-------------------------|-------------------------|--------------------------|
| 41. $\frac{71}{13}$ (4) | 42. $\frac{4}{27}$ (3) | 43. $\frac{3}{100}$ (2) | 44. $\frac{7}{9}$ (4) | 45. $\frac{61}{19}$ (3) |
| 46. $\frac{13}{11}$ (4) | 47. $\frac{7}{23}$ (3) | 48. $\frac{110}{9}$ (4) | 49. $\frac{9}{110}$ (3) | 50. $\frac{318}{17}$ (3) |

51. $\frac{11}{27}$ (4) 52. $\frac{118}{7}$ (5) 53. $\frac{300}{11}$ (5) 54. $\frac{5}{11}$ (3) 55. $\frac{97}{6}$ (4)
 56. $\frac{89}{19}$ (3) 57. $\frac{78}{23}$ (3) 58. $\frac{79}{29}$ (8) 59. $\frac{197}{31}$ (4) 60. $\frac{409}{37}$ (6)

Find the decimal equivalent of:

61. $\frac{97}{6}$ 62. $\frac{9}{110}$ 63. $\frac{11}{27}$ 64. $\frac{300}{11}$ 65. $\frac{78}{33}$
 66. $\frac{17}{19}$ 67. $\frac{15}{13}$ 68. $\frac{53}{21}$ 69. $\frac{9}{19}$ 70. $\frac{11}{19}$
 71. $\frac{7}{17}$ 72. $\frac{3}{13}$ 73. $\frac{13}{23}$ 74. $\frac{11}{23}$

SECTION 9 Aliquot Parts

An aliquot part of a number is a simple fraction of that number. Some aliquot parts, such as aliquot parts of 10 and powers of 10, are very useful and when mastered can aid in many calculations.

Aliquot Parts of a Dollar

Example: Find the cost of:

6 pounds of candy at 50¢ a pound	\$ 3.00
24 packets of frozen vegetables at 25¢ a packet	6.00
18 boxes of napkins at 12½¢ a box	2.25
	<u>\$11.25</u>

*Aliquot part
of a dollar*

- 50¢ = $\frac{1}{2}$ Instead of multiplying by 50: divide 6 by 2, or multiply by $\frac{1}{2}$.
 25¢ = $\frac{1}{4}$ Instead of multiplying by 25: divide 24 by 4, or multiply by $\frac{1}{4}$.
 12½¢ = $\frac{1}{8}$ Instead of multiplying by 12½: divide 18 by 8, or multiply by $\frac{1}{8}$.

To find the aliquot part of a dollar, place the cents over 100 and reduce to the lowest terms:

$$\frac{12\frac{1}{2}}{100} = \frac{12 \times 2 + 1}{2 \times 100} = \frac{25}{200} = \frac{1}{8}$$

Example: Find the cost of:

36 pairs of socks at $62\frac{1}{2}\phi$ a pair	\$ 22.50
30 pairs of hosiery at 75ϕ a pair	22.50
12 sets of underwear at $87\frac{1}{2}\phi$ a set	10.50
24 cotton vests at $83\frac{1}{3}\phi$ each	20.00
	<u>\$ 75.50</u>

$62\frac{1}{2}\phi = \$ \frac{5}{8}$ Instead of multiplying by $62\frac{1}{2}$, multiply 36 by 5, and divide the product by 8.

$75\phi = \$ \frac{3}{4}$ Instead of multiplying by 75, multiply 30 by 3, and divide product by 4.

$87\frac{1}{2}\phi = \$ \frac{7}{8}$ Instead of multiplying by $87\frac{1}{2}$, multiply 12 by 7, and divide the product by 8.

$83\frac{1}{3}\phi = \$ \frac{5}{6}$ Instead of multiplying by $83\frac{1}{3}$, multiply 24 by 5, and divide the product by 6.

In each case the answer is expressed in dollars and fractions of dollars.

Aliquot Parts of 100

Examples: Multiply 1,164 by 25:

$$25 = \frac{100}{4} \text{ Multiply by 100 and divide by 4.}$$

$$1,164 \times 25 = \frac{1,164 \times 100}{4}$$

$$\begin{array}{r} 4 \overline{) 116,400} \\ = \underline{\underline{29,100}} \end{array}$$

Divide 6,319 by $12\frac{1}{2}$:

$$12\frac{1}{2} = \frac{100}{8}$$

$$6,319 \div 12\frac{1}{2} = 6,319 \div \frac{100}{8}$$

$$= 6,319 \times \frac{8}{100}$$

$$= 505.52$$

Rule to use an aliquot part:

Change the number which is an aliquot part to an equivalent fraction with 100, or some other power of ten, as the numerator. In the case of dollars and cents express the multiplier as a fraction of a dollar. Use this new fraction to multiply or divide in the usual way.

Some Important Aliquot Parts of 100 or of \$1.00 (100 cents).

$\frac{1}{12}$	$\frac{2}{12}$ ($\frac{1}{6}$)	$\frac{3}{12}$ ($\frac{1}{4}$)	$\frac{4}{12}$ ($\frac{1}{3}$)	$\frac{5}{12}$	$\frac{6}{12}$ ($\frac{1}{2}$)	$\frac{7}{12}$	$\frac{8}{12}$ ($\frac{2}{3}$)	$\frac{9}{12}$ ($\frac{3}{4}$)	$\frac{10}{12}$ ($\frac{5}{6}$)	$\frac{11}{12}$	TWELFTHS
$8\frac{2}{3}$	$16\frac{2}{3}$	25	$33\frac{1}{3}$	$41\frac{2}{3}$	50	$58\frac{1}{3}$	$66\frac{2}{3}$	75	$83\frac{1}{3}$	$91\frac{2}{3}$	number or cents
$\frac{100}{12}$	$\frac{100}{6}$	$\frac{100}{4}$	$\frac{100}{3}$	$\frac{500}{12}$	$\frac{100}{2}$	$\frac{700}{12}$	$\frac{200}{3}$	$\frac{300}{4}$	$\frac{500}{6}$	$\frac{1,100}{12}$	equivalent fraction
$\frac{1}{10}$	$\frac{2}{10}$ ($\frac{1}{5}$)	$\frac{3}{10}$	$\frac{4}{10}$ ($\frac{2}{5}$)	$\frac{5}{10}$ ($\frac{1}{2}$)	$\frac{6}{10}$ ($\frac{3}{5}$)	$\frac{7}{10}$	$\frac{8}{10}$ ($\frac{4}{5}$)	$\frac{9}{10}$	TENTHS		
10	20	30	40	50	60	70	80	90	number or cents		
$\frac{100}{10}$	$\frac{100}{5}$	$\frac{300}{10}$	$\frac{200}{5}$	$\frac{100}{2}$	$\frac{300}{5}$	$\frac{700}{10}$	$\frac{400}{5}$	$\frac{900}{10}$	equivalent fraction		
$\frac{1}{8}$	$\frac{2}{8}$ ($\frac{1}{4}$)	$\frac{3}{8}$	$\frac{4}{8}$ ($\frac{1}{2}$)	$\frac{5}{8}$	$\frac{6}{8}$ ($\frac{3}{4}$)	$\frac{7}{8}$	EIGHTHS				
$12\frac{1}{2}$	25	$37\frac{1}{2}$	50	$62\frac{1}{2}$	75	$87\frac{1}{2}$	number or cents				
$\frac{100}{8}$	$\frac{100}{4}$	$\frac{300}{8}$	$\frac{100}{2}$	$\frac{500}{8}$	$\frac{300}{4}$	$\frac{700}{8}$	equivalent fraction				
$\frac{1}{6}$	$\frac{2}{6}$ ($\frac{1}{3}$)	$\frac{3}{6}$ ($\frac{1}{2}$)	$\frac{4}{6}$ ($\frac{2}{3}$)	$\frac{5}{6}$	SIXTHS						
$16\frac{2}{3}$	$33\frac{1}{3}$	50	$66\frac{2}{3}$	$83\frac{1}{3}$	number or cents						
$\frac{100}{6}$	$\frac{100}{3}$	$\frac{100}{2}$	$\frac{200}{3}$	$\frac{500}{6}$	equivalent fraction						
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	FIFTHS							
20	40	60	80	number or cents							
$\frac{100}{5}$	$\frac{200}{5}$	$\frac{300}{5}$	$\frac{400}{5}$	equivalent fraction							
$\frac{1}{4}$	$\frac{2}{4}$ ($\frac{1}{2}$)	$\frac{3}{4}$	FOURTHS								
25	50	75	number or cents								
$\frac{100}{4}$	$\frac{100}{2}$	$\frac{300}{4}$	equivalent fraction								
$\frac{1}{3}$	$\frac{2}{3}$	THIRDS									
$33\frac{1}{3}$	$66\frac{2}{3}$	number or cents									
$\frac{100}{3}$	$\frac{200}{3}$	equivalent fraction									
$\frac{1}{2}$	HALVES										
50	number or cents										
$\frac{100}{2}$	equivalent fraction										

Figure 3-15

Figure 3-15

Table 3-15 also serves as a table of aliquot parts of \$100 if the numbers are read as dollars.

Aliquot Parts and Percentages

This will be covered in Chapter 5.

WORKOUT EXERCISE XI

1. Calculate the aliquot parts, of one hundred dollars, of the following amounts: (a) $\$6\frac{1}{4}$ (b) $\$8\frac{1}{3}$ (c) $\$41\frac{2}{3}$ (d) $\$83\frac{1}{3}$ (e) $\$66\frac{2}{3}$

(f) $\$56\frac{1}{4}$ (g) $\$81\frac{1}{4}$ (h) $\$87\frac{1}{2}$ (i) $\$93\frac{3}{4}$ (j) $\$43\frac{3}{4}$

2. Calculate the following aliquot parts of one dollar: (a) $\frac{5}{12}$

(b) $\frac{15}{16}$ (c) $\frac{17}{24}$ (d) $\frac{3}{8}$ (e) $\frac{11}{16}$ (f) $\frac{17}{32}$ (g) $\frac{4}{5}$.

3. Make out the following invoice, as of the end of last month, using aliquot parts to perform the multiplication:

To Mr. Slim Chance, Gayest Heights Haberdashery, for:

12½ dozen white handkerchiefs at \$1.60 a dozen,
 33⅓ dozen neckties at \$3.99 a dozen,
 62½ dozen pairs of socks at \$4.20 a dozen,
 25 shirts at \$1.90 each,
 75 pairs of shorts at \$1.29 each,
 87½ dozen coloured handkerchiefs at \$1.38 a dozen,
 50 sport shirts at \$1.98 each,
 125 bathing trunks at \$1.29 each,
 175 tie clips at 75¢ each.

(Note: $125 = 100 + \frac{1}{4}$ of 100; $175 = 100 + \frac{3}{4}$ of 100.)

4. Extend the table in figure 3-15 to include sixteenths.

5. Textile Jobbers purchase from Hidate Clothing the following bolts of material:

7 bolts of velvet, containing $27\frac{1}{8}$, $31\frac{1}{4}$, $47\frac{1}{2}$, $29\frac{3}{4}$, $33\frac{3}{8}$, $30\frac{5}{8}$, $28\frac{7}{8}$ yards, respectively, at 75 cents a yard.

3 bolts of printed nylon, containing $25\frac{3}{8}$, $27\frac{1}{4}$, $24\frac{7}{8}$ yards, respectively, at \$1.80 a yard.

4 bolts of drip-dry cotton, containing $25\frac{1}{2}$, $31\frac{7}{8}$, $27\frac{5}{8}$, 28 yards, respectively, at \$1.62½ a yard.

Make out the invoice to Textile Jobbers, using aliquot parts for the calculations.

6. The Troy Toy Corporation sold to Mrs. Vi Count of 1718 Monarch Street, King, the following items:

- 37 electric train sets @ \$37.50 (Note: \$37.50 is $\frac{3}{8}$ of \$100),
 17 dozen assorted rubber balls at 62½ cents a dozen,
 24 dozen baseballs at \$2.50 a dozen (Note: \$2.50 is $\frac{1}{4}$ of \$10),
 17 beauty dolls at \$12.50 each,
 31 dozen baseball bats at \$16.66⅔ a dozen,

Make out the invoice to Mrs. Vi Count using aliquot parts for the calculations.

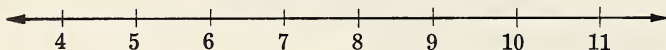
7. Find the value of: 8. Find the value of: 9. Find the value of:
- (a) 36 lbs. at 25¢ (a) 41 bu. at 66⅔¢ (a) 97 yds. at 81¼¢
 (b) 48 lbs. at 50¢ (b) 18 pieces at 33⅓¢ (b) 181 lbs. at 62½¢
 (c) 29 lbs. at 75¢ (c) 39 yds. at 16⅔¢ (c) 118 doz. at 12½¢
 (d) 108 lbs. at 41⅔¢ (d) 103 lbs. at 83⅓¢ (d) 74 yds. at 43¾¢
 (e) 37 bu. at 91⅔¢ (e) 33 lbs. at 16⅔¢ (e) 210 ft. at 31¼¢
 (f) 27 yds. at 8⅓¢ (f) 74 pieces at 33⅓¢ (f) 47 pieces at 18¾¢
 (g) 83 pieces at 58⅓¢ (g) 713 ft. at 66⅔¢ (g) 110 lbs. at 56¼¢
 (h) 63 pieces at 41⅔¢ (h) 42 bu. at 83⅓¢ (h) 37 yds. at 87½¢

Note: For sixteenths see your own solution to problem 4.

SECTION 10 More on Sets

Symbols: < means "is less than",
 > means "is greater than",
 = means "is equal to",
 ≠ means "is not equal to"

Figure 3-16 The Number Line, or Scale



A number line is frequently used to compare numbers. It is usual for these numbers to increase in magnitude from left to right as shown in figure 3-16.

It follows, therefore, that any figure to the right of another figure is greater than that figure; any figure to the left of it is less than that figure.

Reading the line we see that:

$$10 > 8, \quad 6 < 8, \quad 4 < 9.$$

If two numbers are represented by x and y , if $x \neq y$, then $x < y$ or $x > y$.

Example:

True Statements
 $3 + 1 = 2 + 2$
 $4 + 5 > 6 + 2$
 $5 + 5 < 6 + 5$

False Statements
 $3 + 1 = 2 + 3$
 $3 + 1 \neq 2 + 2$
 $9 + 2 > 6 + 6$

Examples: $A = \{2, 3, 4, 5\}$

Find whether it is true or false that each number $< 3\frac{1}{2}$:

$$2 < 3\frac{1}{2}, \text{ True,}$$

$$3 < 3\frac{1}{2}, \text{ True,}$$

$$4 < 3\frac{1}{2}, \text{ False,}$$

$$5 < 3\frac{1}{2}, \text{ False.}$$

WORKOUT EXERCISE XII

Indicate whether the following statements are true or false:

1. $3 + 4 = 6 + 1$ 2. $5 + 3 \neq 7 + 1$ 3. $6 + 0 = 6 \times 0$

4. $\frac{14}{4} < \frac{27}{8}$ 5. $\frac{5}{4} < \frac{10}{11}$ 6. $\frac{1}{4} + \frac{1}{3} > \frac{9}{10}$

7. $\frac{1}{2} \times \frac{1}{4} \neq \frac{1}{2} + \frac{1}{4}$ 8. $\frac{3}{11} + \frac{4}{10} > \frac{3}{4}$ 9. $\frac{3}{4} \times 0 \neq 0$

10. $\frac{3}{4} \times 1 \neq 0$ 11. $\frac{6}{8} - \frac{2}{3} > \frac{1}{2} + \frac{1}{4}$ 12. $\frac{11}{7} + 1 < 4 - \frac{6}{5}$

Write the lowest natural number to make the following statements true:

13. $? + 6 = 13$ 14. $7 + 3 = 9 + ?$ 15. $5 + 2 > 5 + ?$

16. $\frac{4}{7} + \frac{3}{7} = ?$ 17. $18 - 3 > 19 - ?$ 18. $36 \div 4 > 24 \div ?$

19. $6 + 7 + 8 = 7 + 6 + ?$ 20. $\frac{3}{8} + \frac{8}{3} = \frac{1}{24} + ?$

21. $54 \div 2 < ?$ 22. $2 + 1\frac{3}{4} = 2\frac{3}{4} + ?$

23. $10 \div 4 < 1\frac{1}{2} + ?$ 24. $3\frac{1}{2} + 2\frac{1}{4} \neq 7 - ?$

Write the lowest natural number to make the following statements false:

25. $11 + ? = 12$ 26. $3 \times 4 = 6 \times ?$ 27. $7\frac{1}{4} - 5 = 1\frac{1}{4} + ?$

28. $17 - 3 < 12 + ?$ 29. $? - 8 > 1$ 30. $22 \div 3 < 7 + ?$

31. $4 + 5 - 2 = 5 - 2 + ?$ 32. $8 + \frac{10}{5} > 7 + ?$

For each member of the following sets state whether the statement is true or false:

33. $S = \{1, 2, 3, 4\}$ member > 3 .

34. $T = \{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3\}$ member is a mixed number.

35. $P = \{0, 2, 4, 6\}$ member > 0 .

36. $V = \{1, 3, 5, 7, 9\}$ member $\neq 5$.
 37. $A = \{2, 4, 6, 8\}$ member is an even number.
 38. $B = \{1, 2, 3, 4, 5\}$ member is an odd number.
 39. $C = \{4, 5, 6, 7, 8, 9\}$ member < 7 .
 40. $R = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2\}$ member is a natural number.

List the members of the following sets:

41. The proper fractions whose numerators and denominators are made from the digits 1, 2 and 3.
 42. The prime numbers between 10 and 30.
 43. The natural numbers less than 12 and greater than 11.
 44. The even natural numbers less than 12.
 45. The improper fractions whose numerators and denominators are made from the digits 1, 2 and 3.

SECTION 11 Figuring for Fun

1. Arrange four 4's in such a way that, together with any necessary signs, they will express the following integers:

1, 2, 3, 7, 8, 9, 10, 11, 12, 15, 16, 17.

(Example: $4 + 4 + 4 + 4 = 16$.)

2. Do you know on which day of the week you were born? Here is a way to find out. We have to use code numbers for the different months, and from January to December they are, in order:

0, 3, 3, 6, 1, 4, 6, 2, 5, 0, 3, 5.

We now add together:

- (a) The last two digits of the year in which you were born,
 (b) A quarter of (a) and ignore any fraction,
 (c) The code number for the month,
 (d) The number of the day in the month.

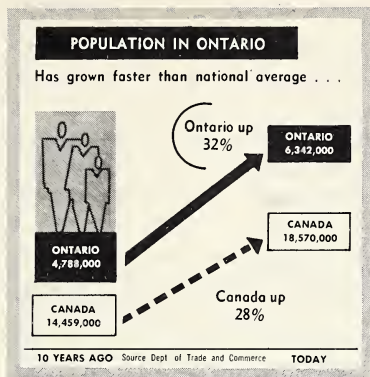
Suppose you were born on February 8, 1949.

Add together:	(a) $4 + 9$	= 13
	(b) $\frac{1}{4}$ of 13	= 3
	(c) 3 for February	= 3
	(d) Day of month	= 8
		<u>27</u>

Divide this number by 7: $27 \div 7 = 3\frac{6}{7}$.

The remainder 6 gives the position of the day of the week on which you were born. In this case the required day is Saturday.

If there is no remainder from the division, the required day is Sunday. (If the required date falls in January or February of a leap year, deduct 1 before you divide).



Graphs

4

A Business Problem

Mr. Ben Friendly, the comptroller of Breeze Pine Products, made a survey over a period of years showing the cost of labour paid for the manufacture of Breezy Maumaus. He was concerned that the increase in this expense was out of proportion to other expenditures and to sales. He sent the results of his survey in the form of a table to the plant manager, Mr. Irwin, but failed to impress him. Mr. Friendly then sent Mr. Irwin a table showing the total expenses compared to labour expenses, broken down into departments. Mr. Irwin was interested in these figures but could see no need for action.

What other means could Mr. Friendly use to convince Mr. Irwin that he thought action was needed? What particular charts could be prepared? What pitfalls should be avoided in both drawing up and reading the proposed charts?

A graph is a symbolic diagram showing the relationships existing between numerical quantities. When skillfully prepared, graphs are a most economical and effective way to present numerical information. They are economical because they can be read and understood directly, without taxing the mind with the tables of figures from which they are usually prepared. They are effective because they assist in presenting important information, which might otherwise be obscured, with force and simplicity.

Although graphs or, as they are sometimes called, charts are of various types, there are important rules for construction which apply to all of them.

General Construction Rules

1. Graphs are usually drawn on graph paper. This is a special type of paper ruled horizontally and vertically to form small squares called *grids*. The horizontal and vertical lines are equidistant and are called *grid lines*. The left-hand vertical grid line and the lower horizontal grid line are called *scale lines*. To make it easier to prepare and read graphs, graph paper usually has every tenth grid line more heavily printed than the rest.

2. Graphs should be neat, attractive, and drawn in ink, preferably a black India ink. All wording on a chart or graph should be printed horizontally.
3. The type of graph to be used must be selected with care, in order to convey the message to the particular audience who will read it. Keep it clear and simple. Use your imagination and ingenuity to give variety to your graphs.
4. The graph must have a title which is informative and, preferably, short. A sub-title may be used if additional explanation is required. This may be placed above or below the graph, although, in this writer's opinion, it is more effective at the top.
5. Scales. Equal distances on the scales should always represent equal values. Only a range of values necessary to present the data should be used. To obtain a representative graph that is correctly centered on the page, great care should be taken in choosing scales.

Horizontal Scale. The lowest horizontal grid line used should be drawn as a heavy line and this *scale line* divided to indicate the value, the units, and the basis of classification of the vertical grid lines. If the graph represents a passage of time, the time element is *always* on the horizontal scale.

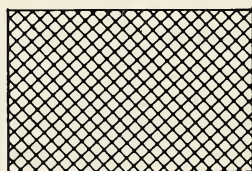
Vertical Scale. The left-hand vertical grid line should be drawn as a heavy line and this scale line divided to indicate the value of the horizontal grid lines. The scale unit chosen should be given at the top of the column. The lowest number on the vertical axis must always be zero. When a graph is not reproduced on graph paper, only those grid lines which are used need be shown. These should be drawn more lightly than the scale lines.

6. **Source.** The source from which the data has been taken to make the graph should always be quoted. This is usually placed below the horizontal scale line on the left-hand side.

7. **Emphasis.** Particular aspects of charts or graphs may be emphasized by the use of colour or shading. Two simple methods of shading are hatching and cross-hatching:



Hatching



Cross-hatching

The Types of Graphs

There are many different kinds of graphs, but when examined closely most of them may be placed in one of the following groups:

Line graph

Single line

Multiple line

Component line

Bar graph

Vertical bar

Horizontal bar

Component bar

Rectangle graph

or 100% chart

Circle graph

or Pie chart

Picture graph

or Pictogram

These are the kinds of graphs or charts most used by business men, and all are dealt with in this chapter.

SECTION 1 Line Graphs

A. Straight Line Graph

A straight line graph *plots* each point of the data given and joins the points together by a series of straight lines.

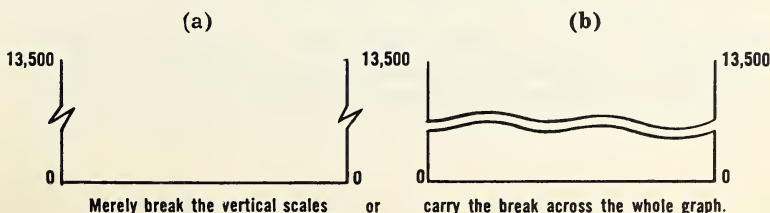
Example: Figures of Canada's population for the ten years from 1950-1959, as published by the Dominion Bureau of Statistics, were as follows:

1950	13,712,000	1955	15,698,000
1951	14,009,000	1956	16,081,000
1952	14,459,000	1957	16,589,000
1953	14,845,000	1958	17,048,000
1954	15,287,000	1959	17,422,000

Draw a single-line graph depicting this information.

The horizontal scale must show ten years from 1950 to 1959. The vertical scale should be in thousands of persons. The *range* in thousands of persons will be 17,422 minus 13,712 equals 3,710. As noted, our vertical scale must start at 0, but our first recorded figure is 13,712. We do not want our graph up at the top of the page so we 'break' the vertical scale. There are two devices used to break the vertical scale:

Figure 4 - 1

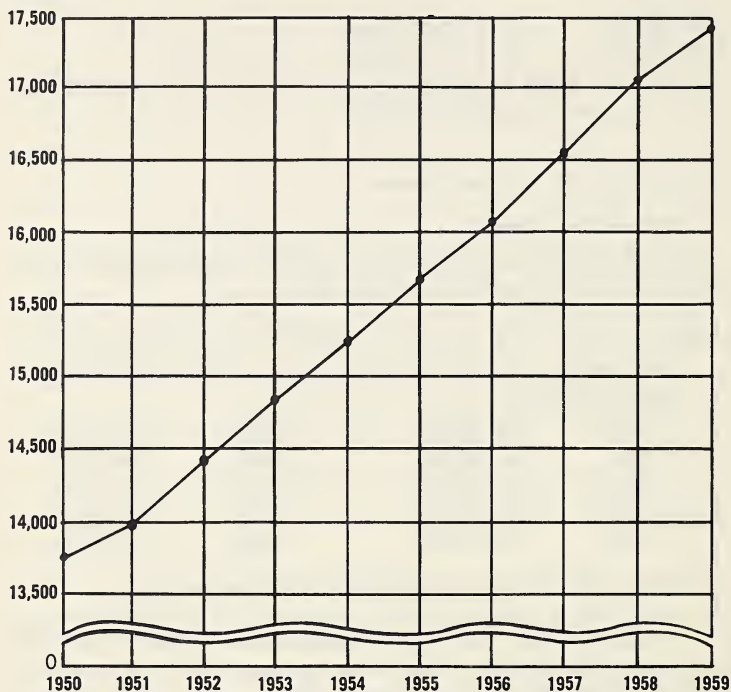


It is always important to indicate the break, otherwise someone reading the graph might be misled.

Graph 1

**Canada's Population
From 1950 to 1959**

THOUSANDS
OF PERSONS



Source: Dominion Bureau of Statistics.

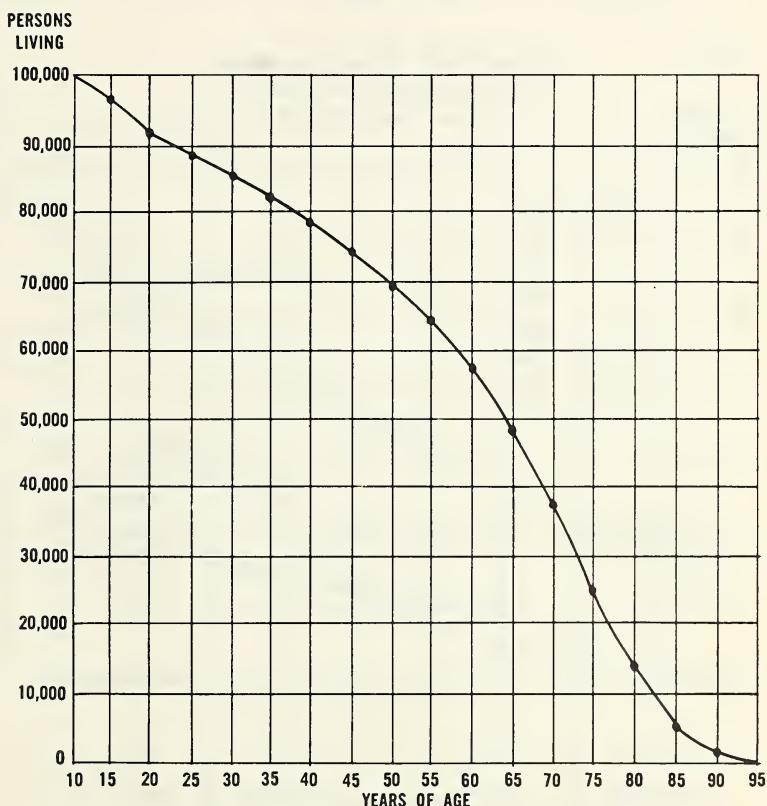
B. Smooth Line Graph

When statistical data varies uniformly or consistently and a number of points are plotted, these are sometimes joined by a smooth curve. When we do this, we are assuming the in-between values.

Example: The American Experience Table of Mortality lists the number of people living at each age from 10 to 95 in intervals of 1 year starting with 100,000 ten-year-olds.

Graph 2

**Number of Persons Living
From Ages Ten to Ninety-Five
Of Original 100,000**



Source: American Experience Table.

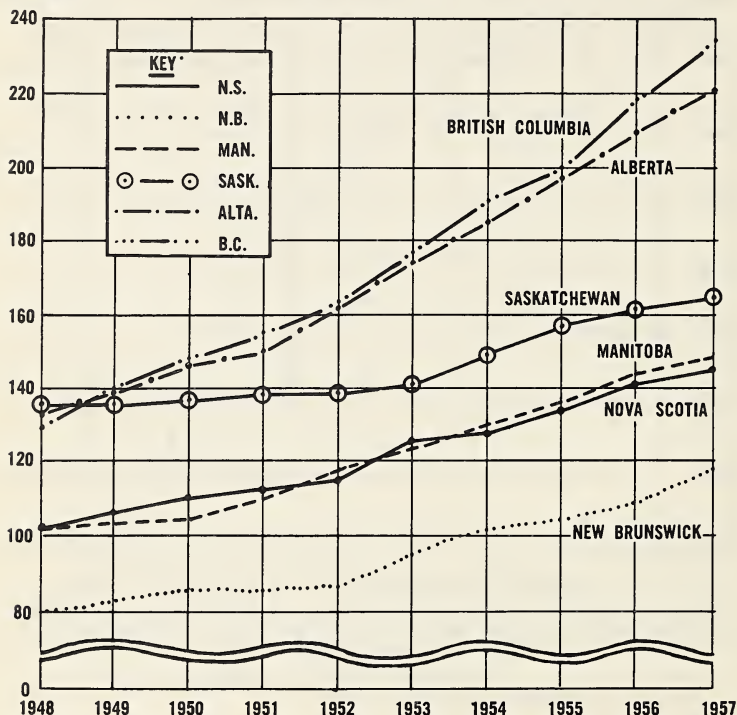
C. Multiple Line Graph

A multiple line graph is prepared in the same way as a single line graph. It is used to compare two or more sets of data. The lines can be distinguished by using various types of lines, for example, broken lines, dotted lines, and coloured lines. Frequently, the left-hand vertical scale will indicate the scale for one line, and the right-hand vertical scale will represent the scale for a second line. Each line must be labelled or a *key* given on the graph.

Example: A multiple line graph showing the average daily attendance in provincially controlled schools over a ten year period from 1948 to 1957 is shown below for the six 'middle value' provinces.

Graph 3 **Average Daily School Attendance**
In Six Canadian Provinces
For the Years 1948-1957

THOUSAND
PUPILS



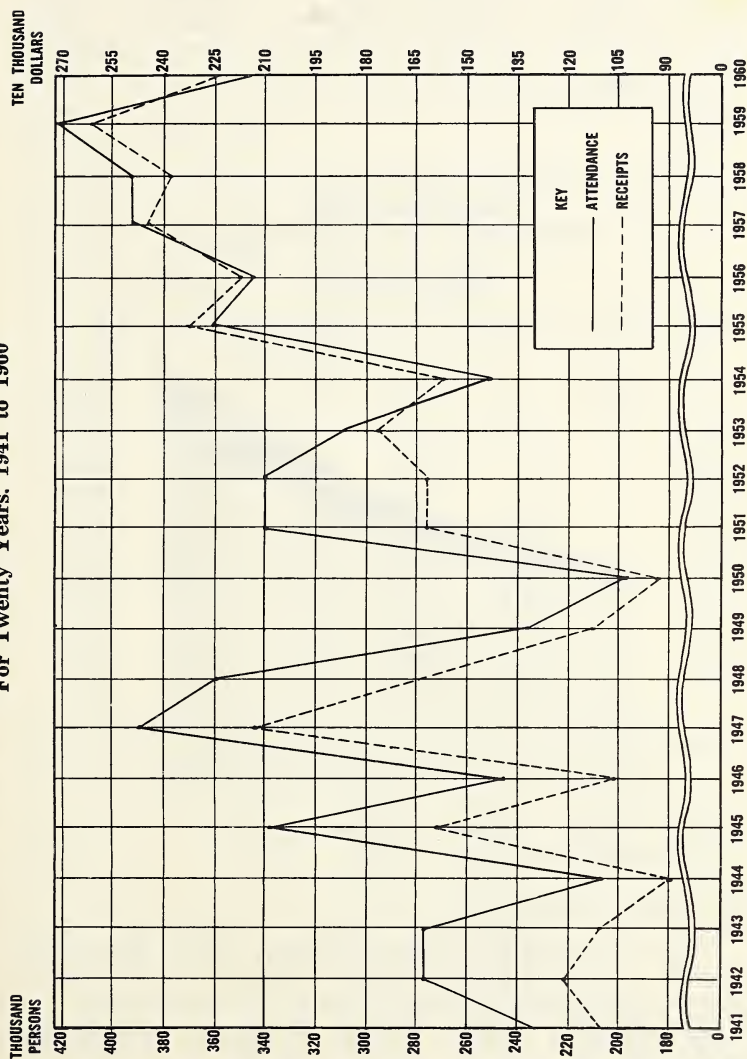
Source: Canada Year Book, 1959.

Over the same ten year period, attendance rose in P.E.I. from 14,774 to 18,573, in Newfoundland from 59,520 to 96,060, in Quebec from 545,841 to 914,479, and in Ontario from 613,586 to 1,040,684.

EXERCISE

Plot the other four provinces, making a break between upper and lower values.

Example: Graph 4 is a multiple line graph with different scales on the right and left vertical axes and shows the attendance and the gate receipts at the World Series Games from 1941 to 1960.



Source: World Almanac, 1960.

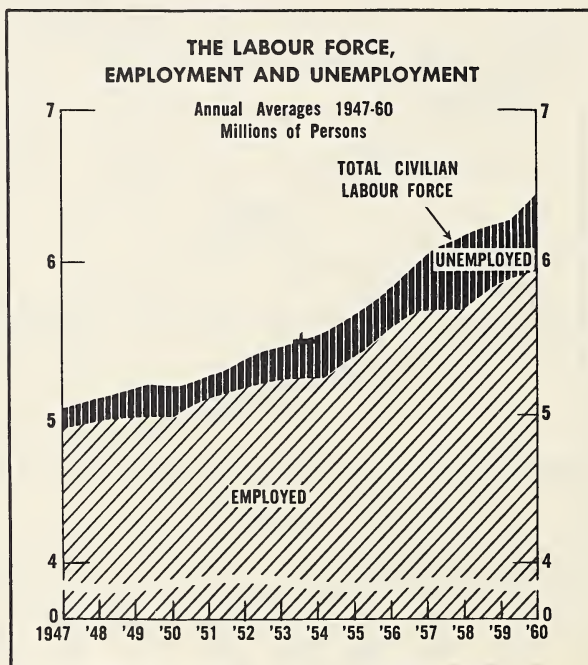
Graph 4

D. Component Line Chart

Component line charts are used to show the total and the breakdown of the total into its component parts. Shading and colouring is used to differentiate between the component parts.

Example: The total civilian labour force in Canada for the years 1947 to 1960 is shown in Graph 5. This is broken down into the number who were employed and the number unemployed for each year.

Graph 5



Source: Financial Post.

WORKOUT EXERCISE

1. From statistical records in the public library make a table of the population of your province for the last ten years, or of your town if the figures are available, and from it draw up a broken line graph.
2. Average figures of income received by Canadian taxpayers is given below:

<i>In-</i> <i>come</i>	<i>No.</i> <i>Tax-</i> <i>payers</i>	<i>In-</i> <i>come</i>	<i>No.</i> <i>Tax-</i> <i>payers</i>	<i>In-</i> <i>come</i>	<i>No.</i> <i>Tax-</i> <i>payers</i>
\$ 500 ..	20,606	\$1,500 ..	629,651	\$2,500 ..	893,868
3,500 ..	1,016,281	4,500 ..	695,875	5,500 ..	340,429
6,500 ..	165,242	7,500 ..	88,901	8,500 ..	49,370
9,500 ..	32,298	12,500 ..	68,351	17,500 ..	22,731
22,500 ..	9,330	37,500 ..	12,181	75,000 ..	2,463

Draw a smooth line graph to record this information. Note: Your x-axis figures should ascend in multiples of \$1,000 although the information is not broken down.

3. Canadian exports of oilseed commodities in the last ten years were, in thousands of dollars: \$18,781; \$25,367; \$24,091; \$24,730; \$51,643; \$83,434; \$110,851; \$78,705; \$77,684; \$85,696. Draw a line graph to indicate this information.

Source: Department of Trade and Commerce.

4. If each of the total exports above is divided as follows, draw a component line graph to show the breakdown, in thousands of dollars.

<i>United</i> <i>Kingdom</i>	<i>Europe</i>	<i>Latin</i> <i>America</i>	<i>United</i> <i>States</i>	<i>Asia</i>	<i>Other</i>	<i>Total</i>
\$ 1,029	\$ 8,425	\$ 689	\$2,956	\$ 4,095		\$ 18,781
2,610	14,414	941	5,437	78		25,367
9,336	9,156	537	3,294	1,382		24,091
9,215	8,402	422	2,710	3,782		24,730
23,453	20,706	279	1,502	4,996		51,643
46,964	24,391	192	2,447	9,218		83,434
47,420	48,597	215	1,639	12,739		110,851
34,619	34,142	121	1,448	7,580		78,705
40,417	22,584	125	1,419	13,027		77,684
43,116	24,178	67	1,534	14,554		85,696

5. By means of a multiple line chart, compare the total sales of shares during a recent week on the Toronto Stock Exchange with those sold on the New York Stock Exchange.

	<i>Mon.</i>	<i>Tues.</i>	<i>Wed.</i>	<i>Thur.</i>	<i>Fri.</i>
Toronto	6,015,000	3,884,000	5,175,000	6,679,000	8,101,000
New York	2,970,000	3,050,000	3,180,000	2,890,000	2,520,000

6. Compare, by means of a multiple line chart, the volume of stock sales on the Montreal, Vancouver, and American Stock Exchanges for a recent week as reported in the Financial Post.

	<i>Mon.</i>	<i>Tues.</i>	<i>Wed.</i>	<i>Thur.</i>	<i>Fri.</i>
Montreal	586,700	448,000	649,200	982,200	868,600
Vancouver	279,919	374,028	334,921	567,796	396,115
American	980,000	860,000	890,000	840,000	820,000

7. Use a component line chart to show the total student enrolment in your school for the last ten years. Show the total enrolment and break it down into: (a) male and female students, (b) grades, (c) courses.

8. Make two component line charts to show the total number of stocks listed on the Toronto Stock Exchange and the Montreal Stock Exchange broken down into the number whose price went up, the number whose price went down, and the number unchanged during the last six weeks:

	Toronto			Montreal		
	Up	Down	No change	Up	Down	No change
1 week ago	321	348	160	113	121	114
2 weeks ago . . .	349	336	158	102	128	145
3 weeks ago . . .	333	301	194	117	112	130
4 weeks ago . . .	378	281	177	113	90	167
5 weeks ago . . .	327	281	187	137	109	141
6 weeks ago . . .	379	290	161	157	89	109

9. Ontario have estimated their needs for electricity, in kilowatts, and the source of this power to be as follows:

	1965	1970	1975	1980
Hydraulic	5,300	5,300	5,650	7,240
Thermal	3,000	4,350	5,300	7,240
Nuclear	1,250	2,300	5,000	7,240

Draw a component line chart to illustrate this information.

SECTION 2 Bar Graphs

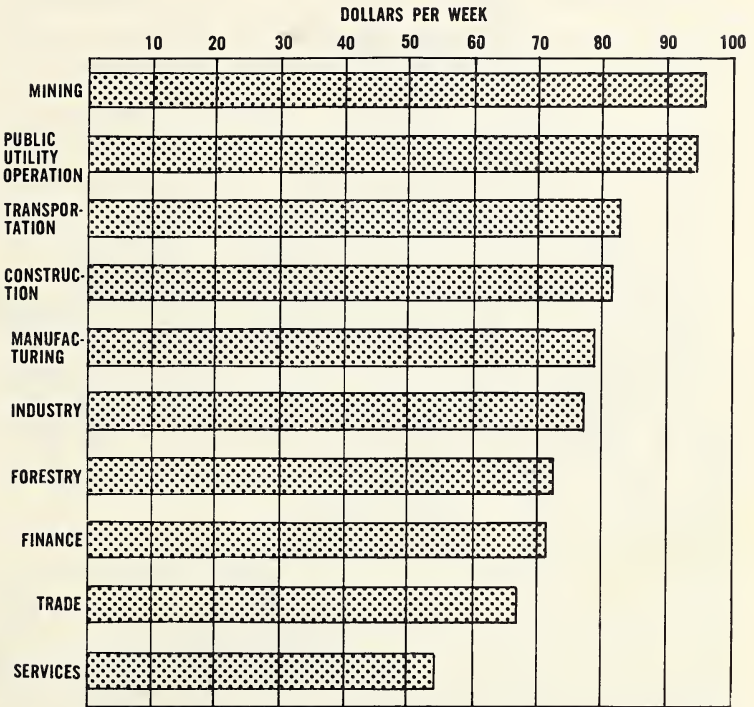
A. Horizontal Bar Graph

Horizontal bar graphs can best be used to compare different kinds of things. Only a horizontal numerical scale is used. The categories are listed vertically. The longest bar is usually placed at the top. The spaces between the bars vary in width from one-half to one full bar. Occasionally the spaces are omitted for emphasis. The bars can be coloured with coloured pencils, hatched, cross-hatched, or otherwise shaded.

Example: The Dominion Bureau of Statistics Employment and Pay-roll figures for 1961 give the following weekly wages by industry: Forestry, \$73.18; Mining, \$96.36; Manufacturing, \$79.64; Construction, \$82.00; Transportation & Communication, \$82.70; Public Utility Operation, \$94.69; Wholesale & Retail Trade, \$66.63; Finance, Insurance & Real Estate, \$72.30; Services (hotels, restaurants, etc.), \$54.85; Industry, \$76.99.

Graph 6

**Average Weekly Wage in Canadian Industry
In 1961**



Source: Dominion Bureau of Statistics.

B. Vertical Bar Graph, or Column Chart

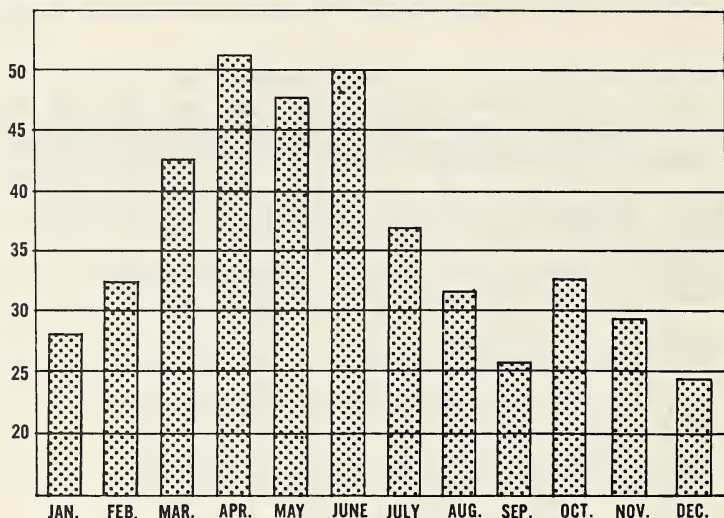
Vertical bar graphs should always be used for time sequences, with the horizontal scale representing the time in days, months, years, or periods.

Example: The sale of new passenger automobiles in Canada in a typical year is given below.

**Canadian Sale of New Passenger Cars
In a Typical Year**

Graph 7

THOUSANDS
OF CARS

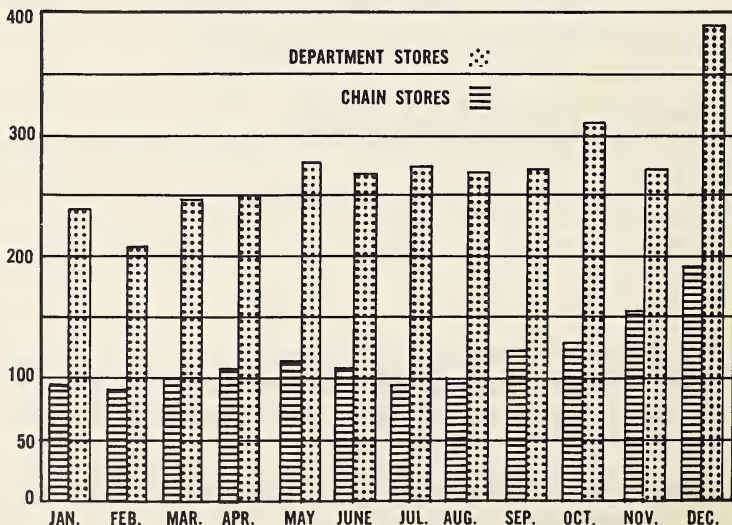


Source: Dominion Bureau of Statistics.

MILLIONS
OF DOLLARS

**Canadian Retail Sales
In a Typical Year**

Graph 8



Source: Dominion Bureau of Statistics.

C. Multi-Column Bar Graph

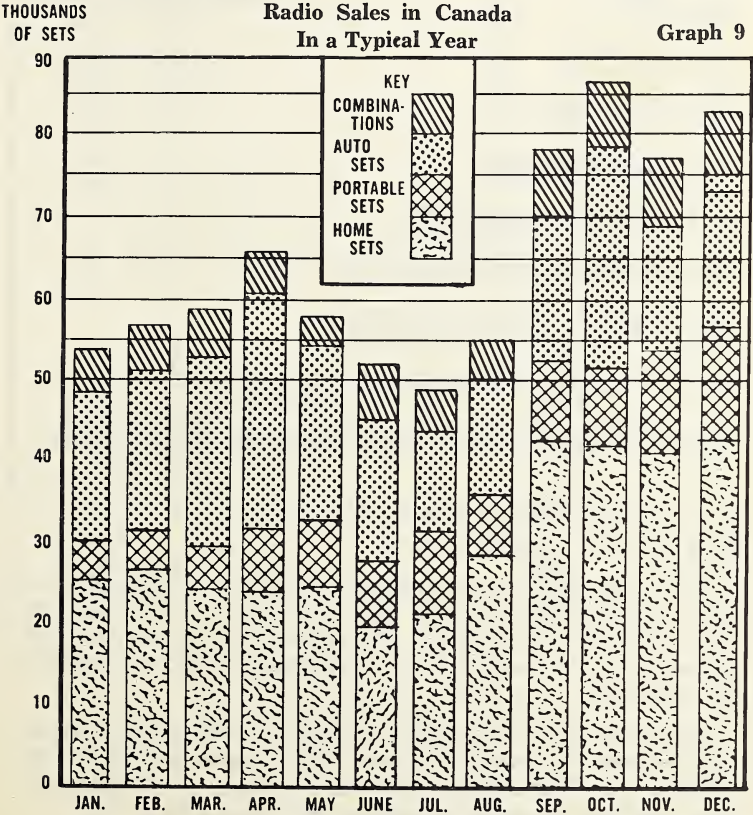
Bar graphs, both horizontal and vertical, can show a number of columns under each category or date. This affords an excellent method of comparison.

Example: The retail sales in Canadian department stores and Canadian chain stores are shown in graph 8.

D. Component Bar Graph

Component bar graphs, either horizontal or vertical, break the bar into its component parts.

Example: Radios produced and sold in Canada can be broken down into: home radio sets, portable sets, automobile sets, and combination sets. The total column represents the total sales.

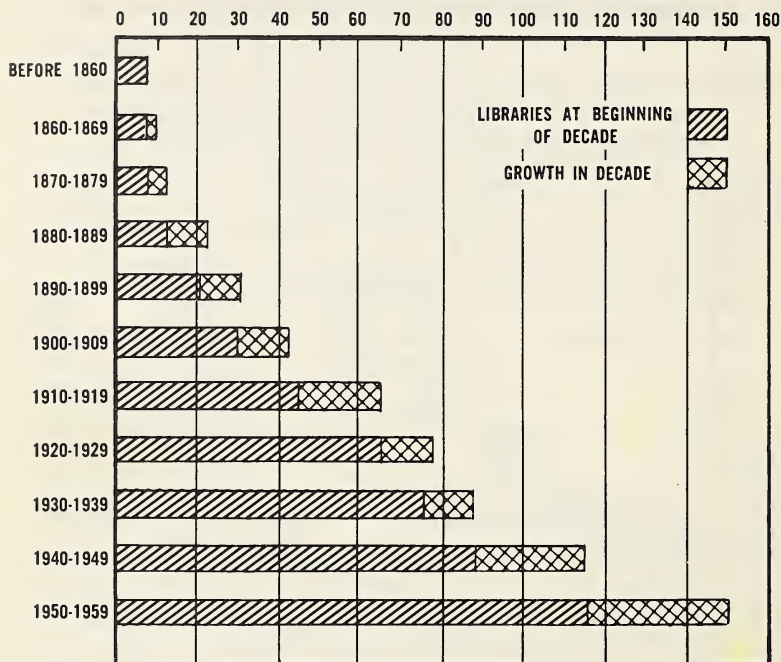


Source: Dominion Bureau of Statistics.

Example: The growth of public library services in the last one hundred years is shown in Graph 10 in ten year (one decade) spans.

Graph 10

**Growth of Public Library Service in Canada
During the Last 100 Years
NUMBER OF LIBRARIES**



Source: Dominion Bureau of Statistics.

WORKOUT EXERCISE II

1. Each dollar that a woman spends on apparel is distributed as follows: underwear 18%, dresses 16%, sportswear 14%, shoes 14%, coats and suits 10%, hosiery 6%, other accessories 5%, hand bags 5%, millinery 5%, all other 7%. Draw a horizontal bar graph to illustrate this information.

2. Each dollar that a man spends on apparel is distributed as follows: suits and coats 29%, shirts 16%, shoes 14%, trousers and slacks 12%, underwear 5%, hosiery 5%, nightwear 3%, ties 2%, all other 14%. Draw a vertical bar graph to illustrate this information.

3. Canada exports to more than 40 foreign markets approximately 1.7 billion dollars worth of newsprint each year supplying about half the world's needs. The total world needs and the United States imports, in thousands of tons, are given for five years. Draw a multi-column bar chart to illustrate this information:

	19-5	19-4	19-3	19-2	19-1
United States	5,670	5,445	5,426	5,426	4,936
World	8,586	8,346	8,303	7,794	7,473

4. Design a bar graph to show the total pounds of fish and their respective dollar value, taken from the following fishing grounds:

	<i>In thousands of pounds</i>	<i>In thousands of dollars</i>
Lake Ontario	1,969	\$ 381
Lake Erie	36,350	2,516
Lake St. Clair	982	207
Lake Huron	4,551	1,173
Georgian Bay	277	81
North Channel	237	72
Lake Superior	2,071	203
Other Inland Waters	7,932	1,003

Information from the Department of Lands and Forests.

What was the average price per pound obtained in each district?

5. From the following table of Canadian trade with the United Kingdom design a vertical bar chart showing exports above and imports below the x-axis, or to the left and right of the y-axis.

	<i>In thousands of dollars</i>	
	<i>Imports</i>	<i>Exports</i>
Agricultural products	\$ 38,014	\$283,234
Animals & animal products	18,380	38,717
Fibre & textile products	86,078	2,567
Wood products & paper	8,069	133,403
Iron & products	205,193	24,592
Metals & products	63,997	225,802
Minerals & products	30,113	16,280
Chemicals & products	23,553	35,752
Miscellaneous commodities	53,253	14,559

6. Draw a horizontal bar graph to illustrate the information given in Workout Exercise 1, problem 2.

7. Draw a component bar graph to illustrate the information given in Workout Exercise 1, problems 3 and 4.

8. Draw a component bar graph to show the total sales of shares sold by the Toronto and New York Stock Exchange for each day of the week from the information given in Workout Exercise 1, problem 5.

9. Draw a component bar graph to show total sales of shares and shares sold by Montreal, Vancouver, and American exchanges on each day of the week from information given in Workout Exercise 1, problem 6. Which of these two graphs you have drawn tells the clearest story?

10. Draw component bar graphs to show the sale of cigarettes by type for the following three selected years. Information is given in billions of cigarettes.

	Year 1	Year 2	Year 3
King size	71	115	85
Regular	365	315	155
Filter	25	35	210

SECTION 3 Rectangle or 100% Chart

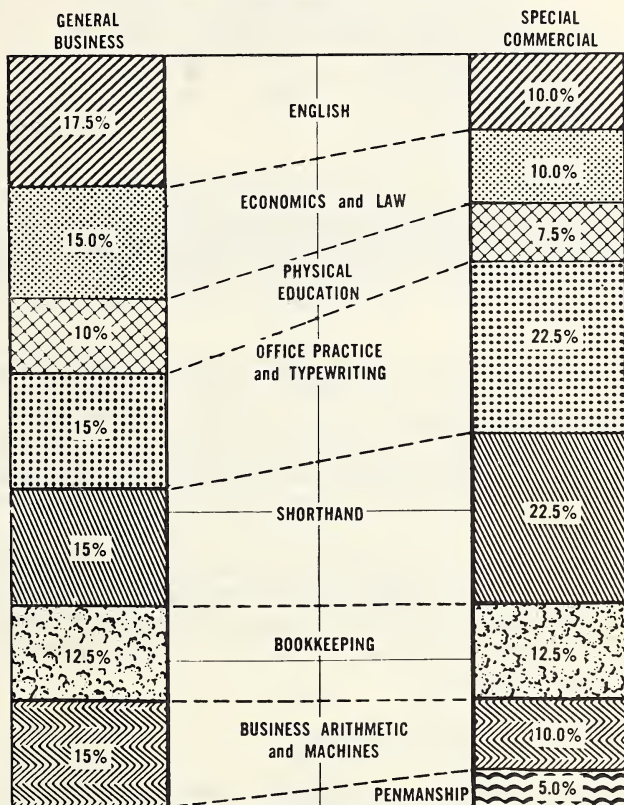
A rectangle chart is similar to a component bar graph but is usually expressed in percentages. In this way the relative values of the component parts of two or more complete units can be measured even though the units themselves may have different values. Or a single item in a group can be compared with the whole group.

Example: Compare the time spent in various studies in a one-year special commercial course with the final grade general business course if the respective curricula are as follows:

	<i>General Business</i>		<i>Special Commercial</i>	
	<i>Periods</i>	<i>Per- centage</i>	<i>Periods</i>	<i>Per- centage</i>
English, Correspondence	7	17.5	4	10.0
Economics and Law	6	15.0	4	10.0
Physical Education	4	10.0	3	7.5
Office Practice, Typewriting	6	15.0	9	22.5
Shorthand	6	15.0	9	22.5
Bookkeeping	5	12.5	5	12.5
Business Arithmetic & Business Machines	6	15.0	4	10.0
Penmanship	—	—	2	5.0
	<u>40</u>	<u>100.00</u>	<u>40</u>	<u>100.0</u>

Graph 11

**Comparison of Special Commercial High School Course
With General Business High School Course**



Source: Department of Education, Ontario.

WORKOUT EXERCISE III

1. Draw a rectangle or 100% chart to illustrate the distribution of a dollar spent by women on apparel and by men on apparel found in Workout Exercise II, problems 1 and 2.
2. Fish caught in Canada by commercial fishermen number in thousands of pounds, according to the source: Lake Ontario 1,969; Lake Erie 36,350; Lake St. Clair 982; Lake Huron 4,551; Georgian Bay 277; North Channel 237; Lake Superior 2,071; and other inland waters 7,932. Bring each of these figures to a percentage of the whole catch, and illustrate the information by means of a rectangle graph.

3. Educational attainment in terms of years of school completed for 1950, 1960, and estimated for 1970 is given below:

	1950	1960	1970 <i>projected</i>
Elementary: Under 5 years	11%	8%	6%
5-8 years	37	31	24
High School: 1-5 years	38	45	52
College: 1-3 years	8	8	9
4 years and over	6	8	9

Draw three rectangle charts to compare these figures.

4. In three selected years the sale of cigarettes by type, in billions of cigarettes, was: King size 71, 115, 85; Regular 365, 315, 155; Filter 25, 35, 210. Bring these figures to percentages of the yearly sales for each of the three years, and compare the distribution by means of a 100% graph.

5. The media used for business advertising is divided as follows: newspapers 31%, direct mail 15%, television 13%, magazines 8%, business papers 5%, radio 6%, other 22%. Draw 100% graphs to indicate this information. If the television advertising is divided into network 7%, spot 4%, local 2%; and the radio advertising into network 1%, spot 2%, local 3%, compare these two sources by supplementary graphs.

6. A portion of our income is spent in purchasing services. This income is estimated to be spent as follows: housing 32%, medical 11%, utilities 10%, transportation 8%, recreation 5%, domestic service 3%, clothing 5%, personal business 15%, other 11%. Draw a 100% graph to show this distribution.

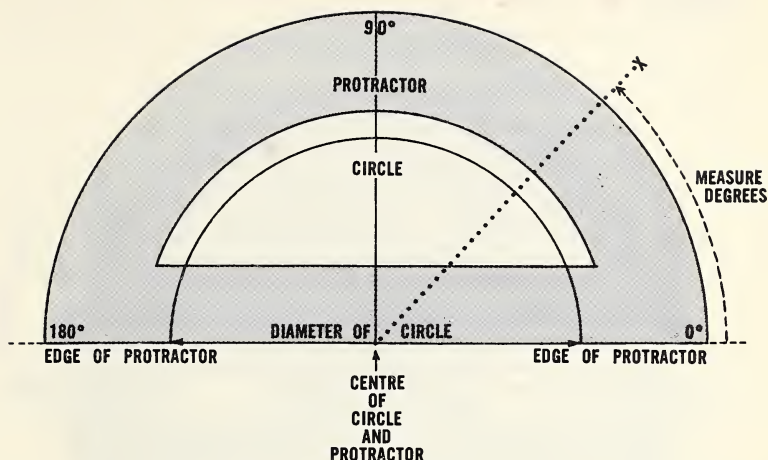
SECTION 4 Circle Graphs

Pie chart is the popular name for a circle graph. It is used to show the relationship of a single part to the whole. For example, it is often used to show the way in which a sales dollar is broken up into expenses and profit.

Once again, we must list all the component parts of the whole and calculate them as percentages of the whole. We then divide our pie or circle into one hundred (per cent) pieces and divide these pieces in the same percentage ratio as the component parts. Alternatively, a protractor may be used. A circle is 360° ; so 100% is represented by 360° and $1\% = 3.6^\circ$.

Use of Protractor:

Place centre of protractor on centre of circle with edge of pro-



tractor lying along diameter of circle; measure required number of degrees on protractor scale and mark. Join centre of circle to this mark and extend the line to cut the circumference of the circle. This will be the required piece of pie.

Example: So-Rite Limited showed a profit last year of \$50,000. If their cost of sales amounted to \$300,000 and their net sales were \$500,000, show by means of a pie chart the distribution of the sales dollar.

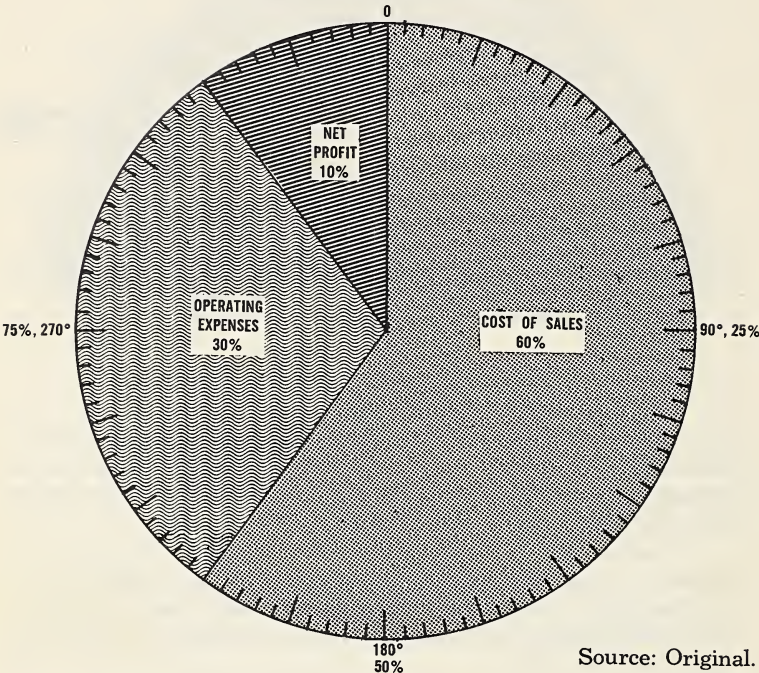
	Amount	Percentage	Degrees
Sales	<u>\$500,000</u>	<u>100%</u>	<u>360</u>
Cost of sales	<u>300,000</u>	<u>60%</u>	<u>216</u>
Operating expenses	<u>150,000</u>	<u>30%</u>	<u>108</u>
Net profit	<u>50,000</u>	<u>10%</u>	<u>36</u>

See Graph 12.

Frequently, the largest piece of pie commences at 12 o'clock, although this rule is not rigidly adhered to; it depends what you are trying to emphasize. The various segments should be colored or shaded. Hatching is very effective. Each segment should be labelled with the name and the percentage. Frequently, the actual figures are also included. The pie can also be drawn with one or more pieces separated from the main pie. When drawn in three dimension, this can be very striking.

Graph 12

So-Rite Limited
Distribution of Sales Dollar



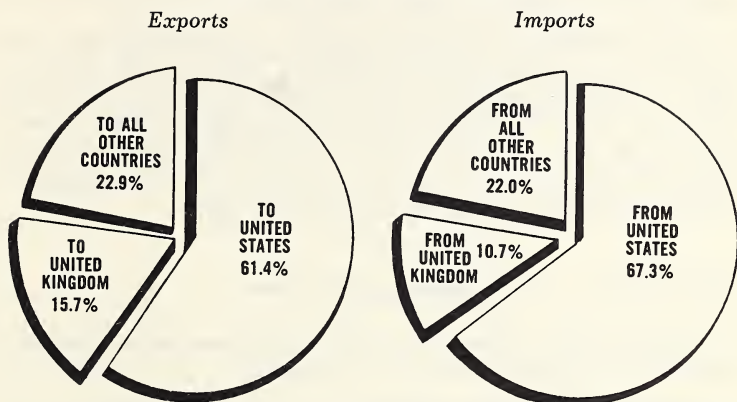
Source: Original.

Example: Canadian exports in a recent year totalled \$5,021.9 million exported to: United States, \$3,083.5 million; United Kingdom, \$785.8 million; all other countries, \$1,152.6 million. Canadian imports amounted to \$5,508.7 million imported from: United States \$3,709.1 million; United Kingdom, \$588.6 million; all other countries \$1,211.0 million. Draw two pie graphs showing these imports and exports.

Exports			Imports	
	Amount	Percentage	Amount	Percentage
United States	\$3,083.5	61.4	\$3,709.1	67.3
United Kingdom	785.8	15.7	588.6	10.7
Other Countries	1,152.6	22.9	1,211.0	22.0
	<u>\$5,021.9</u>	<u>100.0%</u>	<u>\$5,508.7</u>	<u>100.0%</u>

Graph 13

Canadian Exports and Imports



Source: Dominion Bureau of Statistics.

WORKOUT EXERCISE IV

1. Draw a pie chart to show how a woman distributes each dollar she spends on apparel if it is divided as follows: dresses and sportswear 30%, underwear 18%, shoes and hosiery 20%, coats and suits 10%, handbags and accessories 10%, millinery 5%, miscellaneous 7%.
2. Draw a pie chart to show how a man distributes each dollar he spends on apparel if it is divided as follows: suits and coats 29%, shirts 16%, underwear 8%, shoes and hosiery 19%, trousers and slacks 12%, ties 2%, miscellaneous 14%.
3. The total population of Canada is divided by activity as follows: 36% employed, 2% unemployed, 19% keeping house, 23% in school, 12% pre-school, 2% in the armed forces, 6% in other spheres.
 - (a) Draw a pie graph to illustrate this information.
 - (b) From the latest available information find the total population of Canada and draw up a table to show the numerical division by activity.
4. Assume that the average household spends \$4,000 per annum divided as follows: food \$1,000, medical \$220, recreation \$210, transportation \$580, clothing \$480, home operation \$740, appliances \$330, miscellaneous? Reduce each figure to a percentage and draw a graph to illustrate this division of expenditure.
5. From your school life or home life calculate some original data, such as the division of your 24 hour day, and draw a pie graph to show this.

6-9. Draw pie graphs for problems 3, 4, 5, and 6 in Workout Exercise III. Compare these graphs to the 100% graphs. Which do you think the most effective in each case? State your reasons.

10. The distribution of the dollar spent for drugs and personal care products in the average family is divided as follows: Drugs 59%, divided as follows: medication 22%, prescriptions 20%, health aids 10%, baby needs 7%; Personal care 41%, divided as follows: hair products 9%, shaving products 6%, cosmetics 6%, toiletries 15%, hygiene 5%. Draw a pie graph to show this information making a break to distinguish between drugs and personal care figures.

SECTION 5 Pictographs or Pictograms

Pictographs are an extension of the bar graph and lend themselves to ingenuity, unusual treatment, and eye appeal.

Self-explanatory symbols are used, with one symbol representing a designated number of the items depicted. A change in number is represented by more or fewer symbols, not by a difference in size of symbol. Pictographs do not show details but help to make comparisons of approximate quantities.

Figure 4-2



Let each aeroplane represent 1,000 aeroplanes.

Then one-half aeroplane would represent 500 aeroplanes.

Pictograms are usually arranged in horizontal rows, with the exception of money which may be 'stacked' vertically.

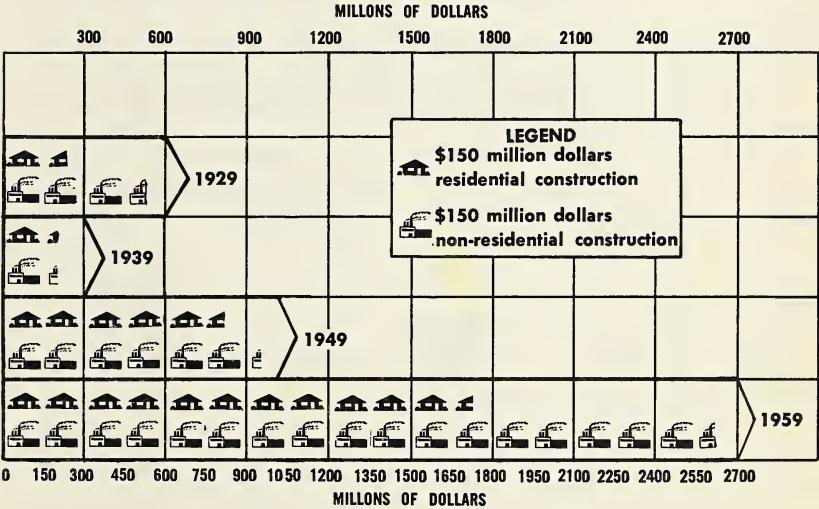
Figure 4-3

Some Symbols Used For Pictograms



Example: New construction in Canada, in millions of dollars, amounted to:
1929: residences, \$230; non-residences, \$490;
1939: residences, \$174; non-residences, \$164;
1949: residences, \$794; non-residences, \$920;
1959: residences, \$1,743; non-residences, \$2,592.
Source: Dominion Bureau of Statistics.

Graph 14
Comparing New Construction in Canada
For Selected Years



Source: Dominion Bureau of Statistics.

SECTION 6 Pitfalls to Avoid in Preparing Graphs

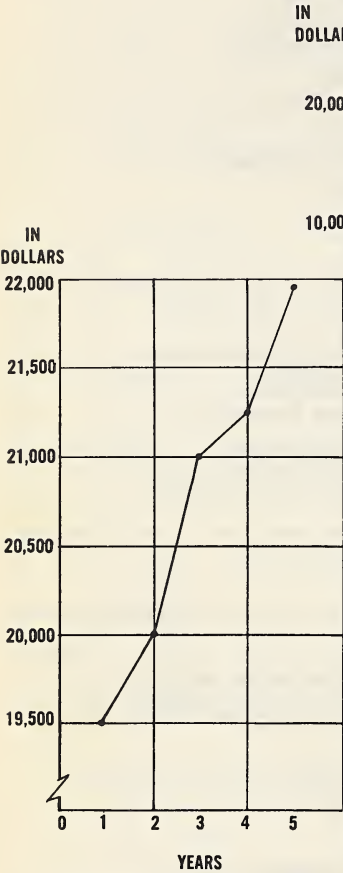
A. Misrepresentation by Exaggeration of Scale

The vertical and horizontal scales must be chosen so that there is no distortion, deliberate or otherwise, of the original data.

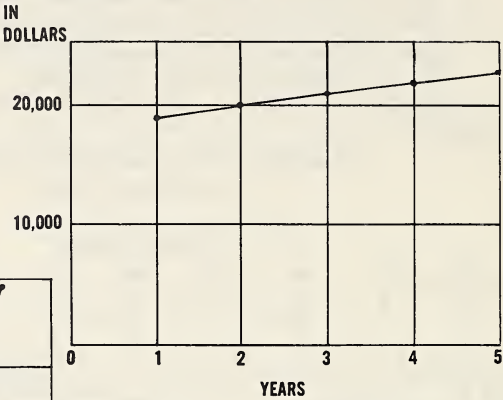
Example: The net profit of the Bubble and Squirt Syphon Company for the past five years was: \$19,500; \$20,000; \$21,000; \$21,225; and \$22,000.

Graph 15

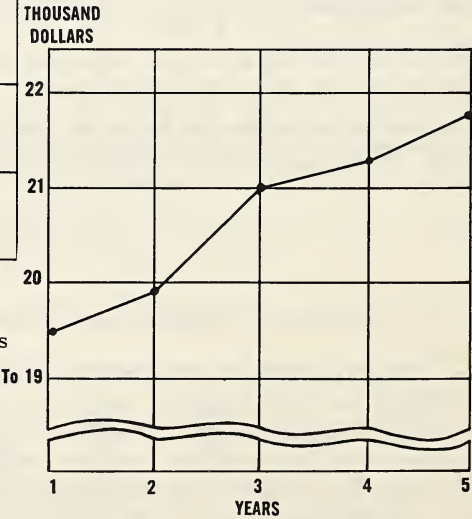
The Bubble and Squirt Syphon Company
Record of Net Profit
For the Past Five Years



(a) Too steep and appears too optimistic.



(b) Too flat and indicates little change.

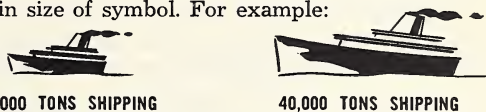


(c) More indicative of an approximate 13% increase over a five year period.

B. Incorrect Use of Symbols

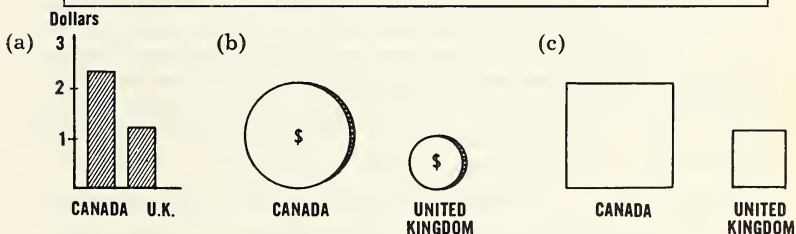
Many illustrations will be seen where the difference in number is shown by a difference in size of symbol. For example:

Figure 4-4



This is not a true graph and its use should be avoided because the viewer will be confused as to whether the basis of comparison is length, height, or volume. Even if the basis of comparison is given, it is difficult to compare the apparent size. If the above two ships compare one year's shipping with that of another year, it could be interpreted that the ships themselves were larger, which might or might not be the case, and has no bearing on the data being compared.

Example: A recent survey indicated that the average wage paid to Canadian labour was \$2.25 per hour, whereas the average wage paid to labour in the United Kingdom was \$1.20 per hour.



(a) represents the facts better than (b) or (c).



(d) is a better pictogram than (e).

WORKOUT EXERCISE V

1. Draw a pictogram to show the enrolment of boys and of girls in your school according to:

- the number of boys and the number of girls in each grade,
- the number of boys and the number of girls in each course offered by your school.

2. Draw a pictogram to depict the number of fish caught commercially in the waters given in Workout Exercise II, problem 4.
3. Choose one problem from each exercise in this chapter and compile a pictogram to illustrate the information. Is this as effective as the original graph you drew? Give your reasons for your choice.
4. Find some original material from the sports field that you can depict by means of a pictogram.
5. Discuss the value of graphic presentation in business reports pointing out the disadvantages as well as the advantages.

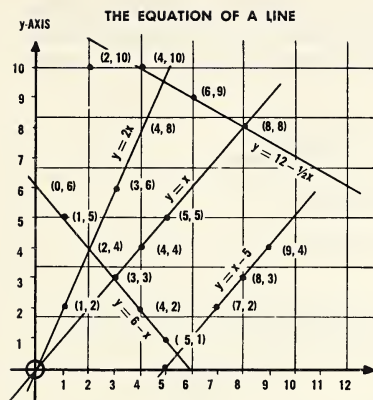
SECTION 6 Figuring for Fun

1. When Torbin returned to school in September, he said he had caught the largest fish in the class. He claimed it had a head 5 inches long and a tail as long as its head and half of its body. Its body, he claimed, was as long as its head and tail. How long was the fish Torbin caught?
2. A two-table bridge party was made up of Mr. and Mrs. Waterhouse, Mr. and Mrs. Cromwell, Mr. and Mrs. Ottman, and Mr. and Mrs. McGraw. No husband and wife were partners. The Waterhouses and the McGraws sat at different tables. Mr. McGraw played with his daughter, who was sitting next to her husband. Mrs. Waterhouse played with her father, who sat next to Mrs. Cromwell. Draw up the two bridge tables showing the person sitting in each seat.
3. Make up your own substitute alphabet to send secret messages. Here is one way you can make one which is difficult to "crack": Start with any word that has no letter repeated in it. Fill in the squares with the rest of the alphabet, number each column at random, and then copy down the columns in order from top to bottom under the regular alphabet.

1	4	6	3	2	7	5
P	R	O	B	L	E	M
A	C	D	F	G	H	I
J	K	N	Q	S	T	U
V	W	X	Y	Z		

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z — regular

P A J V L G S Z B F Q Y R C K W M I U O D N X E H T — cypher



Algebra and its Allies

5

A Business Problem

The Spin-a-Web Knitting Company manufacture Jer-C-Spin Swim Suits. They have found that their fixed costs for manufacturing and selling these suits amount to \$150,000 a year irrespective of the amount of sales they have. They have further expenses which vary according to the amount of sales. These total one-tenth of their sales volume.

The company have asked you to show them graphically how much their total expenses will amount to for the following sales volumes: \$200,000, \$250,000, \$300,000, \$350,000 and \$400,000. (Hint: let y = total expenses and x = sales volume. Then $y = \text{fixed expenses} + \frac{1}{10}x$.)

They would also like you to tell them at what volume of sales they will start to lose money. (Profit = Sales — Total Expense.)

Algebra is the shorthand of arithmetic. We make letters and symbols represent numbers and then apply our calculations to these numbers or symbols. An understanding of the fundamentals is essential in order to interpret the formulae, graphs, and tables found in all business records. It enables us to solve complicated problems by the use and understanding of equations. It is, in fact, one of the most useful tools of the mathematician and the business man.

SECTION 1 Simple Equations

An equation is the statement that one algebraic expression equals another and is formed when two quantities or expressions are equal in value and are connected by the sign $=$ meaning "is equal to".

<i>The left side of the equation</i>	$=$	<i>The right side of the equation</i>
A	$=$	B
5	$=$	5
$2x + 3x$	$=$	$5x$

If we have a problem to solve, we frequently call the unknown quantity in the problem 'x'. 'x' is then an *algebraic symbol*, and we are employing algebra to solve our problem. The expression $2x$ is understood to mean $2 \times x$.

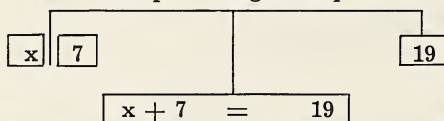
Example: 7 added to what number will give 19?

Required to find the unknown number.

So we let 'x' = the unknown number.

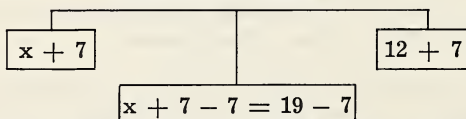
Then $x + 7 = 19$.

Figure 5-1 A Balance Representing the Equation $x + 7 = 19$



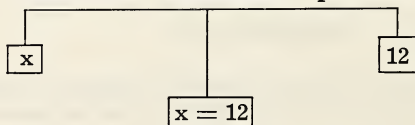
If we represent this equation by weights on either side of a balance, to solve the equation, we need only find what number would just balance x . If we remove 7 from both sides, our balance will still balance, and we shall have our unknown x alone on one side. In equation solving we always isolate the unknown on the left-hand side of the equation.

Figure 5-2 Isolate the Unknown



On the left hand side 7 is first added to, then subtracted from, the unknown number, leaving the unknown number alone. On the right hand side 7 is subtracted from 19, leaving 12. The result is shown in figure 5-3.

Figure 5-3 Perform the Mathematical Operations

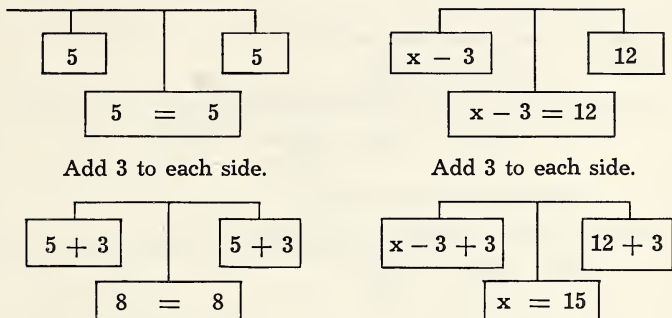


The answer 12 is called the *root* of the equation $x + 7 = 19$.

In solving this equation we have made use of a fundamental truth which we call an *axiom*. We have subtracted the same number (7) from both sides of the equation but have retained the equality of the two sides. There are four axioms used to solve equations.

Axiom Number One: *The equality of the two sides of an equation will be maintained if the same number is added to both sides.*

Figure 5-4 Add the Same Number to Both Sides of an Equation



Example: (Axiom One.) Solve the equation and prove the answer:

$$x - 8 = 12$$

$$x - 8 = 12$$

Add 8 to both sides (Axiom One).

$$x - 8 + 8 = 12 + 8$$

$$\therefore x = 20$$

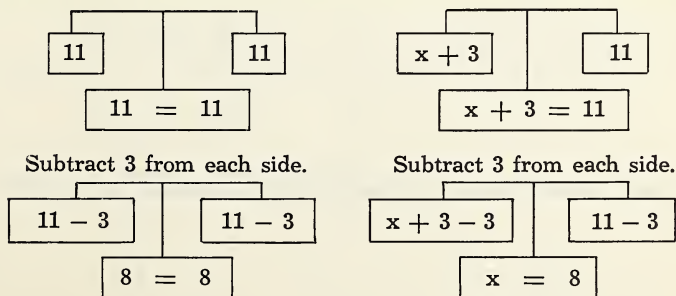
Proof: Substitute $x = 20$ in original equation.

$$\text{Then } 20 - 8 = 12$$

$$12 = 12$$

Axiom Number Two: *The equality of the two sides of an equation will be maintained if the same number is subtracted from both sides of an equation.*

Figure 5-5 Subtract Same Number from Both Sides of an Equation



Example: (Axiom Two.) Mabel has saved \$1.21 toward a new record which will cost \$2.89. How much more must she save in order to purchase this record?

Let x = the amount required.

Then $x + \$1.21 = \$2.89$

Subtract \$1.21 from each side (Axiom Two).

$$\therefore x + \$1.21 - \$1.21 = \$2.89 - \$1.21$$

$$\therefore x = \$1.68$$

$\therefore$ Mabel still requires \$1.68.

Proof: Substitute \$1.68 in the original equation:

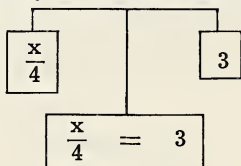
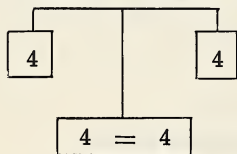
$$\$1.68 + \$1.21 = \$2.89$$

$$\therefore \$2.89 = \$2.89$$

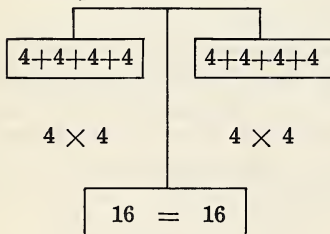
Axiom Number Three: *The equality of the two sides of an equation will be maintained if both sides are multiplied by the same number.*

Figure 5-6

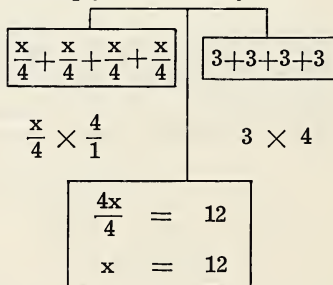
Multiply Each Side of an Equation by the Same Number



Multiply each side by four.



Multiply each side by four.



Example: (Axioms One and Three.) Solve the equation $\frac{3}{5}x - 4 = 17$.

$$\frac{3}{5}x - 4 = 17$$

Add 4 to each side (Axiom One).

$$\therefore \frac{3}{5}x - 4 + 4 = 17 + 4$$

$$\therefore \frac{3}{5}x = 21$$

Multiply both sides by $\frac{5}{3}$ (Axiom Three).

$$\therefore \frac{5}{3} \times \frac{3}{5}x = \frac{21}{1} \times \frac{5}{3}$$

$$\therefore x = 35$$

Proof: Substitute $x = 35$ in original equation:

$$\frac{3}{5} \times \frac{35}{1} - 4 = 17$$

$$17 = 17$$

Axiom Number Four: *The equality of the two sides of an equation will be maintained if both sides are divided by the same number.*

Figure 5-7 Divide Each Side of an Equation by the Same Number

$$\begin{array}{c} \boxed{33} \qquad \boxed{33} \\ \hline \boxed{33 = 33} \end{array}$$

Divide both sides by 3.

$$\begin{array}{c} \boxed{\frac{33}{3}} \qquad \boxed{\frac{33}{3}} \\ \hline \boxed{11 = 11} \end{array}$$

$$\begin{array}{c} \boxed{3x} \qquad \boxed{27} \\ \hline \boxed{3x = 27} \end{array}$$

Divide both sides by 3.

$$\begin{array}{c} \boxed{\frac{3x}{3}} \qquad \boxed{\frac{27}{3}} \\ \hline \boxed{x = 9} \end{array}$$

Example: (Axioms Two and Four.) Solve the equation $17 + 3x = 26$

$$17 + 3x = 26$$

Subtract 17 from each side (Axiom Two).

$$17 - 17 + 3x = 26 - 17$$

$$\therefore 3x = 9$$

Divide each side by 3 (Axiom Four).

$$\therefore \frac{3}{3}x = \frac{9}{3}$$

$$\therefore x = 3$$

Example: Peter has P marbles, and David has D marbles. Peter has 6 more than three times as many marbles as David.

- (a) Express this information as an equation.
 (b) If Peter has 12 marbles, how many marbles has David?

- (a) Three times David's marbles = $3D$

An amount six more than this = $3D + 6$

$$\therefore P = 3D + 6$$

- (b) As D is the unknown we have to find, we use the form

$$3D + 6 = P.$$

Subtract 6 from both sides:

$$3D + 6 - 6 = P - 6$$

$$\therefore 3D = P - 6$$

Substitute 12 for P.

$$\therefore 3D = 12 - 6 = 6$$

Divide each side by 3.

$$\therefore D = 2$$

$\therefore$ David has 2 marbles.

Like Terms

If Tom has 6 apples, Paul has 2 apples, and Charles has 3 apples, the boys together have:

$$6 + 2 + 3 = 11 \text{ apples.}$$

If we let the letter a = apple, then the equation would be:

$$6a + 2a + 3a = 11a.$$

These are *like terms* as they all involve the same unknown a. Like terms can therefore be added together. Before we solve an equation involving like terms, we must first add the like terms.

Example: Solve $6x - 4 + 3x - 2x = 10$.

Collect like terms. $\therefore 7x - 4 = 10$

Add 4 to each side. $\therefore 7x - 4 + 4 = 10 + 4$

$$\therefore 7x = 14$$

Divide each side by 7. $\therefore x = 2$

WORKOUT EXERCISE I

Solve the following equations using axiom one and check each result:

- | | | |
|---------------------------------------|-------------------------------------|--------------------------------------|
| 1. $t - 6 = 8$ | 2. $x - 12\frac{1}{2} = 25$ | 3. $y - 3\frac{1}{4} = 3\frac{1}{4}$ |
| 4. $a - 4.5 = 9.3$ | 5. $b - \frac{1}{2} = 7\frac{1}{2}$ | 6. $z - 5\frac{1}{4} = 3\frac{3}{4}$ |
| 7. $x - 6.5 = 3.4$ | 8. $d - \frac{7}{8} = 1$ | 9. $x - 21 = 176$ |
| 10. $b - 4\frac{1}{4} = 5\frac{1}{4}$ | | |

Solve the following equations using axiom two and check each result:

- | | | |
|--------------------------------------|-----------------------------|----------------------|
| 11. $x - 6 = 10$ | 12. $A + 3 = 16\frac{1}{2}$ | 13. $P + 2.5 = 7.5$ |
| 14. $y + \frac{3}{4} = 2\frac{1}{2}$ | 15. $7 = d + 2$ | 16. $z + 8.1 = 17.3$ |
| 17. $x + 1.33 = 4.13$ | 18. $4.6 = y + 3.1$ | 19. $a + 1.8 = 2.2$ |
| 20. $V + 0.8 = 1.1$ | | |

Solve the following equations using axioms three and four and check each result:

- | | | |
|-------------------------------------|--------------------------|-------------------|
| 21. $2y = 36$ | 22. $\frac{4}{5}x = 44$ | 23. $7a = 70$ |
| 24. $\frac{y}{3} = 21$ | 25. $4\frac{1}{4}d = 51$ | 26. $8.3A = 24.9$ |
| 27. $\frac{V}{10} = 7.34$ | 28. $\frac{7}{8}P = 9.1$ | 29. $65 = 13s$ |
| 30. $4\frac{1}{2}b = 29\frac{3}{4}$ | | |

Find the root of the following equations and check each result:

- | | | |
|-------------------------|--------------------------|-------------------------|
| 31. $4x = 16$ | 32. $8T = 32$ | 33. $17 = 5d$ |
| 34. $3.5a = 17.5$ | 35. $8\frac{1}{4}y = 33$ | 36. $\frac{1}{3}p = 11$ |
| 37. $\frac{4}{5}b = 32$ | 38. $4.25s = 17$ | 39. $\frac{x}{3} = 7$ |
| 40. $\frac{1}{3}h = 5$ | | |

Solve and verify:

- | | |
|---|----------------------------|
| 41. $4x + 5x = 45$ | 42. $16y - 36 = 4y$ |
| 43. $7d - 3 + 4d = 30$ | 44. $3A - 2 = 7$ |
| 45. $6d - 3 = 3$ | 46. $4s - 4 + 3s - 3 = 14$ |
| 47. $3B + 7 = 22 + 9$ | 48. $6T - 8 = 4T + 10 - T$ |
| 49. $9P - \frac{3}{4}P + 10 = 1\frac{1}{2}P + 20 - 3$ | |
| 50. $1\frac{1}{6}b + \frac{5}{12}b + 1 = 15 - b$ | |

Transposition of Terms

Consider axiom one, figure 5-4.

Solve $x - 3 = 12$.

Add 3 to each side, but perform the left side addition mentally.

$$\therefore x = 12 + 3$$

Consider axiom two, figure 5-5.

Solve $x + 3 = 16$.

Subtract 3 from each side, but perform the left side subtraction mentally.

$$\therefore x = 16 - 3$$

If you are very observant, you will be able to see an important rule resulting from axioms one and two. In each of the above cases the term 3 has been *transposed*, that is, moved, from one side of the equation to the other. But in each case what happened to the + or - sign? Look again at the two examples, and you will understand the following rule which is just a short cut for performing axiom one or axiom two.

We can transfer any term of an equation from one side to the other if we change the sign of the term.

We normally transfer the unknown terms to the left side of the equation and the known numbers to the right side of the equation.

Example: Solve and verify: $17 - 3y = 47 - 13y$

$$17 - 3y = 47 - 13y$$

Transpose + 17 to the right side and change to - 17.

Transpose - 13y to the left side and change to + 13y.

$$\therefore 13y - 3y = 47 - 17$$

$$\therefore 10y = 30$$

$$y = 3$$

Proof: Substitute $y = 3$ in the original equation.

$$17 - 3y = 47 - 13y$$

$$\therefore 17 - (3 \times 3) = 47 - (13 \times 3)$$

$$\therefore 17 - 9 = 47 - 39$$

$$\therefore 8 = 8$$

WORKOUT EXERCISE II

Solve by transposing unknown terms to the left side and known numbers to the right side. Verify all answers.

1. $2 + 5x = 4 - x$

2. $16f - 3 = 8 - 5f$

3. $3 + 2y = 19 - 6y$

4. $32p - 72 = 8p$

5. $4L - 4 + 3L - 3 = 14 + 2L$

6. $6d - 8 = 4d + 10 - d$

7. $1\frac{1}{6}x + \frac{5}{12}x + 1 = 15 + x$

8. $50 - m + 16m = 43 - 10m + 57$

9. $18 + p + 18p - 4p = 38 + 5p$

10. $4.5a - 3.1a + 67 = 116 - 8.4a$

11. $4.9x - 18.1 = 3.8x - 2.7$

12. $2\frac{1}{2}y + y + 4\frac{1}{4} = \frac{1}{2}y + 24 + 1\frac{1}{4}$

13. $\frac{y}{4} + 12y - 16 = 8y + 18$

14. $3.5 + 4.2d = 4.9 - 3.3d + 1.6$

15. $18 = 4n + 2$

16. $7\frac{1}{2}m - 3 = 2\frac{1}{2}m + 27$

17. $17.8 = 9t - 0.2$

18. $\frac{1}{2}g + \frac{1}{4}g + \frac{1}{3}g + 8 = 80$

19. $10N - 2N - 16 = 3N + 4$

20. $5p - 3p + 8 = 50 + 2p$

SECTION 2 Problem Solving with Simple Equations

Algebraic equations will help to solve many problems more quickly than is possible by arithmetic means. There are a few basic steps which should be followed for successful problem solving.

Steps to Solve Algebraic Problems

1. Read the problem slowly two or three times and understand what is required.
2. When helpful, draw a diagram and label it with the given facts.
3. Write down what facts are given, what you are looking for and what relationship exists between the two.
4. Let the unknown be represented by a letter, such as x .
5. Using the same letter, express all the given facts in terms of the same algebraic symbol.
6. Write the equation expressing the given relationship.
7. Solve the equation and give the solution.
8. Verify the answer by substituting this value in the original problem and solving arithmetically.

Example: Sylvia saves $\frac{1}{3}$ of her weekly allowance for her holidays. Last week she spent \$6.00; how much is her weekly allowance?

Let x = weekly allowance in dollars.

Then $\frac{2}{3}x$ = amount of allowance spent.

$$\therefore \frac{2}{3}x = 6$$

$$\therefore x = 9$$

$\therefore$ Sylvia's weekly allowance is \$9.00.

$$\text{Proof: Sum saved} = \frac{1}{3} \times \frac{9.00}{1} = \$3.00$$

$$\text{Total allowance} = \$6.00 + \$3.00 = \$9.00$$

WORKOUT EXERCISE III

In each of the following problems, let x = the unknown number, form the equation, solve the equation, state the answer, and verify.

1. If 6 is added to a certain number, the result is 14.
2. I am thinking of a number. If I double the number and add 7, the result is 5.3. Find the number.
3. 13 less than eight times a certain number is 19.
4. Ronald has saved \$85 less than four times the amount Paul has saved. Ronald has saved \$15. How much has Paul saved?
5. Last Saturday Graham spent 74 cents more than three times the amount Sally spent. Graham spent \$1.25. How much did Sally spend?
6. What number increased by 11 is 57?
7. A number increased by one-third that number is 36. What is the number?
8. Hazel is 3 times as old as Harry. The sum of their ages is 16 years. Find the ages of Hazel and Harry.
9. Sylvia is 4 times as old as Peter. The difference in their ages is 18 years. Find the ages of Sylvia and Peter.
10. Louise and Roland received 144 marks between them on a mathematics test. If Louise had twice as many marks as Roland, how many marks did each have?
11. Samuel put half his weekly pay in the bank. He spent \$12 and still had \$17 left. How much weekly pay did Samuel receive?
12. Abie needed \$60 to buy a bicycle. His father promised that if Abie doubled his present savings he would give him the last \$12.50. How much had Abie saved?
13. Bubbles Bimeby sells cosmetics and is paid a commission of $\frac{1}{4}$ of her total sales. Last month she earned \$372.50. What were her total sales?
14. Alice has three times as many plums as Penelope, and Liz has two more than four times as many plums as Penelope. If the three girls have 34 plums between them, find how many each girl has.
15. Peter has a penny less than Sally, and Sally has a penny less than Colin. If their total wealth is \$2.88, how much does each have?
16. Lorraine bought a pencil, a ballpoint pen, and a notebook. She paid three times as much for the pen as for the pencil, and 1 cent more than four times as much for the notebook as for the pencil. If she spent 25 cents for the three items, how much did each item cost?

SECTION 3 Powers, Exponents and Roots
Powers and Exponents

Let us consider the product of x times x . This can be written as:

$$x \times x$$

If we want to find the product of x times x times x , we can write:

$$x \times x \times x,$$

and so forth. However if we want to find the product of x multiplied by x twenty-five times, this method would become rather tedious and a time and space waster. So again we employ a shorthand form to express this particular mathematical operation.

x times x becomes x^2 = second power of x
(commonly called x squared).

x times x times x becomes x^3 = third power of x
(commonly called x cubed).

x multiplied by x , 25 times becomes x^{25} = 25th power of x .

If $x = 3$, we can write $3^4 = 81$.

This is just a special way of writing the product of four 3's. Three is called the *base*, four indicates the number of times 3 is taken and is called the *exponent*, and 81 is the product or the fourth power of 3, or

$$\text{Base}^{\text{Exponent}} = \text{Product}.$$

The exponent is also called the *index*, but note it is not the power. The product is a power (indicated by the value of the exponent) of the base.

When no exponent is expressed, the exponent 1 is implied, i.e.,

$$b^1 = b,$$

$$4^1 = 4.$$

The first power of b is b ; the first power of 4 is 4.

The exponent always refers only to the number or to the letter immediately to the left of the exponent.

$$xy^2 = x^1y^2 = x.y.y$$

$$abc^3 = a^1b^1c^3 = a.b.c.c.c.$$

Multiplication Signs

When two letters or a letter and a number are multiplied together, the multiplication sign can be replaced by a dot or eliminated altogether.

$$x^4 = x \times x \times x \times x = x.x.x.x = xxxx$$

$$2x^2 = 2 \times x \times x = 2.x.x = 2xx$$

$$a^3 = a \times a \times a = a.a.a. = aaa$$

$$\text{But } 3a = a + a + a$$

Brackets

If the exponent appears outside a bracket, it indicates that the whole contents of the bracket must be raised to the required power. First

perform the operations required inside the bracket; then raise the result to the required power.

$$(3 + 4 + 6)^2 = (13)^2 = 169$$

Examples: Express 16 as a power of 2.

$$2^4 = 16$$

Write $t \times t \times t \times t \times t$ in the shortest way.
 $= t^5$

Express in the shortest form:

$$x + x + x + y + y = 3x + 2y$$

$$x \times x \times y \times y = x^2 y^2$$

$$(2 + 3 + 1)^3 = (6)^3 = 216$$

Given $x = 1$, $y = 2$, $z = 3$, find the quotient when

$x^2 + y^2 + z^2$ is divided by $6x + y^2 - z$.

Substituting the values given:

$$\begin{aligned} x^2 + y^2 + z^2 &= (1 \times 1) + (2 \times 2) + (3 \times 3) \\ &= 1 + 4 + 9 \\ &= 14 \end{aligned}$$

$$\begin{aligned} 6x + y^2 - z &= (6 \times 1) + (2 \times 2) - (1 \times 3) \\ &= 6 + 4 - 3 \\ &= 7 \end{aligned}$$

$$\therefore \frac{14}{7} = 2$$

Roots

One of the equal factors of the product is called the root, i.e., x^4 , x is the 4th root of x^4 because $x \cdot x \cdot x \cdot x = x^4$,

$3^2 = 9$, 3 is the second, or square, root of 9 because $3 \times 3 = 9$,

$4^3 = 64$, 4 is the third, or cubed, root of 64 because $4 \times 4 \times 4 = 64$.

Radical Sign

We have a symbol to indicate a root $\sqrt{\quad}$ called the radical sign; thus

$\sqrt[3]{64}$ is read, the cube root of 64.

$\sqrt{9}$ is read, the square root of 9.

$\sqrt[25]{x}$ is read, the 25th root of x .

Note: In the case of the square root, the 2 is usually omitted.

If we are asked to solve $\sqrt[3]{64}$, it means we have to find one of the *three equal factors* of 64: $4 \times 4 \times 4 = 64$; that is, 4 is the cube root of 64. The 3, indicating the root to be found, is called the *index*. If no index is present, the square root is indicated.

Examples: Write the following in symbol form:

the 81st root of y	$\sqrt[81]{y}$
the 4th root of 10,000	$\sqrt[4]{10,000}$
the square root of 64	$\sqrt{64}$
the cube root of 64	$\sqrt[3]{64}$
the 10th root of x	$\sqrt[10]{x}$

WORKOUT EXERCISE IV

- Find the values of (a) $2^3 \times 3^2$, (b) $11^2 \times 9^3$, (c) $2^3 \times 4^2 \times 5^2$.
- Calculate (a) $5^3 \times 4^2 \times 2^3$, (b) $8^2 \times 7^3 \times 3^4$, (c) $2^8 \times 5^3 \times 9^2$.
- Express in the shortest form possible:
 - $b + b + b$,
 - $g + g + g + g$,
 - $b \times b \times b$,
 - $g \times g \times g \times g$,
 - $b \times b \times g \times g$.
- What is the shortest way of writing:
 - $x + x + x + x$,
 - $x + y + x + y$,
 - $x \times x \times y \times x \times y$,
 - $a \times x \times a \times x \times a \times x$,
 - $x.x.x.x.y$.
- Express (a) 64 as a power of 4, (b) 81 as a power of 3, (c) 125 as a power of 5, (d) 10,000 as a power of 10.
- Express (a) 729 as a power of 3; (b) 3,125 as a power of 5; (c) 1,000,000 as a power of 10.
- If $a = 4$, what is the difference in value between $3a$ and a^3 ?
- If $b = 6$, what is the difference in value between the square of $4b$ and 4 times b squared?
- If 1 boy can deliver 1 batch of handbills in 1 hour: (a) How many batches could 4 boys deliver in 3 hours? (b) How many batches could x boys deliver in 5 hours? (c) How many batches could x boys deliver in y hours?
- Given $x = 5$, $y = 1$, $z = 2$, find the product of $x^2 + y^2 + z^2$ and $x^2 + y^2 - z^2$.
- Given $a = 4$, $b = 3$, $c = 2$, find the quotient of $a^2 + b^2 + c^2$ divided by $a + b + c$.
- If $s = 2$ and $t = 3$, how much greater is the value of $3s^2t^2$ than $s^3 + t^3$?
- Write, using exponents: (a) x taken as a factor 3 times, (b) y taken as a factor 10 times, (c) a taken as a factor x times, (d) b taken as a factor c times, (e) four times p taken as a factor 5 times.

14. Calculate (a) $3^2 \cdot 4^2$; (b) $2^3 \cdot 3^3$; (c) $4^3 \cdot 6^2 \cdot 3^4$; (d) $8^3 \cdot 7^3 \cdot 6^3 \cdot 5^3$.
15. If your allowance in Grade 9 is \$ x a week and it is doubled each year, how much will you have received in four years?
16. What is a root? Read the following: $\sqrt{9}$, $\sqrt[10]{10}$, $\sqrt[3]{64}$, $\sqrt[x]{y}$, $\sqrt[4]{32}$, $\sqrt[5]{3,125}$, $\sqrt{81}$, $\sqrt[3]{27}$, $\sqrt[3]{216}$. How many of the above can you solve?
17. What is the difference between 3^2 and 2^3 , 5^4 and 4^5 ?
18. What is the value of x when: (a) $4^x = 16$; (b) $3^x = 27$; (c) $2^x = 64$; (d) $5^x = 125$?
19. Solve: (a) $\sqrt[3]{8^2}$; (b) $\sqrt{16^3}$; (c) $\sqrt[4]{1,296}$; (d) $\sqrt[3]{\frac{32}{4}}$;
(e) $\sqrt{5^2 - 3^2}$; (f) $\sqrt{\frac{125}{5}}$.
20. Solve: (a) $\sqrt[3]{27}$, (b) $\sqrt{144}$, (c) $\sqrt[4]{625}$, (d) $\sqrt{121}$, (e) $\sqrt{4^2 + 3^2}$,
(f) $\sqrt{\frac{36}{3}}$, (g) $\sqrt[4]{\frac{243}{3}}$, (h) $\frac{\sqrt{5^2 - 4^2}}{3}$.
21. Express the following facts in algebraic symbols: (a) Tom's age 15 years from now; (b) The number of weeks in p years; (c) The total cost of 16 pencils at m cents each and n pens at 35 cents each; (d) The age of Peter if Peter is 7 years younger than Anne; (e) $7\frac{1}{2}$ times Carol's age is 15.

SECTION 4 Addition and Subtraction

Term

A *term* is an algebraic expression which is not separated by $+$ or $-$ signs, such as $6b$, $2ay$, $7xyz$.

$$6b = 6 \cdot b = 6 \times b$$

$$2ay = 2 \cdot a \cdot y = 2 \times a \times y$$

$$7xyz = 7 \cdot x \cdot y \cdot z = 7 \times x \times y \times z$$

A *like term* is a term which contains the same components; $2ay$, $6ay$, $\frac{1}{4}ay$ are like terms, but ay , ax , xy are unlike terms.

Addition

As in arithmetic, like terms may be added together. Johnny has 3 apples, 2 bananas and 3 oranges. Mary has 2 apples, 3 bananas and 4 oranges. Paul has 1 apple, 2 bananas and 2 oranges. We can add these up:

	Apples		Bananas		Oranges
Johnny	3	+	2	+	3
Mary	2	+	3	+	4
Paul	1	+	2	+	2
	6	+	7	+	9

Therefore altogether there are 6 apples + 7 bananas + 9 oranges. We cannot add them any further unless we give them some common name, such as fruit. If we now let a = apple, b = banana, and c = orange, we can express this algebraically:

$$3a + 2b + 3c; 2a + 3b + 4c; 1a + 2b + 2c.$$

Collect and add like terms:

3a	+	2b	+	3c
2a	+	3b	+	4c
1a	+	2b	+	2c
6a	+	7b	+	9c

We cannot add unlike terms; therefore we can only indicate the result of our addition by using the plus sign: $6a + 7b + 9c$.

Examples: Add: $7x, 8y, 7d, 4x, \frac{1}{2}y, 10x, 11d, 4\frac{1}{2}d, 7y, 10y$.

Collect like terms:

$$\begin{aligned} 7x + 4x + 10x &= 21x \\ 8y + \frac{1}{2}y + 7y + 10y &= 25\frac{1}{2}y \\ 7d + 11d + 4\frac{1}{2}d &= 22\frac{1}{2}d \\ \therefore 21x + 25\frac{1}{2}y + 22\frac{1}{2}d &\text{ is the answer.} \end{aligned}$$

Add: $7x + 2y + 3xy; 4x + 3y + xy; 10x + 11y + 9xy;$
 $9x + 13y + 7xy$. For proof, let $x = 1, y = 2$.

				Proof							
7x	+	2y	+	3xy	7	+	4	+	6	=	17
4x	+	3y	+	xy	4	+	6	+	2	=	12
10x	+	11y	+	9xy	10	+	22	+	18	=	50
9x	+	13y	+	7xy	9	+	26	+	14	=	49
30x	+	29y	+	20xy	30	+	58	+	40	=	128
$\therefore \text{Answer} = 30x + 29y + 20xy$											

Subtraction

In algebra like terms may also be subtracted from each other in the same way they are in arithmetic. If you have 10 records and give away 3, you have left:

$$\begin{array}{r} 10 \text{ records} \\ - 3 \text{ records} \\ \hline 7 \text{ records} \end{array}$$

Or, expressed algebraically: $10r - 3r = 7r$

Example: Subtract $6x + 2y + 3z$ from $9x + 4y + 5z$.

$$\begin{array}{r}
 9x + 4y + 5z \\
 6x + 2y + 3z \\
 \hline
 3x + 2y + 2z
 \end{array}$$

Proof: Let $x = 1$, $y = 2$, $z = 3$; then substituting:

$$\begin{array}{r}
 9 + 8 + 15 = 32 \\
 6 + 4 + 9 = 19 \\
 \hline
 3 + 4 + 6 = 13
 \end{array}$$

WORKOUT EXERCISE V

Add and prove the answer:

1.	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	6a	17b	11.9x	$21\frac{1}{2}xy$	2.14r	$2\frac{7}{8}t$	8.76m
	5a	21b	17.4x	$7\frac{1}{4}xy$	0.01r	$8\frac{7}{16}t$	4.98m
	4a	14b	21.9x	$15\frac{3}{4}xy$	5.76r	$9\frac{7}{32}t$	7.89m

2.	(a)	(b)	(c)
	6a + 2b	2.5x + 9.11y	71.9s + 13.8t
	8a + 7b	7.6x + 2.09y	10.8s + 8.7t
	4a + 11b	17.9x + 11.78y	17.7s + 9.6t

(d)	(e)
17.9m + 11.2n	312.7f + 1.01g + 7.92h
71.7m + 2.9n	333.3f + 83.02g + 19.79h
38.76m + 2.0n	743.5f + 29.03g + 17.34h

- 6y, 10x, 4z, 8z, 9y, 7x, 11x, 4y, 7y, 18z.
- 7b, 18a, 4.7c, 11d, 3.5a, 7.4d, 11.3c, 9.7b, 8.11d.
- 4ac, 10cd, 18ac, 4bc, 17bc, 43ac, 19cd, 17cd, 4ac, 15bc, 7cd, 21bc.
- 18s, $4\frac{1}{2}t$, $11\frac{1}{4}r$, $7\frac{1}{2}r$, $9\frac{1}{4}s$, $14\frac{1}{8}t$, $17\frac{3}{8}s$, $4\frac{3}{4}r$, $9\frac{1}{4}t$, $21\frac{1}{2}t$, $18\frac{1}{4}r$, $7\frac{3}{4}s$.
- 98.34de, 101.47fg, 47.38ef, 78.72de, 121.79ef, 103.81ef, 13.78de, 42.79fg, 731.21de, 231.47ef, 85.38fg.
- Write the algebraic expressions for:
 - 6 times a number plus 7 times another number;
 - Four times a number plus the product of two other numbers;
 - Add to a first number 5 times a second number divided by two, plus five times a third number divided by 3;
 - Add to half the product of two numbers one third the product of one of these numbers with a third number and $\frac{1}{4}$ the product of the other number with the third number.

9. Subtract and prove the answer:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
$7m$	$10\frac{1}{2}x$	$101xy$	$9.6s$	$40\frac{1}{4}p$	$147.95y$	$11.72d$
$3m$	$9x$	$79xy$	$3.2s$	$39\frac{3}{4}p$	$83.79y$	$10.73d$

10. Subtract and find the value when $x = 7$:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
$9x$	$101.8x$	$11\frac{1}{2}x$	$3.732x$	$18.31x$	$573\frac{1}{4}x$	$795\frac{1}{16}x$
$5x$	$79.9x$	$9\frac{1}{4}x$	$2.515x$	$17.84x$	$243\frac{3}{4}x$	$496\frac{1}{8}x$

11. Subtract and prove:

(a)	(b)	(c)
$8x + 9y$	$9x + 7y + 6z$	$11.2x + 7.3y + 6.9z$
$7x + 8y$	$9x + 7y + 6z$	$10.3x + 7.2y + 1.9z$
(d)		
$4\frac{1}{2}x + 7\frac{1}{2}y + 28z$		
$3\frac{7}{8}x + 6\frac{3}{4}y + 20z$		

12. What is the value of the following when $a = 4$, $b = 5$, and $c = 6$?

- $18a + 17b$ subtracted from $20a + 15\frac{1}{2}b$,
- $7ab + 2a + 5b$ reduced by $a + b + ab$,
- $15.8a + 17.3b + 11abc$ minus $6abc + 11.9a + 14.2b$,
- $(79.5a + 37.4b + 21.9a + 13.2b + 7abc)$ minus $(4abc + 14a + 18b + 3.5a + 7.6b)$.

13. A window drape is to be made from four widths of cloth. If three widths are respectively $4a + 5b$, $5a + 6b$, $6a + 4b$, inches, how many inches must the fourth width be in order to have the drape $20a + 18b$ inches wide?

SECTION 5 Multiplication and Division

MULTIPLICATION

Factors

$3xyz$ is an algebraic term. It is composed of the four *unlike factors* 3, x, y, and z, as

$$3xyz = 3 \times x \times y \times z$$

$$3xyz = 3yxz = 3zyx, \text{ etc.}$$

Note: The arrangement of the factors in the term has no effect on the result.

Numerical Coefficient: The factor '3' is called the numerical coefficient of the term $3xyz$.

Like Factors: x^3 is an algebraic term composed of three like factors, x, x, and x or $x \times x \times x$. $4x^2y^3$ is an algebraic term composed of six factors: two like factors of x, three like factors of y, and the numerical coefficient 4.

Unlike Factors: x and y are unlike factors; and if x is multiplied by y , unless we know the numerical value of x and y , we must leave the expression as $x \times y$ or xy .

Multiplication sign has priority of treatment.

John has 6 bananas and James has 3. Mary has twice the number of the total. This can be expressed as:

$$\begin{array}{rcl} 2 \times 6 + 2 \times 3 & = & 12 + 6 = 18 \\ \text{or,} \quad 2 \times 9 & = & 18 \end{array}$$

If the addition were performed first, this would be

$$2 \times \underline{6 + 3} = 2 \times 9 = 18, \text{ which is untrue.}$$

Expressed algebraically, let x = number of Mary's bananas:

$$\begin{aligned} b &= 1 \text{ banana} \\ \text{then, } x &= 2 \times 6b + 2 \times 3b \\ &= 12b + 6b \\ &= 18b \end{aligned}$$

Brackets

When brackets appear in any expression, they indicate that the contents of the brackets are to be considered as one unit and any indicated operation within the bracket must be performed before performing any indicated operation outside the bracket.

Examples: $8 + 3 \times 2 = 8 + 6 = 14$, perform multiplication first

But, $(8 + 3) \times 2 = 11 \times 2 = 22$, perform bracket operation first

$$\begin{array}{rcl} a + b \times c & = & a + bc \\ (a + b) \times c & = & c(a + b) \\ a + b \times c + d & = & a + bc + d \\ (a + b) \times (c + d) & = & (a + b)(c + d) \end{array}$$

Compound Multiplication

Compound multiplication can be performed in the same way as in arithmetic.

A Compound Quantity Multiplied by a Simple Quantity

<i>Arithmetic</i>	<i>Algebra</i>
Multiply: $4 \text{ ft.} + 2 \text{ in. by } 3$ $\begin{array}{r} 4 \text{ ft.} + 2 \text{ in.} \\ \times 3 \\ \hline 12 \text{ ft.} + 6 \text{ in.} \end{array}$	Multiply: $4f + 2i \text{ by } 3$ $\begin{array}{r} 4f + 2i \\ \times 3 \\ \hline 12f + 6i \end{array}$

or, $3(4 \text{ ft.} + 2 \text{ in.}) = 12 \text{ ft. } 6 \text{ in.}$

$3(4f + 2i) = 12f + 6i$

Note: '3' is one factor; $(4f + 2i)$ is the second factor. Therefore each term of the second factor must be multiplied by the first factor.

A Compound Quantity Multiplied by a Compound Quantity*Arithmetic*

Multiply 15 by 23:

15	1 (ten) + 5 (units)
23	2 (tens) + 3 (units)
45	3 (tens) + 15 (units)
30	2 (tens) ² + 10 (tens)
345	2 (hundreds) + 13 (tens) + 15 (units)
200	+ 130 + 15
	= 345

*Algebra*Multiply $(x + 5y)(2x + 3y)$:

$x + 5y$	Multiplicand
$2x + 3y$	Multiplier
$3xy + 15y^2$	Product of $(x+5y)$ and $3y$ or $3y(x+5y)$
$2x^2 + 10xy$	Product of $(x+5y)$ and $2x$ or $2x(x+5y)$
$2x^2 + 13xy + 15y^2$	Sum of the partial products when like terms are added

Examples: Multiply: $(2x + 3y)(3y + 4x)$

Multiplicand	$2x + 3y$	
Multiplier	$3y + 4x$	
	$8x^2 + 12xy$	$(2x + 3y) \times 4x$
	$6xy + 9y^2$	$(2x + 3y) \times 3y$
Product	$8x^2 + 18xy + 9y^2$	Add like terms

Multiply: $(2a + 4b + 3c)(3a + 2b)$

Multiplicand	$2a + 4b + 3c$	
Multiplier	$3a + 2b$	
	$4ab + 6bc + 8b^2$	$(2a + 4b + 3c) \times 2b$
	$6a^2 + 12ab + 9ac$	$(2a + 4b + 3c) \times 3a$
Product	$6a^2 + 16ab + 9ac + 6bc + 8b^2$	Add like terms

Simplify: $(x + y)(x + y)(x + y)(x + y)$
 $= (x + y)^4$, from Section 3.

WORKOUT EXERCISE VI

Find the product of:

1. $2x, 3x, 4y, 5y.$ 2. $4, 3a, 5b, 6a, 4b.$ 3. $4d, 8ac, 7a, 4c, 2ac.$

4. $3x, 4xy, 4y, 7x, 3y, 8xy.$ 5. $4s, 2s, 4t, 8t, 3s, 2t.$

6.
$$\begin{array}{r} 4a + 3b \\ \hline 6 \end{array}$$
 7.
$$\begin{array}{r} 10x + 3y \\ \hline 4 \end{array}$$
 8.
$$\begin{array}{r} 11x + 2t \\ \hline 11 \end{array}$$

9.
$$\begin{array}{r} 10d + 8e + 7f \\ \hline 4 \end{array}$$
 10.
$$\begin{array}{r} 21x + 2xy \\ \hline 2x \end{array}$$
 11.
$$\begin{array}{r} 7x + 2y \\ \hline 2y \end{array}$$

12.
$$\begin{array}{r} 7a + 2ab + 3b \\ \hline 3a \end{array}$$

Simplify:

13. $5(x + y) + 6(y + z) + 7(x + z)$

14. $4(x + 2y) + 11(3y + z) + 3(6x + 2z)$

15. $3(2x + 4y) + 3(4y + z) + 3(6x + 2z)$

16. $5x(x + 3y) + 2x(4y + z) + 2x(3x + 2z)$

17. $2ab(a + b + c) + 2bc(a + b + c) + 2ac(a + b + c)$

18. $a^2b(2a + 3b) + ab^2(3a + 4b) + ab(a^2 + b^2)$

19. $xy(x^2 + y^2) + x^2y(x + y) + xy^2(xy + y)$

Find the product of the following, and verify the answer:

20.
$$\begin{array}{r} x + 3 \\ \hline x + 2 \end{array}$$
 21.
$$\begin{array}{r} 2x + 3 \\ \hline 4x + 2 \end{array}$$
 22.
$$\begin{array}{r} 3y + 2 \\ \hline 4y + 3 \end{array}$$
 23.
$$\begin{array}{r} 8a + 4 \\ \hline 7a + 2 \end{array}$$

24.
$$\begin{array}{r} 4b + 1 \\ \hline 2b + 1 \end{array}$$
 25.
$$\begin{array}{r} 7d + 3 \\ \hline 4d + 7 \end{array}$$
 26.
$$\begin{array}{r} 2a + 3b \\ \hline 3a + 2b \end{array}$$
 27.
$$\begin{array}{r} 4x + 2y \\ \hline 3x + y \end{array}$$

28.
$$\begin{array}{r} 8x + 3y \\ \hline 3y + 8x \end{array}$$
 29.
$$\begin{array}{r} 4.5d + e \\ \hline d + 2e \end{array}$$
 30.
$$\begin{array}{r} 4d + 5e \\ \hline 5d + 4e \end{array}$$
 31.
$$\begin{array}{r} 6g + 11f \\ \hline 2g + 7f \end{array}$$

32.
$$\begin{array}{r} a + b + c \\ \hline a \end{array}$$
 33.
$$\begin{array}{r} x + y + z \\ \hline x + y \end{array}$$
 34.
$$\begin{array}{r} a + b + c \\ \hline a + b \end{array}$$
 35.
$$\begin{array}{r} x + y + z \\ \hline x + y + z \end{array}$$

36. Subtract $(x + 3)(x + 4)$ from $(x + 4)(x + 7)$. Prove and find the value when $x = 2$.37. From $(3a + 2)(5a + 7)$ subtract $(a + 1)(2a + 2)$. Prove and find the value when $x = 3$.38. Subtract the product of $2x + 1$ and $3x + 2$ from the product of $4x + 2$ and $3x + 4$. Prove and find the value if $x = 4$.39. Simplify $(x + 1)^2 + (x + 2)^2 + (x + 3)^2$ and verify.40. Simplify $(2 + a)^2 + 3(4 + a)^2 + (5 + a)^2$ and verify.41. Simplify $(x + y)^2 + 2(x + 1)^2 + 3(y + 2)^2$ and verify.

DIVISION

A Simple Quantity Divided by a Simple Quantity

Division can be performed in the same way as in arithmetic. x divided by y can be written: $x \div y$, $\frac{x}{y}$, or x/y .

Examples:

Arithmetic

$$16 \div 8$$

Cancel like factors in numerator and denominator.

$$\frac{2 \times 8}{8} = 2$$

$$32 \div 6$$

$$\begin{aligned} \frac{2 \times 16}{2 \times 3} &= \frac{16}{3} \\ &= 5\frac{1}{3} \end{aligned}$$

Algebra

$$10ab \div 2b$$

$$\frac{10 \times a \times b}{2 \times b} = 5a$$

$$20ab \div 8b$$

$$\begin{aligned} \frac{4 \times 5 \times a \times b}{4 \times 2 \times b} &= \frac{5a}{2} \\ &= 2\frac{1}{2}a \end{aligned}$$

WORKOUT EXERCISE VII

Simplify:

- | | | | | |
|-----------------------------|-----------------------------|---------------------------|------------------------------|--------------------------|
| 1. $\frac{3x}{9x}$ | 2. $\frac{6a}{6a}$ | 3. $\frac{10y}{5y}$ | 4. $\frac{12xy}{3x}$ | 5. $\frac{24x^2y}{12xy}$ |
| 6. $\frac{4ab}{12b}$ | 7. $\frac{7x^2y^2}{14xy}$ | 8. $\frac{15ab}{15a^2b}$ | 9. $\frac{21c^2d}{7cd^2}$ | 10. $\frac{4gh^2}{2gh}$ |
| 11. $\frac{100xyz}{10y}$ | 12. $\frac{44a^2b^2}{11ab}$ | 13. $\frac{8xyz}{4x^2z}$ | 14. $\frac{250ab^2d}{25abc}$ | |
| 15. $12m^2n \div 4mn$ | | 16. $169t^2s \div 13ts^2$ | | |
| 17. $119x^2y^3z \div 17xyz$ | | 18. $81a^2bd^2 \div 3abc$ | | |
| 19. $19cde^2 \div 13de$ | | 20. $11p^2r \div 5pr^2$ | | |

Divide:

- | | |
|--|---------------------------|
| 21. $64x^2y$ by $8xy$ | 22. $144x^2y^2$ by $12xy$ |
| 23. $21xyz$ by $21xyz$ | 24. $243c^2d$ by $27cd^2$ |
| 25. Find the quotient when $328a^2bc^3$ is divided by $16a^3bc$. | |
| 26. Find the quotient when $17.25p^2q^3x$ is divided by $4.25pqx^2$. | |
| 27. Find the quotient when $40.75xy^2z^2$ is divided by $3.25x^2yz$. | |
| 28. Abie Flyaway travels from Montreal to New York in $16a^2b^2$ hours. If the aeroplane travels at $4ab$ miles an hour, how many miles did he travel? | |
| 29. The cost of $8xy$ gallons of gasoline is $48x^2y$. How much per gallon does it cost? Check your answer if $x = 6\phi$ and $y = 2\phi$. | |

A Compound Quantity Divided by a Simple Quantity

Divide each term of the compound quantity by the divisor.

*Examples:**Arithmetic*

Divide 12 ft. 9 in. by 3.

$$\begin{array}{r} 3) \ 12 \text{ ft. } 9 \text{ in.} \\ \underline{4 \text{ ft. } 3 \text{ in.}} \end{array}$$

Divide 484 by 4.

$$\begin{array}{r} 4) \ 4 \text{ (hundreds) } 8 \text{ (tens) } 4 \text{ (units)} \\ \underline{1 \text{ (hundred) } 2 \text{ (tens) } 1 \text{ (unit)}} \end{array}$$

$$\text{Simplify: } \frac{3 \text{ lb. } 6 \text{ oz.}}{3}$$

$$\begin{array}{r} 3) \ 3 \text{ lb. } 6 \text{ oz.} \\ \underline{1 \text{ lb. } 2 \text{ oz.}} \end{array}$$

*Algebra*Divide $12a + 9b$ by 3.

$$\begin{array}{r} 3) \ 12a + 9b \\ \underline{4a + 3b} \end{array}$$

Divide $4a + 8b + 4c$ by 4.

$$\begin{array}{r} 4) \ 4a + 8b + 4c \\ \underline{1a + 2b + 1c} \end{array}$$

$$\text{Simplify: } \frac{3x + 6y}{3}$$

$$\begin{array}{r} 3) \ 3x + 6y \\ \underline{x + 2y} \end{array}$$

WORKOUT EXERCISE VIII

Simplify:

1. $3) \ 3x + 6y$

2. $2) \ 12ab - 6bd$

3. $4x) \ 24xy + 16xz$

4. $a^2) \ abc + a^2$

5. $x^2) \ x^2y - x^3z$

6. $3xy) \ 9x^2y^2 - 6xy$

7. $\frac{6x}{3}$

8. $\frac{4x^2 + 2x}{x}$

9. $\frac{xy + xz}{x}$

10. $\frac{ax + bx}{x}$

11. $\frac{6m^2 + 4n^2}{2}$

12. $\frac{24r^2 + 18s^2}{6}$

13. $\frac{49b - 28c}{7}$

14. $\frac{63p^2 - 35p}{9p}$

15. $\frac{36ab^2 - 48a^2b}{12ab}$

16. $\frac{xy + xz}{x} + \frac{yz + xy}{y} + \frac{xz + yz}{z}$

17. $\frac{2a^2 + 4a}{2a} - \frac{4a^2}{4a}$

18. $\frac{x^3 - x^2}{x} + \frac{x^2 - x}{x} + 3$

19. $\frac{(x + 2)(x + 3) + (x + 1)(x + 4)}{x}$

20. $\frac{(x^2 + x)(2x^2 + 3x) + (x^2 + x)(x^2 + 3x)}{x^2}$

21. $(6a + 12b - 18c) \div 6$

22. $(10x^2y + 20xy^2 + 30x^2y^2) \div 10xy$

23. $(39x^3 + 65x^2 - 13x) \div 13x$

$$24. \frac{35 - 63y}{7} + \frac{121 + 11y}{11}$$

$$25. \frac{by + bz}{b} + \frac{py + pz}{p}$$

26. A piece of pipe $48ab + 6ac$ feet long is to be covered with insulation. If each piece of insulation is $3a$ feet long, how many pieces will be required?

27. If $5a$ gallons of gasoline cost $\$(a^2b + ab^2)$, how much would 35 gallons cost?

28. The cost of manufacturing $50x$ widgets is $10(x^2 + xy)$. The raw material costs one quarter as much and the labour three times as much as the raw material. How much is the total cost per widget?

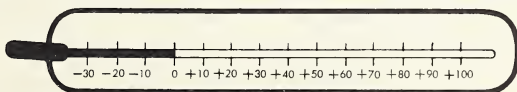
29. If $a = x^2 + x - 1$ and $b = 4x^2 - x + 1$, find the value of $\frac{a + b}{x}$ and $3a + 2b$.

SECTION 6 Signed Numbers

A signed number is a number that is preceded by a $+$ (plus) or a $-$ (minus) sign. A number preceded by a plus sign is a positive number; thus $+4$, $+1\frac{1}{2}$, $+6.5ab$ are positive numbers. A number preceded by a minus sign is a negative number; thus -1 , $-223\frac{1}{2}$, $-10x$ are negative numbers. If no sign is used, a positive sign is understood. Negative numbers, therefore, are opposite in sense to positive numbers. In Algebra the plus and minus signs are called *signs of quality* as a signed number indicates quality as well as quantity.

Examples: The daily weather bulletins give the range of temperatures across Canada and often for other parts of the world. These temperatures are quoted in degrees above zero and degrees below zero. For instance, White River might record a temperature of -20° F on the same day that Montreal records a temperature of $+20^\circ$ F. Both cities have the same numerical or *absolute* value but are opposite in quality, each lying 20° from the 0° mark but in the opposite direction.

The Centigrade thermometer registers the freezing point at 0° , and the positive numbers are above freezing temperatures, and the minus numbers, below freezing temperatures.



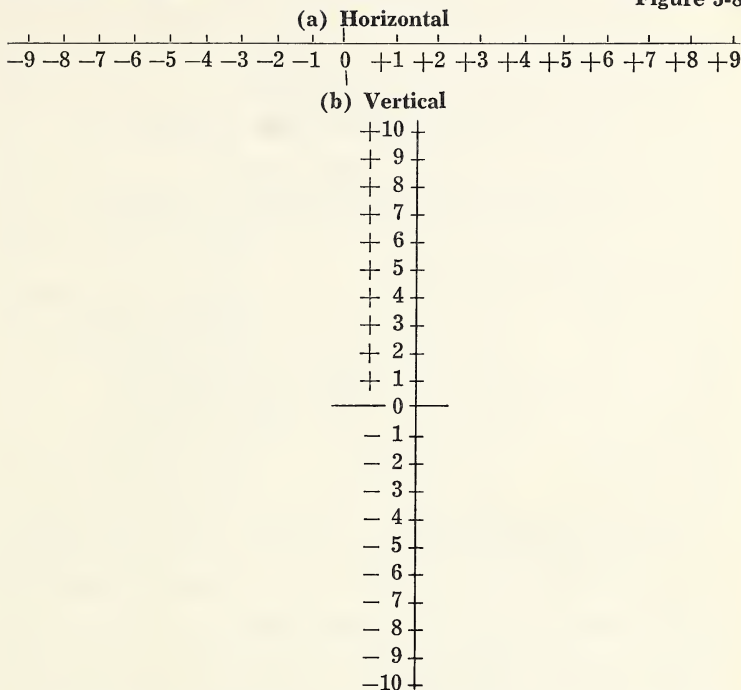
The Ups-and-Downs Trading Post reported the result of their year's trading last year as $+\$7,500$ and this year as $-\$1,900$. This indicates a profit last year of $\$7,500$ and a loss this year of $\$1,900$.

WORKOUT EXERCISE IX

1. Use signed numbers to indicate the following temperatures: (a) 15 degrees above zero, (b) 25 degrees below zero, (c) 18 degrees above zero, (d) 9 degrees above zero, (e) 60 degrees below zero, (f) 90 degrees above zero.
2. Sea level is represented by 0. All locations are represented as being located a number of feet above or below sea level. Mount Everest, the highest point on the earth's surface, is 29,002 feet above sea level. The lowest elevation is found in the Dead Sea at 1,292 feet below sea level. Express these elevations with signed numbers. From an atlas or encyclopedia find and express as signed numbers the elevations of the following places: Death Valley, the deepest part of Lake Superior, Sea of Gallilee, Longs Peak in the Rocky Mountains, Snowdon in Wales, the Matterhorn in the Alps, your own town.
3. The line of 0° of longitude passes through Greenwich, England. Locations east of Greenwich are designated as positive and locations west of Greenwich as negative. Berlin, Germany is $+14^\circ$; Dublin, Ireland, -6° ; and Wellington, New Zealand $+75^\circ$ of longitude. Find and express as signed numbers the degrees of longitude of the following places: Zanzibar, Lisbon, Peking, Moscow, Istanbul, Panama, New York City, Hamilton, Ontario, St. John's, Victoria, Calgary, Regina, Winnipeg, Quebec, Charlottetown, Halifax, Fredericton, Melbourne, Australia, your own town.
4. The line of 0° of latitude passes around the earth at the equator. All locations above the equator may be expressed as positive and all locations below the equator as negative. Berlin, Germany is $+52\frac{1}{2}^\circ$; Dublin, Ireland $+53\frac{1}{2}^\circ$; and Wellington, New Zealand -41° degrees of latitude. Find and express as signed numbers the degrees of latitude of the places indicated in question 3.
5. The normal summer level of Lake Wamasing is $+618$ feet. If the lake rises 3 feet above normal in the spring and drops to 4 feet below normal in the fall, what are the spring and fall heights? What is the difference between the two levels? If the water at your cottage is normally 5 feet deep 15 feet from shore, how high above the bottom of the lake would a dock need to be in order to be 1 foot above the water in the spring? How high is the dock above sea level?
6. The City Cab Co. have a dispatch office located 7 blocks north of the station. A cab leaves the office and drives 4 blocks south where you engage it. The cab then drives you to your home located 12 blocks north of the dispatch office. If a northward direction is positive, state where you engaged and dismissed the cab taking the dispatch office as the zero starting point.

The Number Scales

Figure 5-8



On these number scales, distances measured to the right or vertically upward from zero are positive, and distances measured to the left or vertically downward from zero are negative. Any point on these scales can be represented by a signed number which will indicate its direction from 0 as well as its magnitude or *absolute* value. The absolute value of a number is therefore its magnitude only without reference to its positive or negative quality.

Examples: In figure 5-8(a), what is the change and in what direction is the change from (i) -5 to $+5$; (ii) $+7$ to -4 ?

- (i) -5 to 0 is 5 units of positive change.
 0 to $+5$ is 5 units of positive change.
 $\therefore$ There is a total positive change of 10 points.
- (ii) $+7$ to 0 is 7 units of negative change.
 0 to -4 is 4 units of negative change.
 $\therefore$ There is a total negative change of 11 points.

In figure 5-8(b), what is the change and in what direction is the change from (i) -1 to $+9$; (ii) $+10$ to -3 ?

- (i) -1 to 0 is 1 unit of positive change.
 0 to $+9$ is 9 units of positive change.
 $\therefore$ There is a total positive change of 10 units.
- (ii) $+10$ to 0 is 10 units of negative change.
 0 to -3 is 3 units of negative change.
 $\therefore$ There is a total of 13 units of negative change.

WORKOUT EXERCISE X

1. From figure 5-8 indicate the amount and direction of change from:

- | | | |
|-------------------|-------------------|------------------|
| (a) -9 to $+9$ | (b) -1 to 0 | (c) $+8$ to -3 |
| (d) 0 to $+7$ | (e) -5 to $+3$ | (f) $+3$ to -6 |
| (g) -6 to $+3$ | (h) $+1$ to $+7$ | (i) 0 to -10 |
| (j) -8 to -3 | (k) -4 to $+3$ | (l) $+7$ to $+1$ |
| (m) $+9$ to -1 | (n) -3 to 0 | (o) -7 to -3 |
| (p) $+10$ to $+1$ | (q) -10 to $+9$ | (r) $+6$ to -4 |

2. Indicate by signed numbers:

- (a) assets of \$9.98, liabilities of \$5.75,
 (b) profit of \$13,000, loss of \$7,000,
 (c) bank balance of \$50, bank overdraft of \$20,
 (d) income of \$110, expense of \$56,
 (e) debt of \$23.50, credit of \$35.10,
 (f) bank deposit of \$450, bank withdrawal of \$150.

3. Indicate by a signed number the net change in the following instances:

- (a) The rugby team advances 20 yards and loses 15 yards.
 (b) Tel Fon Common shares rise 14 points and drop 11 points.
 (c) The traffic helicopter rises 500 feet and drops 200 feet.
 (d) A bank deposit of \$175, a withdrawal of \$200.
 (e) A debt of \$10, a credit of \$6.50.
 (f) A profit of \$8,750, a loss of \$7,450.

4. Draw number scales to represent the situations in problem 3 above.

Addition of Signed Numbers

When we add, we combine two or more numbers into a single number to represent the sum of the numbers added. The numbers being added can be either positive or negative, and the resultant sum can be a positive or negative quantity.

Numbers with Like Signs

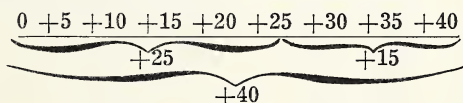
(i) Positive Numbers:

Example: Billy Boy has 25¢, represented by +25, in one pocket and 15¢, represented by +15, in another pocket, or a total of 40¢ in both pockets.

The addition may be written either horizontally or vertically:

$$\begin{array}{r} + 25 \\ + 15 \\ \hline + 40 \end{array} \quad \text{or} \quad (+25) + (+15) = +40$$

This can also be illustrated on a number scale:



We are adding *gains* together, and the result is a gain. Our rule is:
Positive numbers added together result in a positive number.

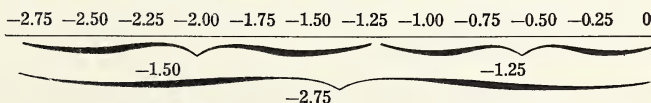
(ii) Negative Numbers:

Example: Mary Girl owes her mother \$1.25, represented by -1.25, and owes her brother \$1.50, represented by -1.50. Her total indebtedness is therefore -2.75 or \$2.75.

The subtraction may be written either vertically or horizontally:

$$\begin{array}{r} - 1.25 \\ - 1.50 \\ \hline - 2.75 \end{array} \quad \text{or} \quad (-1.25) + (-1.50) = -2.75$$

This can also be illustrated on a number scale:



We are adding losses together, and the result is a loss. Our rule is:
Negative numbers added together result in a negative number.

Example: Combine the following algebraic expressions:

(a) $+3b + 7b + 27b$

(b) $2a - 3b + 2c; -8b + 3a + 5c; 7c + 4a - 2b$

Check the addition by letting $a = 1, b = 2, c = 3$.

Check

(a) $+3b \quad 3 \times 2 = 6$

$+7b \quad 7 \times 2 = 14$

$+27b \quad 27 \times 2 = 54$

$+37b$ $37 \times 2 = 74$

(b) Arrange like terms in the same column.

Check

$+2a \quad -3b \quad +2c \quad +2 \quad -6 \quad +6 = +2$

$+3a \quad -8b \quad +5c \quad +3 \quad -16 \quad +15 = +2$

$+4a$ $-2b$ $+7c$ $+4$ -4 $+21$ $= +21$

$+9a$ $-13b$ $+14c$ $+9$ -26 $+42$ $= +25$

WORKOUT EXERCISE XI

1. Add:

(a) $+35$ (b) -18 (c) $-\$10$ (d) $+15$ marbles (e) -7° (f) $+2x$
 $+76$ -17 $-\$15$ $+25$ marbles -8° $+7x$

(g) $+4abc$ (h) $-13xy^2$ (i) $+7a$ (j) $-3ef$
 $+11abc$ $-21xy^2$ $+8a$ $-10ef$
 $-27xy^2$ $+15a$ $-27ef$

2. Illustrate the additions in problem 1 on number scales.

3. Combine the algebraic expressions:

(a) $2x + 3y + 4z; 8z + 2y + 4x; 7y + 3x + 5z$

(b) $-17r - 4s - 7t; -8s - 7t - 4r; -10t - 4s - 18r$

(c) $-14m + 10p + 17n; +13p + 25n - 11m; 21n - 34m + 27p$

(d) $-x^2y + xy^2; 2xy^2 - 3x^2y; -8x^2y + 7xy^2; +11xy^2 - 9x^2y$

(e) $-5e - 9f + 11g; -7f - 8e; +6g - 2e; +4g - 3f$

4. Prove the additions in problem 3 by assigning numerical values to the algebraic symbols.

Numbers with Unlike Signs

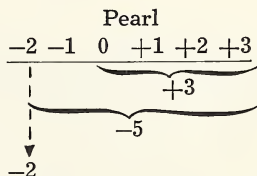
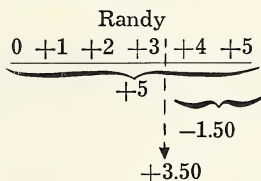
Examples: On Saturday Randy received his weekly allowance of \$5.00, represented by $+5.00$. However he owed his sister Sue \$1.50, represented by -1.50 . The result of combining or adding the two leaves Randy with \$3.50, or $+3.50$, or the difference between what he has and what he owes.

Pearl received her allowance of \$3.00, represented by $+3.00$, but owed her father \$5.00. As a result of combining or adding the two, she would still owe her father \$2.00, or -2.00 , or the difference between what she owes and what she has.

The above additions may be written:

$$\begin{array}{r} + 5.00 \\ - 1.50 \\ \hline + 3.50 \end{array} \qquad \begin{array}{r} + 3.00 \\ - 5.00 \\ \hline - 2.00 \end{array}$$

Again the number scale can be used to illustrate the above problems:



We can now formulate the rule: *To add two signed numbers with unlike signs, find the difference between the two numbers and attach the sign of the number with the greater absolute value.*

Examples: Add: (a) $4x - 5y$; $4y - 5x$; $6x - 6y$; $7y - 3x$

(b) $x^2 - 4x + 2$; $-4 + 3x^2 + 2x$; $3x - 5x^2 + 2$

Check by putting $x = 1$ and $y = 2$.

(a)

Check

$$\begin{array}{r} 4x - 5y \\ -5x + 4y \\ 6x - 6y \\ -3x + 7y \\ \hline 2x + 0 = 2x \end{array} \qquad \begin{array}{r} 4 - 10 = -6 \\ -5 + 8 = +3 \\ 6 - 12 = -6 \\ -3 + 14 = +11 \\ \hline +2 \end{array}$$

1st column: Add positive numbers: $4x + 6x = 10x$

Add negative numbers: $-5x - 3x = -8x$

Difference: $2x$; attach $+$ sign of larger figure.

2nd column: Proceed as above.

(b)

Check

$$\begin{array}{r} x^2 - 4x + 2 \\ 3x^2 + 2x - 4 \\ -5x^2 + 3x + 2 \\ \hline -x^2 + x + 0 \end{array} \qquad \begin{array}{r} -1 \\ +1 \\ 0 \\ \hline 0 \end{array}$$

WORKOUT EXERCISE XII

1. Add and illustrate on a number scale:

- (a) $8 - 3 + 2$ (d) $-7 + 3 + 4$ (f) $4 + 3 + 2$
 (b) $6 + 3 - 8$ (e) $-9 + 6 - 3 + 4$ (g) $4 + 3 - 6 - 2$
 (c) $-11 + 4 + 3$

2. Add:

- (a) $\begin{array}{r} + \$25 \\ - \$10 \\ \hline \end{array}$ (b) $\begin{array}{r} - 10x \\ + 5x \\ \hline \end{array}$ (c) $\begin{array}{r} + 7x^2 \\ - 8x^2 \\ \hline \end{array}$ (d) $\begin{array}{r} - 3.5xy \\ + 3.2xy \\ \hline \end{array}$ (e) $\begin{array}{r} - 14\frac{1}{2}f \\ + 15\frac{3}{4}f \\ \hline \end{array}$
 (f) $\begin{array}{r} + 18b \\ 0 \\ \hline \end{array}$ (g) $\begin{array}{r} - 3\frac{2}{5}t \\ + 5\frac{3}{5}t \\ \hline \end{array}$ (h) $\begin{array}{r} - 16R \\ + 27R \\ \hline \end{array}$ (i) $\begin{array}{r} + 41\frac{1}{2}x \\ - 32\frac{3}{4}x \\ \hline \end{array}$ (j) $\begin{array}{r} - 27n \\ + 17n \\ \hline \end{array}$

3. An elevator goes up 17 floors and deposits passengers; then, goes down 18 floors to pick up passengers. Show by addition how far the elevator is from the starting point. In what direction is the elevator from the starting point?

4. Add and check when $a = 1$, $b = 3$, $c = 2$.

$$\begin{array}{rcl} 3a - 2b + 5c & = & \\ - 6a + 3b - 4c & = & \\ - 5a - 3b + 2c & = & \end{array}$$

5. Add and check when $x = 0$, $y = 1$, $z = 2$.

$$\begin{array}{rcl} 4x - 3y + 6z & = & \\ - 9x + 4y - 9z & = & \\ - 5x + 7y + z & = & \end{array}$$

6. Add and check when $r = \frac{1}{4}$, $s = \frac{1}{2}$, $t = \frac{3}{4}$.

$$\begin{array}{rcl} - \frac{3}{4}r + 1\frac{1}{4}s + 2\frac{1}{2}t & = & \\ + 1\frac{1}{4}r - 2\frac{1}{2}s - 1\frac{1}{4}t & = & \\ + \frac{1}{4}r - 1\frac{3}{4}s + \frac{1}{2}t & = & \end{array}$$

7. Add and check when $x = 1$:

$$\begin{array}{l} -x + 3x^2 - 2; 4 - 3x + 5x^2; -6x^2 + 5 - 8x; 7x - 4x^2 + 2; \\ 2x^2 - 5 + 3x - x^2 \end{array}$$

8. Add and check when $a = 1$, $b = 2$, $c = 3$, $d = 4$:

$$\begin{array}{l} 2ab - a + c; 2a - 3ab + 2d + 4ac; 3ac + 2ab - a - c; \\ 3c + 2a - 4ab + 2ac \end{array}$$

9. Use signed numbers to compute Sarah's bank balance at the end of January. On January 1 she had a balance of \$3.50 on hand. During the first week she deposited \$15 and \$25 and withdrew \$18; the second week she deposited \$11 and withdrew \$12; the third week she deposited \$8 and \$7 and withdrew \$13; the fourth week she deposited \$18 and withdrew \$15.

10. Simplify: $x^2 - 3xy + y^2 + 2 + 4x^2y^2 - 5x^2 + 10x - 3 + x^2y^2 - 5xy + 3x + 8 - 6x - 4y^2 + 10x^2y^2 - 4xy + 5y^2 + 3x^2$

Subtraction of Signed Numbers

Let us suppose that Brian has 25 marbles and that Bill has no marbles and also owes 7 marbles to John. How many marbles must Bill obtain to own as many as Brian? If Brian's marbles are represented by $+25$ and Bill's by -7 , we must find the difference between $+25$ and -7 . In other words, we must subtract -7 from $+25$. We know that Bill must obtain 7 marbles to pay back John and an additional 25 to own as many as Brian; that is, he must obtain $25 + 7 = 32$ marbles. This can be stated as:

$$\begin{array}{r} + 25 = \\ \text{Subtract } - 7 \quad \text{Add } + 7 \\ \hline 32 \end{array}$$

Again, listening to the weather report, we hear the high temperature for the day is 15° . If the record low temperature for the same day is -10° , how many degrees must the mercury drop to record the low temperature? We must find the difference between -10 and $+15$ by subtracting $+15$ from -10 . We know that the temperature must drop 15° to 0° and then drop a further 10° to reach -10° or a total drop of -25° . This can be stated as:

$$\begin{array}{r} - 10 = \\ \text{Subtract } + 15 \quad \text{Add } - 15 \\ \hline - 25 \end{array}$$

These two examples illustrate our general rule for subtraction:

To subtract signed numbers, mentally change the sign of the subtrahend and add.

Examples: Subtract:

-9	$+7x$	$-11xyz$	$+8.5t$	$-15x^2y$
$+3$	$-2x$	$+8xyz$	$-3.7t$	$+16x^2y$
-12	$+9x$	$-19xyz$	$+12.2t$	$-31x^2y$
$6x - 4y$	$-4a + 2b$	$5bc - 6cd$	$-4de + 5ef$	
$-2x + 3y$	$-5a - 3b$	$8bc - 8cd$	$-6de + 6ef$	
$+8x - 7y$	$+a + 5b$	$-3bc + 2cd$	$+2de - ef$	

WORKOUT EXERCISE XIII

1. Subtract:

(a) $+4$	(b) $+6x$	(c) $-7y$	(d) $3.5a$	(e) $-3xy$
-2	$-4x$	$+4y$	$-2.6a$	$-4xy$
(f) $6abc$	(g) $-5x^2y$			
$14abc$	$5x^2y$			

2. Subtract:

$$\begin{array}{lll}
 \text{(a)} \quad \begin{array}{r} 6s - 4t \\ -5s + 3t \\ \hline \end{array} & \text{(b)} \quad \begin{array}{r} 2a - 3b + 4c \\ -5a + 6b - 7c \\ \hline \end{array} & \text{(c)} \quad \begin{array}{r} 9x^2 + 3xy - 4y^2 \\ 6x^2 - 3xy + 4y^2 \\ \hline \end{array} \\
 \text{(d)} \quad \begin{array}{r} 8mn - 2p^2 - 4q^2 \\ 12mn - 4p^2 - 6q^2 \\ \hline \end{array} & \text{(e)} \quad \begin{array}{r} 6d^2e - 4e^2f + 3f^2g \\ -6d^2e - 4e^2f - 3f^2g \\ \hline \end{array} &
 \end{array}$$

3. Subtract:

$$\begin{array}{lll}
 \text{(a)} \quad 6x \text{ from } -5x & \text{(b)} \quad -8ab \text{ from } +8ab & \text{(c)} \quad 3t^2 \text{ from } -6t^2 \\
 \text{(d)} \quad 11abc \text{ from } -15abc & \text{(e)} \quad 2x^2y^2 \text{ from } -6x^2y^2 & \text{(f)} \quad -4.5mn^2 \text{ from } 0 \\
 \text{(g)} \quad 0 \text{ from } -32pm & \text{(h)} \quad -16xyz \text{ from } -14xyz & \\
 \text{(i)} \quad 63p^2 \text{ from } 18p^2 & &
 \end{array}$$

Signed Brackets

As mentioned earlier, any expression contained within a bracket is considered as one unit, and the sign before a bracket is a sign of operation for the complete unit. The signs within the bracket are signs of quality of each component. By utilizing brackets, we can restate our subtraction problems. From the previous example:

$$\begin{array}{ll}
 \begin{array}{r} 7x \\ -2x \\ \hline 9x \end{array} & \text{or } 7x - (-2x) = 7x + 2x \\
 & = \underline{\underline{9x}} \\
 \begin{array}{r} 6x - 4y \\ -2x + 3y \\ \hline 8x - 7y \end{array} & \text{or } (6x - 4y) - (2x + 3y) \\
 & = 6x - 4y + 2x - 3y = \underline{\underline{8x - 7y}} \\
 \begin{array}{r} -4de + 5ef \\ -6de + 6ef \\ \hline 2de - ef \end{array} & \text{or } (-4de + 5ef) - (-6de + 6ef) \\
 & = -4de + 5ef + 6de - 6ef \\
 & = \underline{\underline{2de - ef}}
 \end{array}$$

When we subtract in columns, we must mentally change the sign of the subtrahend and add; when we use brackets to enclose the minuend and the subtrahend, in order to perform the necessary subtraction, we must remove the brackets and collect and add the like terms. When we remove the brackets from an expression, if the sign of operation before the bracket is a plus sign, the terms within the bracket retain their original sign of quality; but if the sign before the bracket is a minus sign, each sign of quality within the bracket must be changed. The rule can be stated: *Brackets preceded by a plus sign may be removed without a further change of signs. To remove brackets preceded by a minus sign, remove the brackets and the sign of operation and change each sign of quality within the brackets.*

When all brackets are removed, combine and add the like terms.

Examples: (a) $(4x - 3y) - (3y + 4x)$

$$= 4x - 3y - 3y - 4x$$

$$= -6y$$

(b) $(8x^2 - 4y^2 + 7z^2) - (-5x^2 + 9y^2 - 3z^2)$

$$= 8x^2 - 4y^2 + 7z^2 + 5x^2 - 9y^2 + 3z^2$$

$$= 13x^2 - 13y^2 + 10z^2$$

(c) $-(3a^2 + b^2) + (-8a^2 - 3b^2) - (10b^2 - a^2)$

$$= -3a^2 - b^2 - 8a^2 - 3b^2 - 10b^2 + a^2$$

$$= -10a^2 - 14b^2$$

(d) Simplify: $-(14s - 13t) - (-6t + 4s) + (-8s - 4t)$

$$= -14s + 13t + 6t - 4s - 8s - 4t$$

$$= -26s + 15t$$

WORKOUT EXERCISE XIV

Simplify:

1. $(a - 3b) + (4a + 2b)$
2. $(-x + 5y) - (4x - 10y)$
3. $-(4x^2 + 2y^2 - 8z^2) - (7x^2 - 2y^2 - 8z^2)$
4. $(-ab - bc - cd) + (3ab - 2bc - 4cd) - (-6ab - 3bc + 2cd)$
5. $6y - 2x - (7x - 2y) + 3x - (14x + 8y)$
6. $(-m^2 + mn + n^2) - (-2m^2 - mn + n^2) - 4mn$
7. $(rs - st - rt) - (-rs + st + rt) - rs$
8. $u^2 - (6w^2 - 2wx + x^2) - (-3w^2 + 8wx + 2x^2) + 3wx$

Affix the correct signs of quality in the brackets on the right hand side of the following equations:

9. $-x - y - 4a + 2b = (\quad x \quad y) + (\quad 4a \quad 2b)$
10. $7m - n + 3l + 4k + 6p = (\quad 7m \quad n) - (\quad 3l \quad 4k) + 6p$
11. $-ab + bc - 3ad + 4ac + d^2 = (\quad ab \quad bc) - (\quad 3ad \quad 4ac) + d^2$
12. Find the value of $(5a^2 - b + c) - (-2a^2 + 3b + 4c) + 2ac$, when $a = 3$, $b = 2$ and $c = 1$.
13. If $x = 2$ and $y = 3$, find the value of:
 - (a) $(x^2 - y^2) - (x + y) + (-x - y) - (3x + 2y) - (-4x - 2y)$
 - (b) $(4xy - 2x) - (3x^2 - 2x^2y^2) + 3xy - (2xy^2 - 2x^2y) - 5$
 - (c) $(-3y^2 - 7) - 0 - (8x^2 - 7y^2) + 8y - (-y^2 + 2xy) + 10$
14. Subtract:
 - (a) $4x^2 - 3x + 3$ from $5x + 10 - 3x^2$
 - (b) $8x - 10y + 3xy$ from $7x - 4xy + 11y$
 - (c) $17a^2 + 10b^2 - 8c^2$ from the sum of $4c^2 - 3b^2 + 6a^2$ and $9b^2 - 11c^2 + a^2$
 - (d) $\frac{1}{3}x^2 + \frac{2}{3}xy - \frac{1}{2}y^2$ from $-\frac{2}{3}x^2 + \frac{2}{3}y^2 - \frac{1}{2}xy$
15. If $a = x + y + z$, $b = 2x + 3y + z$, $c = 3x + 2y - 4z$, find the value of $a - b - c$ in terms of x , y and z .

Multiplication of Signed Numbers

Positive Numbers

Multiplication of positive numbers and algebraic expressions has been dealt with in Section 5.

Negative Numbers

Peter spends 25 cents a day on car fare, represented by -25 . In 6 days he will have spent 150 cents, as the sum of six -25 's is -150 .

This can be written:

$$6(-25) = -150 \quad \text{or,}$$

$$(-25) + (-25) + (-25) + (-25) + (-25) + (-25) = -150.$$

Or, we could say that 25 cents is subtracted six times from Peter's income. To subtract six 25 cent payments, we write

$$-6(+25),$$

$$\text{and } -6(+25) = -150, \quad \text{or}$$

$$-(+25) - (+25) - (+25) - (+25) - (+25) - (+25) = -150.$$

Following the rule for subtraction, we change the signs and add; therefore $-6(+25) = +6(-25) = -150$.

The above examples give us the rule: *If two numbers having unlike signs are multiplied, their product is negative.*

The Creaky Joint Manufacturing Company decide to reduce their staff by one member and in this way reduce their expenses by \$250 a month, and therefore save, or be ahead, \$3,000 in the year. To subtract the twelve \$250 reductions, we would write: $-12(-250)$

Again, following the rule for subtraction, we change the signs and add:

$$-12(-250) = +12(+250) = +3,000, \text{ or}$$

$$-(-250) - (-250) - (-250) - (-250) - (-250) - (-250) = +3,000$$

In Section 5 we found that when we multiply two numbers having positive signs, their product is positive. We can therefore form our second rule: *If two numbers having like signs are multiplied, their product is positive.*

Examples:

$(+4)(+5)$	$=$	20
$(+4)(-5)$	$=$	-20^*
$(-4)(+5)$	$=$	-20^*
$(-4)(-5)$	$=$	20
$(-5)(+4)(-5)$	$=$	100
$(+4)(-5)(+4)(-5)$	$=$	400
$(-5)(+4)(-5)(+4)(-5)$	$=$	$-2,000^*$
$(-5)(-4)(-5)$	$=$	-100^*

* Note: if the number of factors having a negative sign is odd, then the product is negative.

Examples: (a) Multiply: $(+2a^2b^2)(-3bc^2)$

$$\begin{aligned} (+2a^2b^2)(-3bc^2) &= -(2 \times 3 \times a \times a \times b \times b \times c \times c) \\ &= -6a^2b^3c^2 \end{aligned}$$

(b) Multiply $(x - 5y)(2x - 3y)$; verify when $x = 1, y = 2$.

Proceed as in compound multiplication, Section 5, following the rules for multiplying positive and negative signs just learned.

$$\begin{array}{r} x - 5y \\ 2x - 3y \\ \hline - 3xy + 15y^2 \\ 2x^2 - 10xy \\ \hline 2x^2 - 13xy + 15y^2 = 36 \end{array}$$

Verification: $(-9)(-4) = 36$

(c) Multiply $(2x + 3y)(3y - 4x)$ and verify.

$$\begin{array}{r} 2x + 3y \\ 3y - 4x \\ \hline -8x^2 - 12xy \\ + 6xy + 9y^2 \\ \hline -8x^2 - 6xy + 9y^2 = 16 \end{array}$$

Verification: $(8)(2) = 16$

(d) Multiply $(2a + 4b - 3c)(3a - 2b)$ and verify when $a = 1, b = 2, c = 3$.

$$\begin{array}{r} 2a + 4b - 3c \\ 3a - 2b \\ \hline - 4ab + 6bc - 8b^2 \\ + 6a^2 + 12ab - 9ac \\ \hline 6a^2 + 8ab - 9ac + 6bc - 8b^2 = -1 \end{array}$$

Verification: $(1)(-1) = -1$

(e) Simplify: $(-1)^5$

$$(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$$

WORKOUT EXERCISE XV

1. Give the product of:

- | | |
|------------------------|----------------------------|
| (a) $(+4) \times (+4)$ | (f) $(-6)(-7)(+5)$ |
| (b) $(+4)(-5)$ | (g) $(-9)(+6)(+5)$ |
| (c) $(-5)(-6)$ | (h) $(+11)(-3)(-7)$ |
| (d) $(-2)(-2)(-2)$ | (i) $(-8)(+5)(+3)(-6)(-2)$ |
| (e) $(-3)(-2)(+3)(-2)$ | (j) $(+7)(+5)(+3)(+2)(-1)$ |

2. Find the product of:

- | | |
|-----------------------------|--------------------------|
| (a) $(a)(-b)(c)$ | (f) $(-x)^6(-y)^3$ |
| (b) $(4a^2b)(-2bc^2)$ | (g) $(mn)^4(-mn)^5$ |
| (c) $(-3)^3(+1)(-b^2)^3(a)$ | (h) $(-x)^2(-x)^3(-x)^4$ |
| (d) $(5xy)(-1)(4x^2y^2)$ | (i) $(y^2)(-y)^2$ |
| (e) $(-1)^6(-2)^3$ | |

3. If $x = 1$, $y = -1$, find the value of:

- | | |
|----------------------|-------------------------------------|
| (a) $(+x)^2(+y)^2$ | (d) $x^3 + 2x^2y + 3xy^2 - 9y^3$ |
| (b) $-xy$ | (e) $-x^3 - y^3 + 3x^2 + y^2 - xy$ |
| (c) $(x + y)(x - y)$ | (f) $3x^2y + 4xy^3 - 5x^3y + 2xy^2$ |

4. Simplify:

- | | |
|--|------------------------|
| (a) $x(x - 1)(x + 2)$ | (c) $(x - 1)^2$ |
| (b) $(x + y)(x - y)$ | (d) $(ab - 3)(ab + 3)$ |
| (e) $(n - 4)^2 - (n + 4)^2$ | |
| (f) $a(a + 1)(a^2 - a + 1) - (a^2 + 1)(a^2 - 1)$ | |
| (g) $2(x - y) - 3(x + y) - 4(xy - x)$ | |
| (h) $(m + n)^2 + (m - n)^2$ | |

5. Syl Axel used $(x - 8)$ gallons of gasoline on a recent trip. If the gasoline cost $(y - 5)$ cents per gallon, how much did he spend on gasoline?

6. Threads Ltd. manufacture silver threads at a cost of $\$(a + b - c)$. The raw material cost $(a - c)$ times as much as the cost of manufacture and the labour $(a + 1)$ times as much as the raw material. What is the total cost of manufacture?

7. Chris Foxall averaged $(m - 5)$ miles per hour on a recent trip. If he was driving for $(m - n)$ hours each day for a period of $(m + n)$ days, how far did he travel? If $m = 30$ and $n = 20$, how many miles did he actually travel?

8. Mr. Bacon left \$10,000 to his four sons. David was to receive \$1,000 less than Paul, and \$1,500 less than Chris. Chris received \$2,000 less than William. How much did each son receive?

9. Farmer MacNaughton owned $(6x - 8)$ chickens. If each chicken layed $(x + 2)$ eggs in a week, how many eggs would he get in $(50 - x)$ weeks?

10. Pauline spent $(90 - x)$ cents each school day for her lunch, and $(120 - x)$ cents for pleasure on Saturday and Sunday. If her weekly

allowance is \$5.00, how much had she left? If $x = 40$ cents, how many cents had she actually left?

Division of Signed Numbers

As multiplication and division are inverse operations, the rules for dividing are exactly the same as for multiplication.

Examples:

$$\begin{aligned}
 (+3)(+5) &= +15; \text{ then } \frac{+15}{+3} = +5 \\
 &\text{and } \frac{+15}{+5} = +3. \\
 (+x)(+y) &= +xy; \text{ then } \frac{+xy}{+x} = +y \\
 &\text{and } \frac{+xy}{+y} = +x. \\
 (-4)(-5) &= +20; \text{ then } \frac{+20}{-4} = -5 \\
 &\text{and } \frac{+20}{-5} = -4. \\
 (-a)(-b) &= +ab; \text{ then } \frac{+ab}{-a} = -b \\
 &\text{and } \frac{+ab}{-b} = -a. \\
 (-5)(+6) &= -30; \text{ then } \frac{-30}{-5} = +6 \\
 &\text{and } \frac{-30}{+6} = -5. \\
 (-m)(+n) &= -mn; \text{ then } \frac{-mn}{-m} = +n \\
 &\text{and } \frac{-mn}{+n} = -m.
 \end{aligned}$$

The rule may be stated as follows: *When numbers having LIKE signs are divided, the quotient is positive; and when numbers having UNLIKE signs are divided, the quotient is negative.*

Examples: $\frac{-2xy}{-xy} = 2$

$$\frac{4a^3 - 8a^2 - 2a}{2a} = 2a^2 - 4a - 1$$

(As in Section 5, divide each term of the dividend by the divisor.)

$$\frac{-36x^3y^2 - 6xy^3}{-6xy} = 6x^2y + y^2$$

The foregoing rules for signed numbers can be summarized as follows:

1. When adding signed numbers with like signs, add their absolute values and to their sum attach the common sign.
2. When adding signed numbers with unlike signs, find the difference between their absolute values and to this difference attach the sign of the greater number.
3. To subtract signed numbers, change the sign of the subtrahend and add.
4. If two numbers having like signs are multiplied, their product is positive.
5. If two numbers having unlike signs are multiplied, their product is negative.
6. When multiplying, if the number of factors having a negative sign is odd, then the product is negative.
7. When numbers having like signs are divided, the quotient is positive.
8. When numbers having unlike signs are divided, the quotient is negative.

WORKOUT EXERCISE XVI

1. Divide the following:

(a) $3 \overline{) 3x + 6y}$

(e) $x^3 \overline{) -x^5 - 6x^4 + 3x^3}$

(b) $-4 \overline{) 16a + 20c}$

(f) $-x^2 \overline{) +4x^4 - 3x^3 - 4x^2y}$

(c) $x \overline{) xy - x^2y}$

(g) $3xy \overline{) 27x^3y^2 - 36xy^2}$

(d) $-y \overline{) xy^2 - x^2y^2}$

(h) $-12y^2 \overline{) -48xy^3 + 36x^2y^2 + 60y^2}$

(i) $-6a^2bc^2 \overline{) 18a^5b^2c^3 - 24a^3b^2c^2}$

2. Perform the following operations:

(a) $(4x - 8y) \div (-4)$

(b) $(12ab^2 - 6a^2b) \div (-3ab)$

(c) $(-16^3t^2 - 12s^2t^3 - 44s^2t^2) \div (-4st)$

(d) $(-4.5x^2yz^2 + 10.5xy^2z - 13.5xyz) \div (-1.5xy)$

(e) $(\frac{1}{2}p^3 - \frac{1}{4}p^2) - (-\frac{1}{4}p)$

3. Divide:

(a) $12m^2n^2 - 6mn^2 + 4m^2n$ by $2mn$

(b) $\frac{7}{8}a^3b^2c - \frac{5}{8}a^2b^2c^2 + \frac{3}{8}abc^3$ by $-\frac{1}{8}abc$

(c) $14.5x^3y - 10.5xy^3 + 7.5x^2y^2$ by $0.5xy$

(d) $-136r^5s^3 + 187r^2s^4 - 119r^4s^3$ by $17r^2s^3$

(e) $-11.4x^3y^2 - 6.6xy^2 + 3.8x^2y$ by $0.2xy$

4. Simplify:

(a) $\frac{2x + 2}{2} + \frac{6y^2 + 3y}{3y}$

(b) $\frac{10ab^2c - 30a^2bc^2 + 40a^2b^2c}{10abc} - \frac{50a^3b^3c^3 - 40ab^3c + 30a^2b^2c}{10ab^2c}$

$$(c) \frac{-12x^3 - 24x^5}{-6x^3} - \frac{3x^4 + 6x^5}{3x^3}$$

$$(d) \frac{8m^2n^2 - 6mn + 2mn^2}{2mn} - \frac{12n^2 - 6n}{3n} + \frac{24m^2 - 12m^2n + 4n^2}{4n}$$

$$(e) \frac{ax^2 - ax^3}{ax} - \frac{by^2 - by^3}{by} \quad (f) \frac{ax + bx}{x} - \frac{bx - by}{b}$$

5. If Clair Thomas travels x miles per hour, how far will she travel in $3x^3 - 2x^2 + 7x$ hours?

6. The Corner Grocery purchase $4.5x$ pounds of butter for $\$(9x^3 - 13.5x^2 - x)$. How much will 25 pounds cost?

7. The price of $16xy$ gallons of gasoline is $\$(32x^4y^4 - 48x^2y^2 - 16xy)$. How much per gallon does it cost?

8. Tom Tuck travelled from Toronto to Vancouver in $175a^6b^4c^2 - 250a^2b^2c^2 - 75a^2bc^2$ hours. If the aeroplane travelled at $25a^2bc^2$ miles per hour, how many miles is it from Toronto to Vancouver?

9. The Petite Plumbers have to cover a total length of pipe $81m^3n^3 - 33m^2n^2 - 51mn^2$ feet long. If each piece of insulation is $3mn^2$ feet long, how many pieces will be required?

10. In $7b^2$ days Farmer Dales's livestock ate $98b^4 - 49b^3 - 21b^2$ pounds of food. If each pound of food cost $3b$ cents, how much per day did the livestock food bill amount to?

11. The Barrac Club have $3x$ members who pay a fee of \$100 a year to attend all the functions. If at the end of the year they have attended less than 50% of the functions, \$50 is refunded. If $\frac{1}{3}$ of the members did not attend 50% of the functions, what was the total intake of fees? If this was used to pay expenses of $\$25x^2$ and the remainder used to pay the expenses of $25x$ events, how much did each event cost?

SECTION 7 Co-Ordinates

The Ordinate and the Abscissa

It is possible for us to plot algebraic equations graphically. We use two special reference lines called the x -axis and the y -axis as shown in figure 5-9. The axes intersect at the origin 0. Each axis is divided into equal units numbered from the origin outwards. For our purpose, we shall use only one quarter or *quadrant* of the figure, although, for more advanced work, all four quadrants are used.

We can now locate any point in the quadrant by stating its distance from each axis. For instance, in figure 5-10 the point P is 3 units to the right of the y -axis and 2 units above the x -axis. The distance from the y -axis to the point is called the *abscissa* and is stated first, the distance from the x -axis is called the *ordinate* and is stated second, and the two co-ordinates for the point P are stated as (3, 2).

When we locate a point in this way, we say we are *plotting* the point.

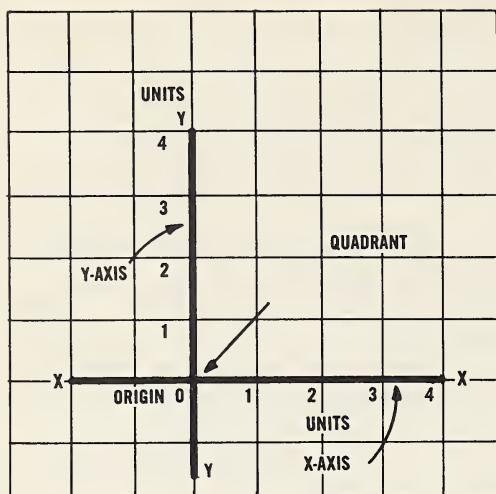


Figure 5-9

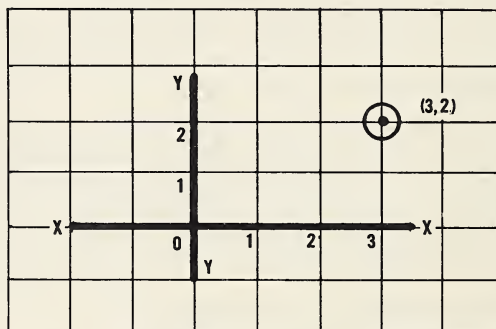


Figure 5-10

Plotting an Equation in Two Variables

If we have an equation such as:

$$y = 2x - 2,$$

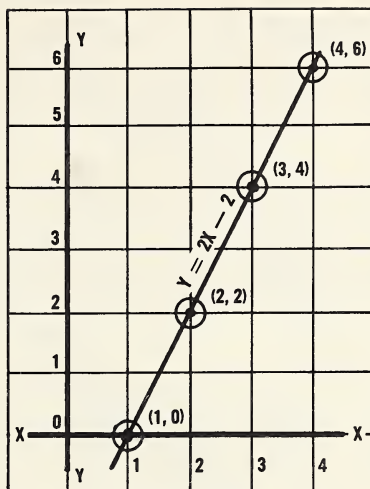
we can see that the value of 'y' will depend on whatever value is assigned to 'x'. If we say $x = 1$, then $y = 2 \times 1 - 2 = 0$. If $x = 2$, then $y = 2 \times 2 - 2 = 2$, and so forth.

In order to 'plot' this equation, we draw up a table in which we assign a series of values to x and solve the equation to find the corresponding value of y.

x	1	2	3	4	5
y	0	2	4	6	8

We are now ready to plot these points as in figure 5-11.

Figure 5-11



These points are now joined and form a straight line. Any equation with the two variables x and y , if only raised to the first power or degree, can be plotted as a straight line. The graph is *the set of all points determined by the equation*.

Steps to follow to construct a graph of a first degree equation in two variables:

- Solve the equation for y in terms of values assigned to x (or x in terms of y if this is more convenient).
- Construct a table to record the corresponding values of x and y .
- Assign values to x , calculate the corresponding value of y and record in the table.
- Plot the points calculated in (c) and join the points.

Examples: Plot the graph for the equations:

(a) $y = 3 + 2x$

(b) $y = x - 1$

(a)

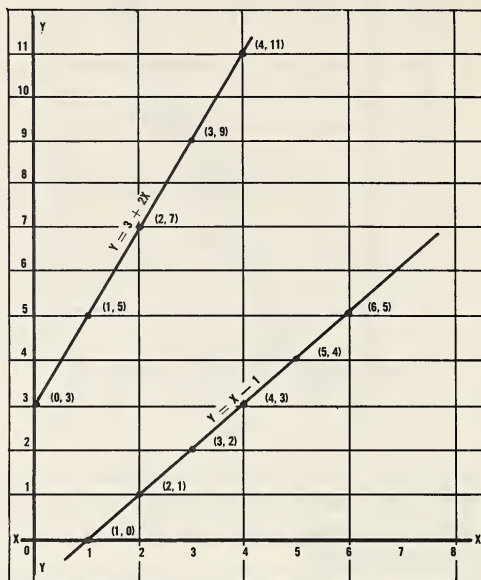
x	0	1	2	3	4
y	3	5	7	9	11

(b)

x	1	2	3	4	5	6
y	0	1	2	3	4	5

Graph of the Equations $y = 3 + 2x$ and $y = x - 1$

Figure 5-12

**WORKOUT EXERCISE XVII**

Using graph paper plot the graphs of the following equations. Label all the co-ordinates.

- | | | |
|-------------------------------|---------------------------|-------------------|
| 1. $y = 3x - 2$ | 2. $y = 2x^*$ | 3. $y = 6 - x^*$ |
| 4. $y = x + 8$ | 5. $y = \frac{1}{2}x - 2$ | 6. $y = 4 - 2x$ |
| 7. $y = \frac{1}{4}x$ | 8. $y = x^*$ | 9. $y = 2x = 0$ |
| 10. $y - 1 = x$ | 11. $y = 3x - 4$ | 12. $y = x - 5^*$ |
| 13. $y = 12 - \frac{1}{2}x^*$ | | |

14. Solve the business problem at the beginning of the chapter.
15. Name the two co-ordinates of a point and explain what they mean and how they are noted when plotting a point.

SECTION 8 Formulae

A formula is a rule or a relationship which is expressed as an algebraic equation. Usually one letter at the left of the $=$ sign is expressed in terms of one or more others at the right of the $=$ sign. If we know the values of every letter in the formula except one, we can solve the equation for that one by the rules we have learned for solving equations.

* See chapter heading.

A very famous formula which led to the discovery of atomic energy is the formula for energy established by Albert Einstein in the early part of this century. This states that energy is equal to mass multiplied by the square of the speed of light, or

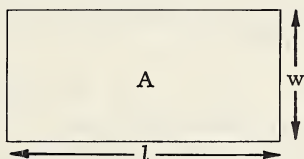
$$E = mc^2,$$

an unpretentious looking formula for such far reaching effects!

Certain letters have, by common usage, become standard symbols and are universally recognized. Some letters, and special symbols, are used in specialized branches of mathematics and science. We shall use a number of these formulae in future chapters.

Examples: $A = l \times w = lw$

Where A = the area of the rectangle
 l = the length of the rectangle
 w = the width of the rectangle
 (See Chapter 7.)



$$I = P \times r \times t = Prt$$

Where P = the amount of money invested
 r = the rate of simple interest per annum
 t = the time, in years, for which the money
 was invested
 I = the amount of interest earned on the
 investment
 (See Chapter 12.)

$$D = R \times T$$

Where D = the distance travelled
 R = the constant rate of travel
 T = the time

Find the distance travelled in $2\frac{1}{2}$ hours by a car moving at 60 m.p.h.

$$\begin{aligned} D &= R \times T \\ &= 60 \times \frac{5}{2} \\ &= 150 \text{ miles} \end{aligned}$$

WORKOUT EXERCISE XVIII

Using the formula $D = R \times T$:

1. Find D when $R = 120$ m.p.h and $T = 1\frac{1}{4}$ hours.
2. Find D when $R = 1.2$ miles per minute and $T = 130$ secs.
3. Find R when $D = 600$ miles and $T = 3$ hours. (Divide each side of the equation by T .)
4. Find R when $D = 10$ yards 2 feet and $T = 4$ secs.
5. Find T when $D = 64$ feet and $R = 16$ feet per sec.
6. Find T when $D = 6,178$ yards and $R = 32$ feet per sec.

Using the formula $S = 16t^2$, where S is the distance in feet an object would fall if dropped from an aeroplane and ' t ' is the time expressed in seconds; find the distance S for the following values of t :

7. 1 sec. 8. 2 secs. 9. 3 secs. 10. 4 secs. 11. 5 secs.
12. 6 secs. 13. 7 secs. 14. 8 secs. 15. 9 secs. 16. 10 secs.
17. Draw up a table for the distance and time for problems 7 – 16 inclusive and plot a graph.

Using the formula $F = \frac{9}{5} C + 32$ (see Chapter 6) where F = Fahrenheit temperature measurements and C = Centigrade temperature measurements, find the temperature in degrees F for the following values of C :

18. $C = 25^\circ$ 19. $C = 40^\circ$ 20. $C = 10^\circ$ 21. $C = 0^\circ$.

Find the temperature in degrees C when F has the following values:

22. $F = 48^\circ$ 23. $F = 41^\circ$ 24. $F = 101^\circ$ 25. $F = 38^\circ$

Using the formula $I = \frac{E}{R}$, where I is the amount of electric current (in amperes), E is the total voltage, and R is the resistance of the conductors (in ohms; this is called Ohm's Law):

26. Find the electric current when the voltage is 120V and the resistance 30 ohms.
27. If the voltage is 500V and the resistance 1,000 ohms, what is the current?
28. Solve Ohm's Law for R and find the resistance in a filament if the voltage is 140V and the current is $2\frac{1}{2}$ amperes.
29. An electric door chime requires a current of $\frac{1}{50}$ ampere; the resistance of the wires, connections, etc. amount to 300 ohms. How many volts will be required to ring the bell?
30. A battery is used to light a flashbulb which requires a current of 0.3 amperes. If the resistance amounts to 5 ohms, how much voltage must the battery produce?

SECTION 9 Sets – Open Sentences and Variables

Symbols Used:	$=$	is equal to
	$\neq$	is not equal to
	$\geq$	is equal or greater than
	$\leq$	is equal or less than
	ϵ	is a member of the set
	ϕ	is an empty set

We will let the universal set which we will use in algebra be $A1$. The same properties will hold true for this set as for others we have studied.

If we say, "There is a number which when added to 7 will be equal to 10," we can express this by:

$$? + 7 = 10$$

In Section 1 we learned to replace this by x ; so

$$x + 7 = 10.$$

This mathematical sentence is called an *open sentence* because it contains a *variable*, named x , which is a place-holder for some replacement from the set $A1$; that is,

$$x \in A1.$$

A *variable* then is a place-holder in an open sentence.

An *open sentence* can be true or false according to the replacement used from the given universe.

Example: Let $A1 = \{ 1,2,3,4,5 \}$ and the sentence be $x + 7 = 10$

The variable x can be replaced by any of the five members of the set $A1$.

If it is replaced by $x = 1$, the sentence becomes

$$1 + 7 = 10, \text{ which is false.}$$

It only becomes true when x is replaced by the number 3:

$$3 + 7 = 10$$

$\therefore$ Only one of the possible replacements from the set $A1$ will make the sentence true.

A Solution Set

Consider the sentence:

$$x + 3 < 6, \text{ where } A1 = \{1,2,3,4,5\} \text{ and } x \in A1.$$

How many replacements for x from the universe will make this sentence true?

$$\begin{array}{rcl}
 1 + 3 < 6 & - & \text{true} \\
 2 + 3 < 6 & - & \text{true} \\
 3 + 3 < 6 & - & \text{false} \\
 4 + 3 < 6 & - & \text{false} \\
 5 + 3 < 6 & - & \text{false}
 \end{array}$$

Of five possible replacements for x , two of these will be true and three will be false.

The set of these true replacements is called the *solution set*.

Examples: $A1 = \{2, 4, 6, 8, 10\}$

Write the solution set for each of the following sentences:

- | | |
|------------------|---------------------|
| (a) $x + 5 = 11$ | (d) $x + 2 > 13$ |
| (b) $x + 5 > 8$ | (e) $x + 4 \leq 8$ |
| (c) $x - 1 < 7$ | (f) $x + 4 \neq 10$ |

In each case the variable x can be replaced by any of the five members of the set $A1$.

- (a) When x is replaced by 2, 4, 8 or 10 the sentence is false
When x is replaced by 6 the sentence is true
 $\therefore$ The solution set is $\{6\}$ a *unit set*
- (b) When x is replaced by 2 the sentence is false
When x is replaced by 4, 6, 8, 10 the sentence is true
 $\therefore$ The solution set is $\{4, 6, 8, 10\}$
- (c) When x is replaced by 8 or 10 the sentence is false
When x is replaced by 2, 4, 6 the sentence is true
 $\therefore$ The solution set is $\{2, 4, 6\}$
- (d) When x is replaced by any member of the set the sentence is false
 $\therefore$ The solution set is empty ϕ
- (e) When x is replaced by 6, 8, 10 the sentence is false
When x is replaced by 2 or 4 the sentence is true
 $\therefore$ The solution set is $\{2, 4\}$
- (f) When x is replaced by 6 the sentence is false
When x is replaced by 2, 4, 8, 10 the sentence is true
 $\therefore$ The solution set is $\{2, 4, 8, 10\}$

WORKOUT EXERCISE XIX

Let $A1 = \{0, 2, 4, 6, 8, 10\}$. For each of the following sentences list all the possible replacements and state which are true:

1. $x + 5 = 9$ 2. $x + 4 = 9$ 3. $x + 6 < 11$ 4. $x + 5 > 12$

Let $U = \{\text{All the positive whole numbers}\}$. Write the solution sets for each of the following sentences:

- | | | |
|---|---------------------|---------------------|
| 5. $x + 7 = 12$ | 6. $x - 4 = 7$ | 7. $x + 11 = 21$ |
| 8. $x + 15 > 30$ | 9. $x + 3 \leq 9$ | 10. $4 + x \neq 7$ |
| 11. $3x = 24$ | 12. $4x + 1 = 17$ | 13. $x + 11 > 15$ |
| 14. $4 - x = 2$ | 15. $x + 3 = 3 + x$ | |
| 16. $x + 1\frac{1}{2} = 1\frac{1}{2} + x$ | | 17. $9 + x \geq 19$ |
| 18. $5x \neq 20$ | 19. $x + 6 \leq 8$ | 20. $9x + 9 = 9$ |

Let $S = \{\text{All the positive whole even numbers}\}$.

Write an open sentence to represent the following:

21. All the numbers which when added to 7 will be greater than 10.
22. All numbers not equal to 20.
23. All the numbers which when added to 5 are equal to or less than 11.
24. All the elements greater than 16 and less than 24.
25. All the elements which when multiplied by 3 and added to x will be equal to or less than 25.
26. All the numbers evenly divided by 3.
27. 21 plus 5 minus a variable is equal to or greater than 15.
28. Seven added to a variable is equal to or less than 20.
29. Write the solution sets for problems 21-27.

Description of Solution Set

Let $A1 = \{\text{set of all the natural numbers}\}$ and the sentence be $x + 3 < 7$. We must select the solution set from the universe $A1$ so that the given condition $x + 3 < 7$ is satisfied. This can be written as follows:

$$\{x \in A1 \mid x + 3 < 7\}$$

Interpreted this means "the set of all the numbers x which are members of the set $A1$ which satisfy the condition that $x + 3 < 7$." The vertical bar means "which satisfy the condition that" or "such that".

The solution set = $\{1,2,3\}$

Examples: $A1 = \{\text{All even whole numbers between 10 and 30}\}$.

Condition: $x + 8 = 34$.

- (a) Describe the solution set.
- (b) Find the solution set.

(a) Description: $\{x \in A1 \mid x + 8 \leq 34\}$

(b) Solution set = $\{10,12,14,16,18,20,22,24,26\}$

WORKOUT EXERCISE XX

23. Describe the solution sets for problems 5 - 27 of previous exercise. If $\{x \in A1 \mid x \text{ is an odd whole number}\}$, give the solution set for the following universes:

$$24. A1 = \{1, 2, 3, 4, 5\}$$

$$25. A1 = \{2, 4, 6, 8, 10, 12\}$$

$$26. A1 = \{1, 4, 7, 10, 13\}$$

$$27. A1 = \{2, 5, 8, 11, 14, 17\}$$

$$28. A1 = \{1\frac{1}{2}, 3, 4\frac{1}{2}, 6, 7\frac{1}{2}, 9, 10\frac{1}{2}, 12\}$$

$$29. A1 = \{\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4\}$$

$$30. A1 = \{13, 18, 23, 28, 33, 38, 43\}$$

SECTION 10 Figuring for Fun

1. Dotty left Toronto for Winnipeg by aeroplane at 8.40 a.m. 1 hour and $14\frac{2}{7}$ minutes later Paul left Winnipeg for Toronto by jet plane. They both arrived at their destinations at exactly the same time. Dotty had travelled 770 miles at 11 a.m. when Paul's plane passed hers. What is the distance in miles between Toronto and Winnipeg?

2. Mary is older than her brother Todd, but both their ages contain the same two digits. One-third the difference between their ages is equal to one-eleventh the sum of their ages. How old are they?

3. Six students sat down at a circular table to hold an executive meeting of the Students Administrative Council. Can you place each student at the table from the following information:

1 student is the treasurer.

John is not the treasurer and sat opposite Paul.

Abie sat at the treasurer's right.

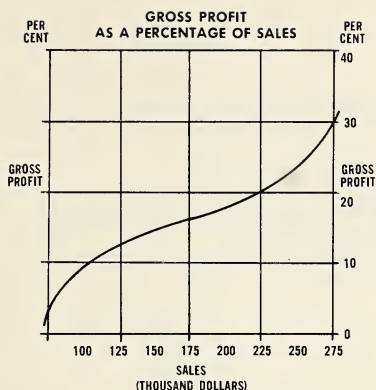
Peter sat next to Robert and opposite Nelson.

Andy sat on John's right.

Adolph sat on Nelson's right.

The secretary sat opposite the treasurer.

Place the students around the table and name the treasurer and the secretary.



Ratio and Percentage

6

A Business Problem

Hem, Haw & Hedge Company of Foggy Vale carry in their store a \$10,000 stock of widgetts. They make 22% profit on this stock and are able to sell five times this amount of stock in one year. (This is called stock turnover.) They also carry a \$15,000 stock of lead balloons which turns over four times a year and on which they make 20% profit.

They are desirous of increasing their total profits and are considering:

- (a) reducing the profit on widgetts to 19% and thereby increasing the stock turnover to six times per year,
- (b) reducing the profit on lead balloons to 17% and thereby increasing the stock turnover to five times a year.

Would these changes give them the desired increase in their profits? Advise Hem, Haw & Hedge Company.

Everyone, from junior to grandfather, employs the device of making numerical comparisons. We compare the number of marbles we have with the number Joe has; the amount of our allowance with the amount of Mary's allowance; how much we earned last summer with how much Tom earned last summer; the score of our school hockey team with the score of Tuddenham High; and many other things.

Important decisions in the business world frequently depend upon the comparison of numbers. Executives analyse their business transactions by comparing this year's profits (or losses) with previous years' profits (or losses); this year's sales with previous years' sales and this year's expenses with previous years' expenses. These comparisons also form the basis for planning next year's operations.

In the investment field, investors compare the earnings and the future prospects of various companies before they invest their money.

In order to compare one number with another, we must study the principles of ratio and percentage.

SECTION 1 Ratio

Finding the Ratio

The ratio of two numbers compares the size, or value, of one number with the size, or value, of another number when both are expressed in the same terms.

Example: If there are 24 boys and 16 girls in your class, the ratio of boys to girls can be expressed as:

24 to 16

24:16

$$\frac{24}{16} = \frac{3}{2}$$

When expressed as a fraction, we can say that the ratio compares the two numbers by dividing one by the other. We can perform this division by dividing both the numerator and denominator of the fraction by 2.

$$\frac{3 \div 2}{2 \div 2} = \frac{1.5}{1}$$

or 1.5:1

The ratio of boys to girls in the class can be stated as 1.5:1.

The rule for finding the ratio of a number to 1 is:

Express the number to be compared as the numerator of the fraction and the number to which it is compared as the denominator of the fraction. Reduce the denominator to 1 by dividing the denominator into the numerator.

Note: The ratio is always a fraction, and so the denominator 1 must be retained.

The Base: When a fraction is used to indicate a ratio, we call the denominator the base. The base always represents the number with which the comparison is being made or the number to which you bring the other number for comparison. The base is always divided into the other number.

These are the fundamental rules when dealing with comparisons. If you thoroughly learn and understand them, you should have no trouble with this type of problem.

Example: There are 24 boys and 16 girls in your class. In the above example we compared the number of boys to the number of girls.

Base: 16 girls

24:16

$$\frac{24}{16} = \frac{3}{2} = \frac{1.5}{1}$$

1.5:1

The ratio of boys to girls is 1.5 to 1.

Now let us compare the number of girls in the class to the number of boys in the class.

Base: 24 boys

16:24

$$\frac{16}{24} = \frac{2}{3} = \frac{0.67}{1}$$

0.67:1

The ratio of girls to boys is 0.67 to 1.

Now let us compare the number of boys *with* the total number of students in the class.

Base: total students in class, 40.

24:40

$$\frac{24}{40} = \frac{3}{5} = \frac{0.6}{1}$$

0.6:1

The ratio of boys to total students is 0.6:1.

Lastly let us compare the number of girls *with* the total number of students in the class.

Base: total students in class, 40.

16:40

$$\frac{16}{40} = \frac{2}{5} = \frac{0.4}{1}$$

0.4:1

The ratio of girls to total students is 0.4:1.

Note: The key words here are “to” the base and “with” the base.

Comparing two Ratios

Referring to the example below, we see that in the next classroom there are 16 boys. We could therefore say that we have 8 more boys

in our classroom. But this would not tell us whether our ratio of boys to girls is higher or lower than that of the next class. If we are also told that there are 12 girls in the next class, we can then find the ratios for both classes and compare these ratios.

Example: Our Class: 24 boys out of a total of 40 students.

Next class: 16 boys out of a total of 28 students.

Bases: total class enrolments.

Ratio of next class

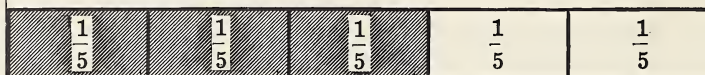
$$\frac{16}{28} = \frac{4}{7}$$

Our ratio

$$\frac{24}{40} = \frac{3}{5}$$

Figure 6-1

Our Class



Next Class



These ratios can be represented graphically as in figure 6-1. We can see that $\frac{3}{5}$ is a little larger than $\frac{4}{7}$; but if we want a more accurate comparison, we must bring the *bases* to the same figure. We learned how to find a common denominator when dealing with fractions.

$$\begin{array}{r} \frac{3}{5} \\ \frac{21}{35} \end{array} \qquad \begin{array}{r} \frac{4}{7} \\ \frac{20}{35} \end{array}$$

We can now compare the number of boys in our class with the number of boys in the next class by stating the ratio as:

21:20 Base: ratio of boys in next class.

$$\frac{21}{20}$$

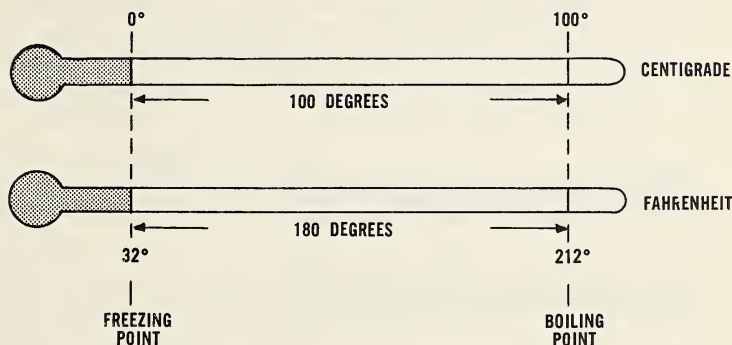
1.05:1

The ratio of our boys to the boys next door is 1.05 to 1.

A Practical Application of Ratio

In our everyday lives we use the Fahrenheit scale on our thermometers to tell us the temperature in degrees Fahrenheit. However, in physics and science laboratories the Centigrade scale is used to measure temperature in degrees Centigrade. It is sometimes important for us to be able to interchange these units of measurement, and this can be done by applying the theory of ratio.

Figure 6-2



(Instead of writing the word “degrees” after the number, we use the symbol “°”.)

From the diagram we can see that there are 100 Centigrade degrees between the freezing point and the boiling point and 180 Fahrenheit degrees between the freezing point and the boiling point. We can therefore find the number of Fahrenheit degrees in terms of Centigrade degrees and the number of Centigrade degrees in terms of Fahrenheit degrees.

$$100^{\circ} \text{ C} = 180^{\circ} \text{ F}$$

Divide both sides of the equation by 20:

$$5^{\circ} \text{ C} = 9^{\circ} \text{ F}$$

Divide both sides of the equation by 5:

$$1^{\circ} \text{ C} = \frac{9}{5}^{\circ} \text{ F}$$

In the same way we can find that

$$1^{\circ} \text{ F} = \frac{5}{9}^{\circ} \text{ C}$$

Applying this knowledge, we can now transfer from one scale to the other.

Example: Find the temperature in degrees Fahrenheit if your Centigrade thermometer reads 20°C .

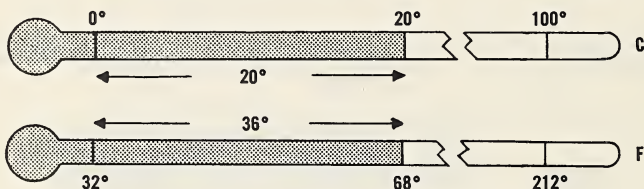
$$1^{\circ}\text{C} = \frac{9^{\circ}}{5}\text{F}$$

$$\begin{aligned}\therefore 20^{\circ}\text{C} &= \frac{9}{5} \times \frac{20}{1}\text{F} \\ &= 36^{\circ}\text{F}\end{aligned}$$

This would be the temperature in degrees F if both scales started at 0° . But we see from the diagram that 0° on the Centigrade Scale is 32° on the Fahrenheit Scale. Therefore to find the temperature in degrees Fahrenheit, add the 32.

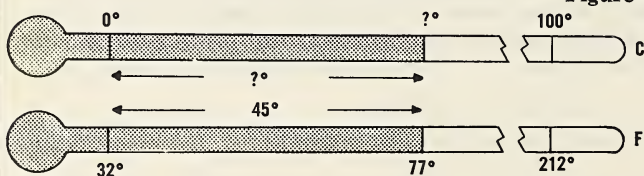
$$\therefore 20^{\circ}\text{C} = 36 + 32 = 68^{\circ}\text{F}.$$

Figure 6-3



Example: Your house thermometer reads 77°F . If you wish to use this thermometer in the laboratory to record an experiment in the Centigrade scale, what room temperature in degrees Centigrade would you record?

Figure 6-4



As the 0° on the Centigrade scale is opposite the 32° on the Fahrenheit scale, we must first subtract the 32° from the $77^{\circ} = 45^{\circ}$, as can be seen from the above diagram.

$$1^{\circ}\text{F} = \frac{5^{\circ}}{9}\text{C}$$

$$\begin{aligned}45^{\circ}\text{F} &= \frac{5}{9} \times \frac{45}{1}\text{C} \\ &= 25^{\circ}\text{C}\end{aligned}$$

WORKOUT EXERCISE I

Express the following ratios as fractions, and then simplify:

1. 78:102
2. 75:125
3. 64:104
4. 121:165
5. 108:156
6. 51:153
7. $\frac{1}{4}:\frac{1}{2}$
8. $\frac{5}{12}:\frac{5}{6}$
9. $\frac{7}{19}:2\frac{1}{6}$
10. $6\frac{1}{4}:9\frac{5}{8}$
11. 5 quarts:2 gallons
12. 9 inches:1 foot
13. 35 minutes:1 hour
14. 85 cents:\$3
15. \$2.75:\$4.50
16. $1\frac{1}{2}$ pints:3quarts
17. $3\frac{1}{2}$ yards:8 feet
18. $2\frac{1}{2}$ hours:100 minutes
19. $4\frac{1}{2}$ yards:11 feet
20. 1 day:8 hours
21. Express the ratios in 7, 11, 13, 15, 17, and 19 above as ratios to 1.

Find two numbers:

22. Whose ratio is 3:2 and whose sum is 30.
(Hint: There are $3 + 2 = 5$ parts in the sum, $\frac{3}{5}$ of 30 and $\frac{2}{5}$ of 30.)
23. Whose ratio is 4:5 and whose sum is 54.
24. Whose ratio is 10:1 and whose sum is 121.
25. Whose ratio is 100:1 and whose sum is 909.
26. Convert the following Centigrade degrees to Fahrenheit degrees:
 $0^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ, 22^\circ, 25^\circ, 50^\circ, 75^\circ$.
27. Convert the following Fahrenheit degrees to Centigrade degrees:
 $34^\circ, 50^\circ, 75^\circ, 95^\circ, 149^\circ, 176^\circ, 199.4^\circ$.

Find the ratio to one and express the result in words:

28. \$3.78 divided by 6 dozen eggs.
29. 165 miles divided by $7\frac{1}{2}$ gallons of gasoline.
30. \$92.75 divided by 35 hours.
31. $484\frac{1}{2}$ miles divided by $8\frac{1}{2}$ hours.
32. $41\frac{1}{4}$ hours divided by $5\frac{1}{2}$ days.
33. \$52.50 divided by 7 boys.
34. A train travels 2,860 miles at an average speed of 55 m.p.h. (miles per hour). How long does it take to complete the journey?
35. 15 students in your class and 12 students from the next room receive first-class honours in an examination. What further information would you need to make a fair comparison. Why?
36. A very important ratio in business is the "current ratio". This is the ratio of the current assets to the current liabilities. The current assets are cash and other assets which become cash within a short period of time; current liabilities are the debts which the company must pay within the same short period of time. Find the current ratios

for the following businesses:

Current Assets

\$ 72,000
\$ 45,000
\$ 60,000
\$ 76,500
\$122,500

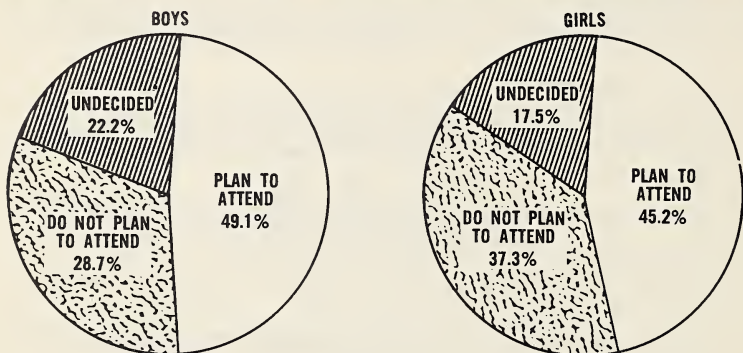
Current Liabilities

\$ 36,000
\$ 37,500
\$ 24,000
\$ 42,500
\$ 52,500

SECTION 2 Percentage

Figure 6-5

College Plans of Senior High School Students



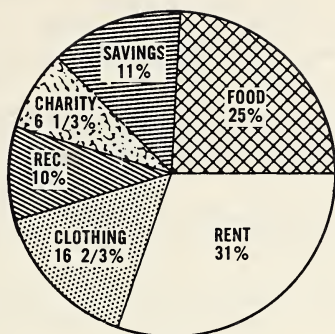
Percentage is the method of comparison most widely used in the business world. It is the method of expressing things as parts of 100. The symbol for percent is %.

Percentage, as a method of comparison, is so important in our daily contacts that it is essential we understand what it is and how it is used. Degree of success or failure in an examination is measured by a per cent score (how many marks you obtained out of a possible 100); there is a per cent increase (increase per 100 students) or a per cent decrease (decrease per 100 students) in this year's school enrolment compared to last year's enrolment; banks pay a per cent interest on deposits and charge a per cent interest on loans; family budgets are expressed in per cents. The rate charged on loans or paid on deposits is quoted as a *rate per cent*.

Example: Mike Boom, a single man, plans to budget his money as follows: food, 25%; rent, 31%; clothing, $16\frac{2}{3}\%$; recreation, 10%; charity, $6\frac{1}{3}\%$; savings, 11%.

Draw a pie graph to represent the disposition of his total income. See figure 6-6.

Figure 6-6



Percentage is a method of expressing numbers and making comparisons of numbers, used by everyone from governments down to the man in the street because it is easier to compare figures when they are expressed as parts of a hundred. The words "per cent" are derived from the Latin "per centum" which mean "by the hundred".

Example: Tab Carter has \$50,000 invested in Provincial bonds. If the bonds pay 5% per year interest, how much does Tab Carter earn?

\$100 pays

\$ 5.00

$\therefore$ \$50,000 pays

$$\frac{5.00}{100} \times \frac{50,000}{1}$$

= \$ 2,500.00

WORKOUT EXERCISE II

- How much does a \$1,000 bond earn each year if it pays 5% per annum interest?
- Tommy Jones spends his average weekday as follows: sleeping, 33 $\frac{1}{3}$ %; school, 25%; recreation, 17%; eating, 8 $\frac{2}{3}$ %; homework, 12%; travelling, 4%. Draw a pie graph showing the distribution of Tommy's day.
- Draw a table showing the percentage of time you spend in your various activities on an average Saturday. Draw a pie graph to show this distribution.
- Statistics tell us that 30% of families do not own a car, 61% own one car, and 9 % own two cars or more. Draw a pie graph illustrating this fact.

SECTION 3 Interchange of Form

A Rate Per Cent Changed to a Common Fraction or a Mixed Number

A rate per cent is also a special case of the common fraction. The whole is divided into one hundred parts and so can be expressed as a common fraction with a denominator of 100.

Rule 1: Drop the per cent symbol, place the number over the denominator 100, and reduce to the lowest terms.

Examples:

Rate Per Cent	Common Fraction	Mixed Numbers
6%	$\frac{6}{100} = \frac{3}{50}$	
37%	$= \frac{37}{100}$	
$8\frac{1}{2}\%$	$\frac{8\frac{1}{2}}{100} = \frac{17}{2} \times \frac{1}{100} = \frac{17}{200}$	
150%	$\frac{150}{100} = \frac{3}{2}$	$= 1\frac{1}{2}$
9.3%	$\frac{9.3}{100} = \frac{93}{1000}$	
212%	$\frac{212}{100} = \frac{53}{25}$	$= 2\frac{3}{25}$

By changing or reducing any fraction so that it has a denominator of 100, we find its percentage value.

Example: 50 students are chosen to take part in a field day celebration. If 37 of the participants are girls, what percentage of boys are chosen?

$\frac{13}{50}$ of the students chosen are boys.

Multiply numerator and denominator by 2.

$\frac{26}{100}$ of the students chosen are boys.

$\therefore$ 26% of the students are boys.

WORKOUT EXERCISE III

Change each of the following rates per cent to a common fraction or a mixed number:

1. 7% 2. 42% 3. $18\frac{1}{2}\%$ 4. 125% 5. 15.5%
 6. 175% 7. 350% 8. $10\frac{1}{2}\%$ 9. 150.55% 10. $66\frac{2}{3}\%$

Find the per cent value of the following fractions:

11. $\frac{7}{25}$ 12. $\frac{11}{25}$ 13. $\frac{23}{50}$ 14. $\frac{17}{50}$ 15. $\frac{26}{25}$
 16. $\frac{51}{50}$ 17. $\frac{3}{10}$ 18. $\frac{7}{10}$ 19. $\frac{3}{4}$ 20. $\frac{1}{4}$
 21. $\frac{7}{5}$ 22. $\frac{13}{5}$ 23. $\frac{61}{20}$ 24. $\frac{97}{300}$ 25. $\frac{300}{200}$

26. John made 21 baskets out of 25 free throws, but Tom made 17 baskets out of 20 throws; who did the better?

27. Hazel made 44 baskets out of 50 free throws, whereas Carol made 22 baskets out of 25; who did the better?

28. Sammy scored 17 questions correct out of 20, and Tab scored 19 questions right out of 20. What percentage did each have correct? What percentage did each have incorrect?

29. At the Dundeen annual picnic there are 300 students. If 180 of the students are boys, what percentage are girls?

30. There are 25 students in a graduating class. 15 of the students own bicycles, 2 own cars, 16 have regular part-time jobs, 14 are proceeding to university, 3 are to study accounting, and 4 are going to Europe. What fraction and what per cent of the class own bicycles, own cars, have jobs, are going to university, are going to study accounting, are going to Europe?

A Common Fraction or Mixed Number Changed to a Rate Per Cent

As a rate per cent can be expressed as a common fraction or a mixed number, so a common fraction or a mixed number can be expressed as a rate per cent.

Rule 2: *Multiply the fraction or mixed number by 100%.*

Example: Change $\frac{6}{7}$ to a rate per cent.

1 whole = 100 per cent parts

$\therefore \frac{6}{7}$ of the whole = $\frac{6}{7}$ of the 100 per cent parts

$\therefore \frac{6}{7} \times 100\% = \frac{600}{7}\% = 85\frac{5}{7}\%$

= 85.7% to one decimal place

Examples:

Common Fraction or Mixed Number	Per Cent
$\frac{3}{4} = \frac{3}{4} \times 100\% = \frac{300}{4} \% =$	75%
$3\frac{1}{2} = \frac{7}{2} \times 100\% = \frac{700}{2} \% =$	350%
$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8} \% =$	$62\frac{1}{2}\%$
$\frac{4}{200} = \frac{4}{200} \times 100\% = \frac{400}{200} \% =$	2%
$\frac{13}{9} = \frac{13}{9} \times 100\% = \frac{1300}{9} \% =$	$144\frac{4}{9}\%$
$7\frac{2}{11} = \frac{79}{11} \times 100\% = \frac{7900}{11} \% =$	$718\frac{2}{11}\%$

WORKOUT EXERCISE IV

Express each of the following fractions or mixed numbers as a rate per cent leaving the answer as a mixed fraction.

- | | | | | |
|---------------------|---------------------|---------------------|----------------------|----------------------|
| 1. $\frac{1}{4}$ | 2. $\frac{7}{8}$ | 3. $\frac{3}{200}$ | 4. $\frac{5}{20}$ | 5. $\frac{5}{8}$ |
| 6. $\frac{11}{8}$ | 7. $15\frac{0}{50}$ | 8. $1\frac{7}{10}$ | 9. $\frac{13}{8}$ | 10. $5\frac{1}{20}$ |
| 11. $\frac{3}{7}$ | 12. $1\frac{7}{9}$ | 13. $8\frac{3}{17}$ | 14. $7\frac{1}{89}$ | 15. $12\frac{1}{29}$ |
| 16. $8\frac{2}{13}$ | 17. $7\frac{2}{7}$ | 18. $\frac{13}{11}$ | 19. $23\frac{1}{41}$ | 20. $8\frac{2}{3}$ |
| 21. $\frac{7}{8}$ | 22. $1\frac{1}{3}$ | 23. $6\frac{7}{11}$ | 24. $\frac{13}{17}$ | 25. $2\frac{7}{13}$ |
| 26. $3\frac{1}{8}$ | 27. $9\frac{7}{7}$ | 28. $4\frac{2}{11}$ | 29. $\frac{13}{25}$ | 30. $4\frac{11}{17}$ |

Give the answer correct to two decimal places:

- | | | | | |
|----------------------|---------------------|---------------------|---------------------|---------------------|
| 31. $3\frac{8}{11}$ | 32. $\frac{7}{13}$ | 33. $\frac{8}{15}$ | 34. $\frac{9}{17}$ | 35. $2\frac{3}{11}$ |
| 36. $1\frac{11}{21}$ | 37. $4\frac{9}{10}$ | 38. $1\frac{9}{17}$ | 39. $\frac{18}{25}$ | 40. $7\frac{2}{5}$ |

A Rate Per Cent Changed to a Decimal Fraction

A rate per cent can also be expressed as a decimal fraction. We know from the place notation of our numbering system that the first place to the right of the decimal point is the tenths column and the second place to the right of the decimal point is the hundredths column. As per cent is another name for hundredths, we can readily express a per cent as a decimal fraction.

Thus 75% means $\frac{75}{100}$ or 75 hundredths or 0.75.

Rule 3: Drop the per cent symbol and move the decimal point two places to the left.

Examples:

Per Cent		Decimal Fraction
83 %	83 hundredths	0.83
27 %	27 hundredths	0.27
$17\frac{1}{2}\%$	$17\frac{1}{2}$ hundredths	0.175
$8\frac{1}{4}\%$	$8\frac{1}{4}$ hundredths	0.0825
2.5%	2.5 hundredths	0.025

WORKOUT EXERCISE V

Bring the following rates per cent to hundredths and then to decimal fractions; set up as in the preceding example:

- | | | | | |
|------------------------|-------------------------|-----------------------|------------------------|-----------------------|
| 1. 78% | 2. 31% | 3. $8\frac{1}{4}\%$ | 4. 7.5% | 5. 94.75% |
| 6. $3\frac{2}{3}\%$ | 7. $41\frac{3}{8}\%$ | 8. $79\frac{3}{8}\%$ | 9. $17\frac{3}{4}\%$ | 10. 17.02% |
| 11. $79\frac{3}{8}\%$ | 12. $82\frac{3}{10}\%$ | 13. $89\frac{8}{9}\%$ | 14. $17\frac{7}{8}\%$ | 15. $8\frac{9}{10}\%$ |
| 16. $7\frac{3}{11}\%$ | 17. $99\frac{9}{10}\%$ | 18. 152.75% | 19. $49\frac{4}{50}\%$ | 20. $83\frac{2}{9}\%$ |
| 21. $59\frac{9}{13}\%$ | 22. $87\frac{11}{13}\%$ | 23. $43\frac{4}{7}\%$ | 24. $176\frac{7}{9}\%$ | 25. 105.57% |

A Decimal Fraction Changed to a Rate Per Cent

As per cent means hundredths, then hundredths can also mean per cent. If, therefore, we substitute the per cent symbol for the "hundredths" in our decimal fraction, we have changed the decimal fraction to a per cent.

Rule 4: *Change a decimal fraction to a rate per cent by moving the decimal point two places to the right and appending the per cent symbol.*

Examples:

Decimal Fraction	Per Cent
0.38 $\left(\frac{38}{100} \times 100\% \right)$	38%
0.5 $\left(\frac{50}{100} \times 100\% \right)$	50%
0.07 $\left(\frac{7}{100} \times 100\% \right)$	7%
0.0015 $\left(\frac{0.15}{100} \times 100\% \right)$	0.15%

Be sure to develop the ability to change from one form to another with facility.

Some of the common fractions and their equivalents are given in figure 6-7.

Example: An investment of \$5,000 earns an income of \$300 a year.
Express this income on the investment as a common fraction,
a decimal fraction, and a rate per cent.

Common fraction	$\frac{300}{5,000} = \frac{3}{50} = \frac{6}{100}$
Decimal fraction	$\frac{6}{100} = 0.06$
Rate per cent	$= 6\%$

Example: Find $12\frac{1}{2}\%$ of \$1,500.

From the table, the common fraction equivalent to
 $12\frac{1}{2}\% = \frac{1}{8}$.

$\therefore 12\frac{1}{2}\%$ of \$1,500

$$= \frac{1}{8} \times \frac{1500}{1}$$

$$= \$187.50$$

WORKOUT EXERCISE VI

Change the following rates per cent to common fractions or mixed numbers:

1. $17\frac{1}{2}\%$ 2. $318\frac{1}{4}\%$ 3. 74.25% 4. $83\frac{2}{3}\%$ 5. $79\frac{2}{11}\%$

Change the following common fractions or mixed numbers to rates per cent, correct to one decimal place:

6. $\frac{5}{6}$ 7. $\frac{5}{27}$ 8. $\frac{110}{99}$ 9. $\frac{3}{17}$ 10. $\frac{8}{13}$ 11. $\frac{5}{7}$ 12. $\frac{4}{11}$

Change the following rates per cent to decimal fractions:

13. $73\frac{1}{2}\%$ 14. $7\frac{7}{8}\%$ 15. $37\frac{1}{41}\%$ 16. $7\frac{3}{7}\%$ 17. $8\frac{3}{91}\%$
18. $3\frac{1}{35}\%$ 19. $4\frac{9}{55}\%$ 20. $3\frac{5}{47}\%$

Change the following decimal fractions to rates per cent:

21. 0.116 22. 1.329 23. 0.742 24. 0.875 25. 0.115
26. 3.01 27. 2.001 28. 0.13 29. 13.02 30. 0.132
31. $7.07\frac{1}{2}$ 32. $0.01\frac{1}{2}$ 33. $0.73\frac{1}{8}$ 34. $0.79\frac{1}{3}$ 35. 3.3

Express the following incomes on the respective investments as a common fraction; a decimal fraction, to 3 decimal places; and the rate per cent as a mixed number:

	Investment	Income		Investment	Income
36.	\$ 5,000	\$ 400	37.	\$ 4,500	\$ 350
38.	550	25	39.	7,750	625

- | | Investment | Income | | Investment | Income |
|-----|------------|--------|-----|------------|--------|
| 40. | 10,380 | 950 | 41. | 20,150 | 1,750 |
| 42. | 30,875 | 2,000 | 43. | 19,845 | 1,650 |
| 44. | 100,000 | 6,000 | 45. | 75,855 | 4,500 |
46. Memorize the equivalent rates per cent of the common fractions given in figure 6-7.
47. Extend figure 6-7 to include sixteenths.
48. Find $59\frac{1}{2}\%$ of \$525.
49. Find $66\frac{2}{3}\%$ of \$1,329.
50. Find $16\frac{2}{3}\%$ of \$7,836.42.
51. Find $42\frac{1}{2}\%$ of \$4,104.48.
52. Find $6\frac{1}{4}\%$ of \$4,960.00.
53. Find $8\frac{1}{3}\%$ of \$8,532.
54. Find $4\frac{1}{4}\%$ of \$8,532.
55. Find $87\frac{1}{2}\%$ of \$3,793.44.
56. Find $93\frac{1}{2}\%$ of \$4,959.36.
57. Find $18\frac{3}{4}\%$ of \$2,828.

Figure 6-7

Some Common Fractions with Equivalent Rates Per Cent and Decimal Fractions

$\frac{1}{2}$ 50% = 0.5	$\frac{1}{4}$ 25% = 0.25	$\frac{1}{3}$ $33\frac{1}{3}\%$ = $0.\dot{3}$	$\frac{1}{5}$ 20% = 0.2	$\frac{1}{6}$ $16\frac{2}{3}\%$ = $0.1\dot{6}$	$\frac{1}{10}$ 10% = 0.1
	$\frac{3}{4}$ 75% = 0.75	$\frac{2}{3}$ $66\frac{2}{3}\%$ = $0.\dot{6}$	$\frac{2}{5}$ 40% = 0.4	$\frac{5}{6}$ $83\frac{1}{3}\%$ = $0.8\dot{3}$	$\frac{3}{10}$ 30% = 0.3
			$\frac{3}{5}$ 60% = 0.6	$\frac{1}{8}$ $12\frac{1}{2}\%$ = 0.125	$\frac{7}{10}$ 70% = 0.7
			$\frac{4}{5}$ 80% = 0.8	$\frac{3}{8}$ $37\frac{1}{2}\%$ = 0.375	$\frac{9}{10}$ 90% = 0.9
				$\frac{5}{8}$ $62\frac{1}{2}\%$ = 0.625	$\frac{1}{12}$ $8\frac{1}{3}\%$ = $0.08\dot{3}$
				$\frac{7}{8}$ $87\frac{1}{2}\%$ = 0.875	$\frac{5}{12}$ $41\frac{2}{3}\%$ = 0.417
					$\frac{7}{12}$ $58\frac{1}{3}\%$ = 0.585
					$\frac{11}{12}$ $91\frac{2}{3}\%$ = $0.91\dot{6}$

SECTION 4 Basic Elements of Percentage Problems

Percentage problems are basically problems of comparison, and you must learn thoroughly the elements involved so that you will recognize and be able to interpret them in any problem you have to solve. They are three in number:

(a) *The Base*: This is the basis or the standard, or the whole, with which we do the comparing. The base in percentage problems has the same function as the base in the ratio problems we have already studied.

(b) *The Percentage*: This is the number or the quantity which is to be compared. It is sometimes called the "amount".

(c) *The Rate Per Cent*: This is the result of comparing the percentage to the base and expressing this ratio in the form of a per cent, a fraction, or a decimal.

Example: Three yards of cloth are cut from a bolt containing 36 yards. What per cent of the bolt of cloth is used?

This can be restated as:

What per cent (rate) of 36 yards (base) is 3 yards (percentage)?

Required to find: the rate.

Elements known: the base and the percentage.

Example: 12% of a class of students failed in mathematics. If there are 50 students in the class, how many students failed?

or

How many students failed (percentage) if 12% (rate) out of a class of 50 (base) failed?

Required to find: the percentage.

Elements known: the base and the rate.

Example: The Free-Craft Hobby Store report that sales this month are only 40% of sales in December last. How much was sold in December last if this month's sales amount to \$15,000?

or

This month's sales of \$15,000 (percentage) are 40% (rate) of last December sales (base).

Required to find: the base.

Elements known: the rate and the percentage.

In the first of the three examples above we have to find the *rate* when the base and the percentage are known; in the second example we are required to find the *percentage* when we know the base and the rate; in the last example we are given the rate and the percentage and are required to find the *base*.

These examples represent the three basic types of percentage problems which you will be asked to solve. If you expect to solve any problem in mathematics, there are two fundamentals you must remember:

1. Understand the problem. Read the problem, analyse it, and then write down: (a) what you are looking for; (b) what is given; (c) what relationship exists between what you are given and what you are looking for.
2. Decide how you can find what you want to know from the information you already have.

If it is a problem in percentage, then the following three steps should be followed:

1. Analyse the problem thoroughly to make sure you know what it means; then write down what you are looking for: a base, a percentage, or a rate.
2. Establish from the problem what is to be compared with what. Write down and identify the elements (base, rate, percentage) given in the problem.
3. Solve the problem for the unknown element by using its relationship with the elements that are known. This technique will be described in the following sections.

WORKOUT EXERCISE VII

In the following problems identify the base, the rate, and the percentage stating the element that is required. (Do not attempt to solve the problems.)

1. Four inches are cut from a 48" length of tubing. What per cent of the length of tubing is left?
2. 20% of a class of students were chosen for the school Christmas play. If there were 75 students in the class, how many students were not chosen?
3. The present year's sales of the Lazy Dazy Products amount to 125% of last year's sales. If last year's sales were \$1,500,000, to how much did this year's sales amount?
4. On a mathematics test you do 20 problems. If you do 80% correctly, how many are incorrect?

5. The school hockey team wins 75% of the games it plays. If it plays 32 games, how many does it win?
6. Your team wins 80% of the games it plays. If it wins 20 games, how many did it play altogether?
7. Sally and Sammy win 60% of the sets of tennis they play in a tournament; if they win 9 sets, how many did they play altogether?
8. Pierre Piper earns \$390 a month and pays out \$105 a month for rent. What per cent of his income does this represent?
9. The Grosvenor family spend \$125 a month on food. If Mr. Grosvenor earns \$575 a month, what per cent of his income is spent on food?
10. Mr. Freeman earns \$475 a month and spends 25% of it on food. How much has he left after purchasing the groceries?

SECTION 5 Finding the Percentage

Let us first obtain a formula to calculate the *percentage* when the base and rate are known. In the second example in Section 4 we are asked to find how many students failed a mathematics examination if 12 per cent of a class of 50 failed. As per cent means "hundredths", the expression "per cent of" means "hundredths of", or "hundredths times". Therefore, if we wish to find a per cent of a number (e.g., 12 per cent of 50), we must multiply the number (base, 50) by the per cent value (rate, $\frac{12}{100}$). We can re-state our problem thus:

The Percentage is equal to the Base $\times$ Rate, or

$$\text{Percentage} = \text{Base} \times \text{Rate}.$$

Using only the first letter of each word, we write the formula:

$$P = B \times R$$

This formula is a very important one to understand and to remember.

Note: In percentage problems it is important to remember that the base, and *only the base*, is multiplied by the rate per cent.

When solving problems using this formula, the rate may be written either as an equivalent fraction or as a decimal fraction.

An Equivalent Fraction

This is a simple common fraction equivalent to the rate per cent. See figure 6-7.

Example: How many students failed a mathematics examination if 12% of a class of 50 failed.

$$\text{Equivalent fraction for } 12\% = \frac{12}{100}.$$

$$P = B \times R$$

$$= 50 \times \frac{12}{100}$$

$$= \frac{600}{100}$$

$$= 6$$

$\therefore$ 6 students out of the class of 50 failed in mathematics.

A Decimal Fraction

See figure 6-7.

Example: A salesman earns 20% on his total sales. If he sells \$150 worth of goods in one week, how much commission does he earn?

$$\text{Decimal fraction for } 20\% = 0.20.$$

$$P = B \times R$$

$$= 150 \times 0.20$$

$$= 30$$

$\therefore$ On sales amounting to \$150, the salesman makes \$30.

WORKOUT EXERCISE VIII

Find the percentage to two decimal places using equivalent fractions:

- | | |
|---------------------------------------|---------------------------------------|
| 1. Base: 155, Rate: $62\frac{1}{2}\%$ | 2. Base: 125, Rate: 125% |
| 3. Base: 728, Rate: 27% | 4. Base: 365, Rate: $6\frac{1}{4}\%$ |
| 5. Base: 711, Rate: $45\frac{1}{3}\%$ | 6. Base: 240, Rate: 18% |
| 7. Base: 36, Rate: 78% | 8. Base: 418, Rate: $16\frac{2}{3}\%$ |
| 9. Base: 408, Rate: $62\frac{1}{2}\%$ | 10. Base: 72, Rate: 27% |

Find the percentage to two decimal places using decimal fractions:

- | | |
|---|---|
| 11. Base: 49, Rate: $62\frac{1}{2}\%$ | 12. Base: 712, Rate: $105\frac{3}{4}\%$ |
| 13. Base: 47.5, Rate: $6\frac{7}{8}\%$ | 14. Base: 76, Rate: $35\frac{1}{8}\%$ |
| 15. Base: 105.45, Rate: $74\frac{1}{4}\%$ | 16. Base: 52, Rate: $110\frac{1}{10}\%$ |
| 17. Base: 333, Rate: $16\frac{1}{16}\%$ | 18. Base: 412, Rate: $112\frac{1}{2}\%$ |
| 19. Base: 365, Rate: $8\frac{1}{2}\%$ | 20. Base: 500, Rate: $15\frac{1}{4}\%$ |

21. Madge Mitten, a saleslady in the glove department of the Nu-Style Ladies' Wear, receives a salary of \$40 a week, plus a commission of 3% on any sales in excess of \$500. Last week her sales amounted to \$864.72. How much did she earn?
22. 12% of a class of students failed in mathematics. If there are 50 students in the class, how many students passed?
23. Problem 5, Workout Exercise VII.
24. Problem 10, Workout Exercise VII.
25. Tom Johnston spends 25% of his \$10 allowance on lunches and 15% on transportation. How much does he spend on each, and how much does he have left?
26. Raymond's father earns $5\frac{1}{2}\%$ on an investment of \$1,500 and $4\frac{3}{4}\%$ on an investment of \$1,200. How much does he earn?
27. Cap Caldwell purchased a red sports car for \$3,950. After Cap drove it for two years, a dealer offered Cap 55% of its original value on a trade-in. How much did the dealer offer him?
28. During a Trans-Canada sale \$1,375,422 worth of shirts were sold at a 20% reduction. What was the amount of the reduction? How much did the sales amount to?
29. 28% of the 3,600 employees of Basher Bros., Ltd., are under 25 years of age. How many employees are over 25 years of age?
30. Trevor Holmes earns \$75.00 a week after taxes. He budgets his money as follows: food, 30%; rent, 25%; clothing, $16\frac{2}{3}\%$; recreation, 10%; charity, $6\frac{1}{3}\%$; savings, 12%. How much can he spend on each? Draw a pie diagram showing both the percentage and the dollar amount.
31. Problem 2, Workout Exercise VII.
32. Problem 3, Workout Exercise VII.

SECTION 6 Finding the Base

Let us transpose our formula so that we may calculate the *base* when the percentage and the rate are known.

Our basic formula is $P = B \times R$.

This time it is the *base* we are seeking; so we must have the B alone on one side of our equation.

Divide both sides of the equation by R.

$$\frac{P}{R} = \frac{B \times R}{R}$$

$$\frac{P}{R} = B$$

$$\text{or} \quad B = \frac{P}{R}$$

That is, the base is equal to the percentage divided by the rate.

A problem which must be solved for the base is a problem in which a number must be found when a certain per cent of it is given.

Example: The profit on the sale of a vacuum cleaner amounted to \$15 which was 12% of the selling price. For how much did the vacuum sell?

$$P = \$15; R = 12\%; B = ?$$

$$B = \frac{15}{0.12} = \frac{15}{0.12} \times \frac{100}{100} = \frac{1500}{12} \text{ or } \frac{15}{\frac{12}{100}} = \frac{15}{1} \times \frac{100}{12}$$

$$= \$125$$

Example: \$24 is 75% of what amount?

$$P = \$24; R = 75\% = 0.75; B = ?$$

$$B = \frac{24}{0.75} = \frac{2400}{75}$$

$$= 32$$

$\therefore$ \$24 is 75% of \$32.

Example: A 2% discount is allowed on cash sales at the Wholesale Haberdashery. If I am allowed a discount of \$2.35, what was the amount of my purchase?

$$P = \$2.35; R = 2\% = 0.02; B = ?$$

$$B = \frac{2.35}{0.02} = \frac{235}{2}$$

$$= \$117.50$$

$\therefore$ \$117.50 was the amount of my purchase.

Note: The base is usually immediately preceded by an expression such as *more than*, *of*, *to*, *as large as*, or *as small as*.

WORKOUT EXERCISE IX

- 15 is 25% of what number? 50% of what number? 75% of what number?
- 30 is $33\frac{1}{3}\%$ of what number? 50% of what number? 200% of what number?

3. 119 is 25% of what number? 35% of what number? 140% of what number?
 4. 50 is 20% as large as what number? 75% as large as what number?
 5. 30 is 25% as large as what number? 125% as large as what number?
 6. 63 is 15% as large as what number? 115% as large as what number?
 7. 40 is 20% more than what number? (Hint: 40 is 120% of the base.)
 8. 14 is 30% more than what number? 60% more than what number?
 9. 21 is 40% more than what number? 75% more than what number?
 10. \$13 is 15% of what amount? 45% of what amount? 120% of what amount?
 11. \$12.50 is 250% of what amount? 75% of what amount?
 12. \$45.20 is 125% of what amount?
 13. 35 is 10% of ? 20% of ? 75% of ? 125% of ?
 14. \$137.50 is 15% of ? 85% of ? $66\frac{2}{3}\%$ of ? 110% of ?
 15. 6% of what number is 60? is 75? is 110? is 250?
 16. The Tru-Craft Hobby Store report that sales this month are only 40% of sales in December last. How much was sold in December last if this month's sales amount to \$15,000?
 17. Problem 6, Workout Exercise VII.
 18. Problem 7, Workout Exercise VII.
 19. The Do-alright Dredge Co. make $18\frac{1}{2}\%$ profit on all their contracts. If their profits for the first three months of the year amounted to \$1,750.50, to how much did their contracts amount?
 20. The Teabuck Department Store allow customers $1\frac{1}{2}\%$ discount on all cash purchases. If Jane July receives \$39.75 in discounts in one year, how much did she spend at the Teabuck Department Store?
 21. Approximately 772,000 people in Canada live in small villages containing 1,000 to 3,000 people. If these people represent 4.80% of the population of Canada, what is Canada's total population to the nearest thousand?
 22. The Clothiers Discount House offer the following discounts:
 - Coats, 80% of the marked price; sell, after deduction of discount, for \$53;
 - Dresses, 90% of the marked price; sell, after deduction of discount, for \$15;
 - Hats, 60% of the marked price; sell, after deduction of discount, for \$5;
 - Suits, 75% of the marked price; sell, after deduction of discount, for \$45;
- What was the marked price of the coats, dresses, hats, and suits?

23. Claude Collins paid the Receiver-General of Canada \$1,320.54 in income tax last year. If this represented 15% of his salary, how much did he earn?
24. Billy Brash receives a salary of \$45.00 a week and $12\frac{1}{2}\%$ commission on any sales in excess of \$500. If he wishes to earn \$120 a week, how much must he sell?
25. Simon Slow is paid a basic wage of \$50.00 a week and a commission of $7\frac{1}{2}\%$ on sales in excess of \$200. If he wishes to earn \$110 a week, how much must he sell?
26. The Realty Sales received \$22,500 commission on the sale of a hotel. If they charged $7\frac{1}{2}\%$ commission, what was the value of the property?
27. The population of a Canadian province is 114.1% of what it was at the previous census. If it is now 4,628,400, what was it at the last census, to the nearest hundred?
28. Royal Flush rented his home for \$125 a month. He paid expenses on the house for: taxes, \$425.15; insurance, \$36.50; repairs, \$215.75. His net income (income less expenses) from the house represents 12% of the cost of the house. How much did the house cost him?
29. Masher, the Mover, hired a collection agency to collect an overdue account. The agency collected 80% of the debt and mailed a cheque for \$76.80 from which they had deducted 20% of the amount collected. How much was the original debt, and how much was the agency's fee?
30. The human body is composed of 65% water. If the water in Paul's body weighs 88 lb., how much does Paul weigh?

SECTION 7 Finding the Rate

Let us again transpose our formula to calculate the *rate* when the percentage and the base are known. Once again we refer back to our basic formula:

$$P = B \times R$$

This time we have to solve the equation for the rate, so we divide each side by B.

$$\frac{P}{B} = \frac{B \times R}{B}$$

$$\frac{P}{B} = R$$

$$\text{or} \quad R = \frac{P}{B}$$

That is, the rate is equal to the percentage, or amount, divided by the base.

Example: \$120 was earned in one year on an investment of \$2,000.
What was the annual rate of interest?

$$P = \$120; B = \$2,000; R = ?$$

$$R = \frac{120}{2,000}$$

$$= 0.06$$

$$\therefore \text{the rate} = 0.06 \text{ or six hundredths} \\ = 6\% \text{ per annum}$$

A problem which asks what per cent one number is of another is looking for the *rate*.

Example: What per cent of 750 pounds is 33 pounds?

$$P = 33; B = 750; R = ?$$

$$R = \frac{33}{750} = \frac{11}{250} = 0.044$$

$$= 4.4\%$$

Example: The Q-T Department Stores sales amounted to \$250,000 this year. If the sales last year were \$200,000, what per cent are this year's sales of last year's sales?

$$P = 250,000; B = 200,000; R = ?$$

$$R = \frac{250,000}{200,000}$$

$$= \frac{5}{4}$$

$$= 1.25$$

$$= 125\% \text{ of last year's sales.}$$

WORKOUT EXERCISE X

Give answers to two decimal places. Estimate your answers first.

1. What per cent of 660 is 44? 60? 120? 275? 420?
2. What per cent of 1,200 is 550? 320? 990? 1,500? 2,080?
3. What per cent of \$306.75 is \$35.00? \$105.50? \$275.75?
4. What per cent of 2,000 pounds is 750 pounds? 1660 pounds? 2400 pounds?

5. What per cent of 350 gallons is 75 gallons? 420 gallons? 390 gallons?
6. 24 is what per cent of 48? of 100? of 145? of 310?
7. 75 is what per cent of 300? of 400? of 500? of 600? of 75?
8. 178 is what per cent of 90? of 100? of 200? of 496?
9. \$88 is what per cent of \$1,200? of \$1,800? of \$2,400?
10. 135 pounds is what per cent of 700 pounds? of 200 pounds? of 100 pounds?
11. Three yards of cloth are cut from a bolt containing 36 yards. What per cent of the bolt of cloth is used?
12. Problem 1, Workout Exercise VII.
13. Problem 8, Workout Exercise VII.
14. Problem 9, Workout Exercise VII.
15. Spike Boots earns a yearly income on the following investments: \$150 on \$1,750; \$250 on \$3,500; \$120 on \$1,500; \$175.50 on \$2,500. What was the annual rate of interest on each investment? What was the total rate of interest on the total investment?

In the following problems solve for the base, the rate, or the percentage as may be required.

16. Cab Clayton earns \$350 a month after paying taxes. He pays each month: \$105 for rent, \$11.95 for electricity, \$5.80 for telephone, and 31% of his income for food. Incidental expenses amount to \$575 a year. If he saves the remainder, what per cent of his income does he save? How much a year does this amount to?
17. Penny Wise received \$77.50 last week as salary. She is paid a basic wage of \$40.00 a week and a commission on all sales over \$2,000. If her sales amounted to \$4,500 last week, what rate of commission does she receive on sales over \$2,000?
18. Ima Clod receives a monthly salary of \$150 and 10% commission on sales over \$5,000 a month. In January, February, and March her sales amounted to \$3,760, \$6,900, and \$6,420 respectively. How much did she earn in the three months? What was her average monthly salary?
19. Flash Back sells three types of shirts; one for \$4.00, one for \$5.50, and one for \$7.50. On the first type he receives a commission of $7\frac{1}{2}\%$, on the second 10%, and on the third $12\frac{1}{2}\%$. In one week he sold 80 at \$4.00, 85 at \$5.50, and 70 at \$7.50. How much commission does he earn?
20. Gadget & Gimmick Ltd., buy \$500 of gimmicks. They sell half of them at a loss of 5%. They sell the second half at a price which gives them a profit of 25% on the entire purchase. For how much did they sell the first half? The second half? What per cent gain is made on the second half?

21. Royal Flush receives a commission of 5% on all his sales and an additional 3% on any sales above \$10,000 in any month. If his sales last month amounted to \$11,500, how much commission did he earn? This month his sales are 30% higher than last month; how much will he earn this month?
22. Tab Talbot receives a commission on all sales of $3\frac{1}{2}\%$. How much must he sell each week in order to earn \$75.00 a week?
23. Spud Masher carries stock valued at \$10,000 in his store. On this stock he makes 25% profit and is able to sell six times this amount of stock in one year. (This is called stock turnover.) If he reduced his profit to 20%, he could turn his stock over eight times. Would it pay him to do so? What would be the difference in his profit?
24. Gimmick Bros., carry a \$75,000 stock of stumbling blocks on which they make $33\frac{1}{3}\%$ profit and which turns over 7 times. By lowering their profit to 25%, they can increase their turnover to 9 times. What effect would this have on their profit?
25. According to a recent survey, 7% of Canada's total labour force is unemployed. If the unemployed persons number 448,000, what is Canada's total labour force, to the nearest thousand? If 74% of the total labour force are male, how many women are included in the total labour force? If 11% of the total labour force are employed in agricultural work, how many individuals are employed in this type of work?
26. Hittem Hard Ltd., collect overdue accounts for individuals and business firms. They charge a fee of 15% of the amount collected. Last month, they collected for a doctor 75% of a debt of \$210, 60% of a debt of \$175, and 80% of a debt of \$95. How much did the agency make, and how much did the doctor receive in payment?
27. Albert Brittle paid \$249.50 for a Freezerup Cooler on which he had paid a 3% Provincial tax. What was the price before the Provincial tax was paid?
28. Miss Take purchased a Kant-Miss set which was reduced 20% from the marked price of \$45.00. If she paid a 3% Provincial tax on her purchase, how much did she pay for the set?
29. Round Holes Ltd., employ 77 men to manufacture square pegs. If this is 140% of the number of men they employed last year, how many were employed last year? Last year the men earned \$1.00 per hour; this year they earn 125% of last year's hourly wage. How much more per hour does the company pay this year?
30. Brick & Brie Company import cheese worth \$195.00. They sell $\frac{1}{3}$ of the stock at a loss of 5%. If they wish to make a profit of 20% on their original purchase, for how much must they sell the remaining $\frac{2}{3}$ of the stock? What per cent of profit does this represent?

31. The Trans-Canada Highway is 4,891 miles long crossing the various Provinces and Parks as follows: Newfoundland, 554 miles; Prince Edward Island, 71 miles; Nova Scotia, 318 miles; New Brunswick, 390 miles; Quebec, 400 miles; Ontario, 1,453 miles; Manitoba, 309 miles; Saskatchewan, 406 miles; Alberta, 282 miles; British Columbia, 568 miles; Parks, 140 miles. What per cent of the highway, to two decimal places, passes through each? Draw a bar graph to illustrate this information.

32. Toronto Maple Leafs won the hockey game last Saturday night by 4 goals to 2 over New York Rangers. Which of the following statements do you consider correct? (A) Maple Leafs won by 50%. (B) Maple Leafs won by 100%. (C) Rangers lost by 2 goals. (D) Maple Leafs are twice as good as the Rangers. State the result in another way which you consider suitable.

SECTION 8 Per Cent of Increase or Decrease

Problems of this type can be troublesome if the basic elements of problem solving outlined in Section 4 are not thoroughly mastered. A change in a few words can change the meaning of a problem and lead to a different solution because a different base is selected.

To find the per cent by which one number is greater, or less than, another number, it is necessary first to find the amount of difference between the two numbers. This represents the *percentage*. Then find the per cent this number is of the whole by comparing it to the correct base.

Example: In 1960 the population of Fieldtown was 72,000. In 1964 it was 84,000.

(a) The 1964 population was what rate per cent of the 1960 population?

$$P = 84,000; B = 72,000; R = ?$$

$$R = \frac{84,000}{72,000} = \frac{7}{6}$$

$$= 1.1667$$

$$= 116.67\%$$

$\therefore$ The 1964 population was 116.67% of that in 1960.

(b) What was the rate per cent increase in the 1964 population?

$$\text{Increase} = 84,000 - 72,000 = 12,000$$

$$P = 12,000; B = 72,000; R = ?$$

$$R = \frac{12,000}{72,000} = \frac{1}{6}$$

$$= 0.16$$

$$= 16.67\%$$

∴ The 1964 population increased 16.67% over the population of the base year 1960.

(c) The 1960 population was what rate per cent of the 1964 population?

$$P = 72,000; B = 84,000; R = ?$$

$$R = \frac{72,000}{84,000} = \frac{6}{7} = 0.8571$$

$$= 85.71\%$$

∴ The 1960 population was 85.71% of the 1964 population.

Example: Mr. Sample is today receiving a salary which is 120% more than he received five years ago when he was paid \$40 a week. How much does he receive today?

As he is paid 120% *more than* he was paid 5 years ago, he must be receiving (100% + 120%) or 220% of the salary he received five years ago.

$$R = 220\%; B = \$40; P = ?$$

$$P = 2.20 \times 40$$

$$= \$88$$

∴ Mr. Sample today receives a salary of \$88 a week.

Example: A dress, priced originally at \$35, is reduced to \$27.50. By how much per cent is the price of the dress reduced?

$$P = \text{difference} = \$7.50; B = \$35; R = ?$$

$$R = \frac{7.50}{35}$$

$$= 0.2143$$

∴ The price is decreased 21.43%.

Example: The total sales of the Fun-Time Clothiers in August this year were 16% more than the total sales in August last year. If the sales this year amounted to \$108,750, what was the sales figure for last year?

Let the sales for last year = 100% because we are comparing this year's sales with last year's sales and it is the unknown base.

Then this year's sales = 116%.

$P = \$108,750$; $R = 116\%$; $B = ?$

$$B = \frac{108,750}{1.16}$$

$$= \$93,750$$

$\therefore$ Sales last year were \$93,750.

Example: The Sentinel Apartments reduced their monthly rent by $12\frac{1}{2}\%$ to \$105. What was the previous rent?

Present rent = 87.5% of previous rent.

$R = 87.5\%$; $P = \$105$; $B = ?$

$$B = \frac{105}{0.875} = \frac{105,000}{875}$$

$$= \$120$$

$\therefore$ Previous rent amounted to \$120.

WORKOUT EXERCISE XI

Find by what per cent the first number is greater than the second:

- | | | |
|-----------------|-----------------|----------------|
| 1. 25 and 20. | 2. 14 and 21. | 3. 10 and 8. |
| 4. 25 and 10. | 5. 10 and 6. | 6. 125 and 75. |
| 7. 750 and 300. | 8. 490 and 280. | 9. 95 and 50. |
| 10. 368 and 42. | | |

Find by what per cent the first number is smaller than the second:

- | | | |
|------------------|------------------|-----------------|
| 11. 20 and 25. | 12. 21 and 14. | 13. 8 and 10. |
| 14. 10 and 25. | 15. 6 and 10. | 16. 75 and 125. |
| 17. 300 and 750. | 18. 280 and 490. | 19. 50 and 95. |
| 20. 42 and 365. | | |
21. What number is 20% greater than 45? than 65? than 95?
22. What number is 5% smaller than 20? than 180? than 760?

23. 60 increased by 20% of itself is what number? Decreased by $8\frac{1}{3}\%$ of itself is what number?

Increase:

24. \$10 by 250%. 25. \$17.50 by 25%. 26. \$99.75 by 30%.
 27. \$76.45 by 115%. 28. \$49.95 by 125%. 29. \$113.75 by 45%.
 30. \$70 by $\frac{3}{4}\%$.

Decrease:

31. \$4.95 by 40%. 32. \$14.95 by 100%. 33. \$10.95 by 33%.
 34. \$17.49 by 45%. 35. \$1,428.15 by 45%.
 36. Bill Body receives a salary of \$95 a week; Chris Cody receives a salary of \$45 a week and earns a commission of $1\frac{1}{2}\%$ on all his sales. At the end of 16 weeks Cody's sales amounted to \$78,420. Who has earned the larger income? By what per cent is it larger?
 37. Sam Tiller purchased a house for \$18,000. He was later transferred to another city and sold his house for \$21,000. What per cent of increase (or profit) did this amount to?
 38. The So-Rite Stores sales for December amounted to \$250,000. If the sales for November amounted to \$200,000, what per cent of increase did the December sales show?
 39. The population of Canada according to a recent survey was 17,442,000 made up as follows: Newfoundland, 449,000; Prince Edward Island, 102,000; Nova Scotia, 716,000; New Brunswick, 590,000; Quebec, 4,999,000; Ontario, 5,952,000; Manitoba, 885,000; Saskatchewan, 902,000; Alberta, 1,243,000; British Columbia, 1,570,000; Yukon Territory, 13,000; Northwest Territories, 21,000. Prepare a table with columns headed: Province, Population, Per Cent of Population; calculate the per cent of population in each province to two decimal places. Use the information in this table to draw a vertical bar chart which will compare the population in the provinces.
 40. Solly Graham withdrew 46% of his savings. He used 70% of the amount he withdrew to purchase an automobile for \$2,675. How much has Solly left in his savings account?
 41. Renfrew Upislunch purchased a car for \$2,500 two years ago. If the car is now worth \$1,000, describe the decrease in three different ways. Which way do you consider is the most satisfactory?

SECTION 9 Sets Again

Variables

In previous sections, a dash has been used to indicate a required member, or members, of a set. Instead of using the expression "— is an even number", or " $— > 2$ ", we use a symbol such as x or y to

represent the member or members of the set to which we refer.

This symbol is called a *variable* as it may be replaced by a member of any specified set and may have varying values. Thus “— is an even number” becomes “ x is an even number”:

“— > 2 ” becomes “ $x > 2$ ”

Example: If T is the set of all the natural numbers, use the letter x as the variable to rewrite “all numbers greater than 9”.

$$x > 9$$

Example: Let $S = \{3, 4, 5, 6, 7\}$ and x be a variable. What members of this set make the following statements true?

x is an even number. Solution (4,6)

$x > 5$ Solution (6,7)

$x < 5$ Solution (3,4)

WORKOUT EXERCISE XII

Rewrite, using the variable z to replace the dash:

- | | | |
|------------|-------------|---------------|
| 1. — > 5 | 2. — $= 4$ | 3. — $\neq 7$ |
| 4. — < 9 | 5. — > 71 | |

If T is the set of all the natural numbers, re-write using the variable x :

- | | |
|---|------------------------------|
| 6. All numbers not equal to 87. | 7. All numbers more than 13. |
| 8. All numbers less than 70. | 9. All numbers equal to 4. |
| 10. All numbers greater than the sum of 7 and 10. | |
| 11. All numbers less than the difference between 12 and 10. | |

Let $S = \{3, 4, 5, 6, 7\}$, and let p = variable. List the members of the set which make the following true:

- | | | |
|----------------------------|---------------|---------------------------|
| 12. $p > 4$. | 13. $p < 5$. | 14. p is an odd number. |
| 15. p is an even number. | | |

Let $S = \{1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{2}\}$, and let s = variable. List the members of the set which make the following true:

- | | |
|------------------------------|------------------------------------|
| 16. $s > 1\frac{1}{4}$. | 17. $s < 2$. |
| 18. s is a natural number. | 19. s is an odd number. |
| 20. s is an even number. | 21. $s < 1$. |
| 22. s is a prime number. | 23. s is exactly divisible by 2. |
| 24. $s \neq 2\frac{1}{4}$. | |

List the members of the following sets:

25. The prime numbers between 1 and 20.
26. The natural numbers between 15 and 16.
27. The names of the months with the first letter J.
28. The names of the Canadian provinces with the first letter N.
29. The numbers less than 125 that are exactly divisible by 17.
30. The months of the year containing 30 days.

Describe the following sets:

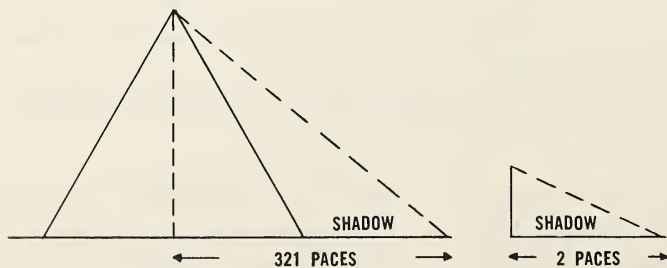
- | | |
|--|--|
| 31. $V = \{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3\}$ | 32. $U = \{2, 3, 5, 7, 11, 13, 17, 19\}$ |
| 33. $S = \{2, 4, 6\}$ | 34. $T = \{10, 12, 14, 16\}$ |
| 35. $R = \{\text{Tuesday, Thursday}\}$ | 36. $P = \{2, 4, 6, 8\}$ |
| 37. $R = \{2\}$ | 38. $N = \{1, 3, 5, 7, 9\}$ |
| 39. $M = \{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}\}$ | 40. $T = \{A, B, C, \dots, Z\}$ |

SECTION 10 Figuring for Fun

1. Once upon a time, between 500 and 600 B.C., there was a philosopher named Thales, one of the Seven Wise Men of Greece, who was renowned as an astronomer and a mathematician. During a holiday in Egypt, he measured the heights of the pyramids in the following manner. He placed a stick upright in the sand, and at noon measured (in paces) the length of the shadow cast by the stick and the length of the shadow cast by the pyramids. The length of the shadow cast by the largest of the pyramids, the Great Pyramid of Khufu or Cheops (built about 2900 B.C.), was 321 paces from the centre of the base. The length of the shadow cast by the stick was 2 paces. If the stick was 36" long and Thales' pace about 30" long, how high is the Great Pyramid, according to Thales?



Figure 6-8



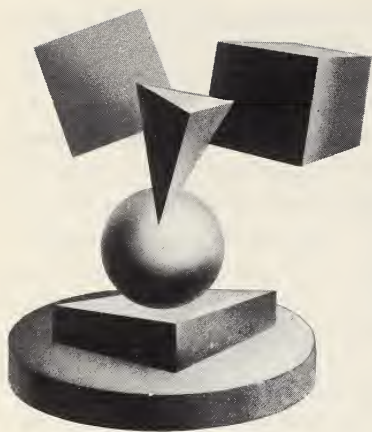
NOT TO SCALE

2. You are given an 8-quart jug full of milk and an empty 5-quart jug and an empty 3-quart jug. You are required to have two containers of milk each holding 4 quarts. If you can only make use of these three vessels, how would you accomplish it?

3. Try Dad on a percentage problem. Ask him to think of any number. (As an example, 12.) Then:

- (a) Take 500% of the number, (60)
- (b) Add 6, (66)
- (c) Take 400% of the result, (264)
- (d) Add 9, (273)
- (e) Take 500% of the result, (1,365)
- (f) Tell you the result.

From this result you will subtract 165, and the answer will be 1% of this answer: $(1,365 - 165 = 1,200)$ 1% of 1,200 = 12, which is the original number.



Mensuration

7

A Business Problem

An exhibition of hardware is to be held in the Bideawee Hotel at which hardware manufacturers are invited to display their products. Space can be rented at \$2.50 a square foot.

The advertising manager for the Long Stand Products has had a number of displays designed and will need the following space:

- a table 8 ft. wide and 23 ft. long,

- a triangular floor area with the 20 ft. base 14 ft. away from the wall, and

- a circular table with a radius of 4 ft. plus a 1 ft. walkway around the table.

A further display is to have a counter 1 ft. wide completely surrounding the display, which itself occupies a 5 ft. square, with $1\frac{1}{2}$ foot walkway around the counter.

How much will the Long Stand Products pay for this space?

SECTION 1 Linear or Line Measurement

The unit of length used in the British Commonwealth and the United States is the yard. The yard was originally the length of the outstretched arm, measured from shoulder to finger-tip, of King Henry I of England (A.D. 1100 - 1135). The foot was the length of the human foot; the inch, the length of the thumb to the first joint. The inch was later taken to be the length of three grains of barley. As can be seen, the measured length would depend upon the dimensions of the person doing the measuring, and a great deal of variation could occur. Nowadays, the standard yard is authorized by Act of Parliament, and an accurately measured bronze bar is preserved in the Standards Office of the British Board of Trade in London, England.

The inch, the foot, the rod, and the mile are units derived from the yard.

British Linear or Line Measurement

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards	= 1 rod (rd.)
320 rods	= 1 mile
1760 yards	= 1 mile
5280 feet	= 1 mile
1 nautical mile	= 1.15 land miles.

Example: The distance between the front door of the school and the entrance to the playing field is measured as 1.362 miles. How far is this in feet and inches? In rods and yards?

$$\begin{aligned}
 1 \text{ mile} &= 5,280 \text{ feet} \\
 1.362 \text{ miles} &= 5,280 \times 1.362 \text{ feet} \\
 &= 7,191.36 \text{ feet} \\
 &= 7,191 \text{ feet} + \frac{36}{100} \times \frac{1}{4} \text{ inches} \\
 &= \underline{7,191 \text{ feet } 4.32 \text{ inches}}
 \end{aligned}$$

$$\begin{aligned}
 1 \text{ mile} &= 320 \text{ rods} \\
 1.362 \text{ miles} &= 320 \times 1.362 \text{ rods} \\
 &= 435.84 \text{ rods} \\
 &= 435 \text{ rods } \frac{84}{100} \times \frac{1}{2} \text{ yards} \\
 &= \underline{435 \text{ rods } 4.62 \text{ yards}}
 \end{aligned}$$

Rules for Conversion

1. To change a measure of length to larger units, divide the given number of units by the number of times the smaller unit is contained in the larger unit.
2. To change a measure of length to smaller units, multiply the given number of units by the number of times the smaller unit is contained in the larger.

Examples: Change: 93 inches to feet. There are 12 inches in 1 foot.

$$\begin{aligned}
 \therefore 93 \div 12 &= 9\frac{3}{2} = 3\frac{1}{4} = 7\frac{3}{4} \text{ ft.} \\
 &= \underline{7 \text{ ft. } 9 \text{ in.}}
 \end{aligned}$$

Change: 10 yards 2 feet 3 inches to feet.

10 yards	=	10×3	=	30 feet
2 feet			=	2 feet
3 inches	=	$\frac{3}{12}$ feet	=	$\frac{1}{4}$ feet
Total			=	<u><u>$32\frac{1}{4}$ feet</u></u>

WORKOUT EXERCISE I

- How many yards are there in: 104 inches, 74 inches, 78 feet, 37 feet, 27.5 rods, 1 mile, 1.5 miles, 1 nautical mile.
- Change to feet: 63 inches, $2\frac{1}{2}$ miles, 91.64 inches, $1\frac{1}{6}$ miles, 15 yards 3 feet, 3 inches, 115 rods, $11\frac{2}{3}$ yards, $1\frac{1}{4}$ nautical miles, 1 mile 920 yards.
- Change to rods: 1 mile, 10 yards, 47 feet, 583 inches, 4 nautical miles, 1 yard, 10 miles 50 yards, 5 miles 60 yards.
- A swimming pool is $12\frac{1}{4}$ yards wide; how many widths must you swim if you swim a quarter of a mile?
- How many yards of wire must a farmer buy to construct a fence which is 375 feet long? If it costs 2¢ an inch, how much will it cost?
- How many running feet of carpet are required to cover a hall 3 yards 4 feet 6 inches long and 21 steps each with a $6\frac{1}{2}$ -inch rise and 11-inch tread?
- The 440 yard track race covers what fraction of a mile?
- Convert to the next larger unit: 136.3 inches, 4,782 feet, 849 yards, 752.3 rods.
- One length of fence measures 330 inches. If Christopher Nickel purchases 15 lengths, how many feet does he buy? If it is priced at \$1.25 a yard, how much does it cost?
- How many yards of fringe are required to trim the edge of a piece of fabric if the sides are 39 inches, 47 inches, 39 inches, and 47 inches.

SECTION 2 Weights

In the English system the unit of mass is the *pound*. As with the yard, the pound is authorized by Act of Parliament, and an accurately measured pound mass (of platinum) is to be found in the Standards Office of the British Board of Trade in London, England. The ounce, the hundredweight, and the ton are units derived from the pound.

Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2,000 pounds	= 1 ton
2,240 pounds	= 1 long ton

SECTION 3 Capacity – Liquid and Dry

Liquid Measure

In the British system the unit of liquid measure is the *Imperial gallon*, and in the United States, the Winchester or *wine-gallon*.

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
1 Imperial gallon	= 1.201 U.S. gallons
1 U.S. gallon	= 0.833 Imperial gallon

Dry Measure

2 pints	= 1 quart
4 quarts	= 1 gallon
2 gallons	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Dry Measure

Dry measure is used in the British system to measure the volume of solids, such as seeds and grain, which are sold by weight. The units of dry measure for the British and the U.S. Standards are the same.

Example: Lois Denominator averaged $22\frac{1}{2}$ miles per gallon while driving her new car in Canada. During her holidays she drove her car down to the United States. How many miles per gallon would she average while purchasing gasoline in the United States?

1 Imperial gallon gives 22.5 miles
 0.833 Imperial gallon gives 22.5×0.833 miles
 $= 18.74$ miles
 $\therefore$ 1 U.S. gallon gives 18.74 miles per gallon.

WORKOUT EXERCISE II

- Change: (a) 5 bu. 3 pk. 3 qt. 2 pt. to pints.
 (b) 3 gal. 2 qt. 1 pt. 3 gi. To gills.
 (c) 307 oz. to pounds
 (d) 8,306 lb. to tons and pounds.
 (e) 127 gi. to gallons, quarts, pints and gills.
 (f) 38 qt. to gallons and quarts.
 (g) 13,784 lb. to long tons and pounds.

2. How many: (a) Pints in 7 bushels, 3 pecks?
 (b) Gills in 5 gal. 1 pt. 2 gi.?
 (c) Gills in 8 gal. 1 pt. 1 gi.?
 (d) Pecks, quarts, and pints in 67 pints?
 (e) Gallons, quarts, pints and gills in 308 gills?
3. Convert: (a) 9 lb. to ounces.
 (b) 14 lb. 8 oz. to ounces.
 (c) 1,121 pt. to bushels, pecks, quarts and pints.
 (d) 793 lb. to hundredweights and pounds.
 (e) 2,984 oz. to hundredweights, pounds and ounces.
4. If one Imperial gallon of water weighs 10 pounds, how many pounds and ounces, to one decimal place, will 1 wine-gallon of water weigh?
5. There are 60 pounds of potatoes in a bushel. How many pounds of potatoes would be contained in a dry gallon? In a peck?
6. Express the Imperial gallon as a percentage of the wine-gallon. Express the U.S. gallon as a percentage of the Canadian gallon.
7. A bushel of oats weighs 34 pounds. What per cent heavier are potatoes than oats? (See question 5.)
8. What per cent less is the American gallon than the Canadian gallon? What per cent more is the Canadian gallon than the American gallon?

SECTION 4 Arithmetic Manipulation

Adding Mixed Units

Example: Tubo Limited sold the following lengths of pipe during one day: 5 ft. 6 in., 4 ft. 2½ in., 8 ft. 4½ in., 7 ft. 8 in., 3 ft. 5 in., 9 ft. 11 in. How many yards, feet, and inches did they sell during the day?

Yards (13)	Feet (3)	Inches
	5	6
	4	2.5
	8	4.5
	7	8
	3	5
	9	11
13	39 = 39⅓ yd. = (13 yd.)	37 = 37⅓ ft. = (3ft.) 1 in.
13	0	1
<u>Total length</u>	<u>13 yd. 0 ft. 1 in.</u>	

Rules For Adding Mixed Units

1. Commence with the right-hand or smallest unit column. Add. Divide the total by the number of units contained in the next larger unit. Carry the number of whole larger units to the next column and enter the remaining number of smaller units in the right-hand column.

2. Add the column of next smallest units, adding in the number carried from the first column. Divide the total by the number of units contained in the next larger unit and proceed as before.

Subtracting Mixed Units

Example: A container of milk is filled with 9 gallons, 1 quart, 1 pint. If 4 gallons, 2 quarts, 1.5 pints are used, how much will remain in the container?

Gallons # -	Quarts + # -	Pints + #
9 (1).....	(4) 1 (1).....	(2) 1
4	2	1.5
4	2	1.5

Starting with the right-hand column, 1.5 pints cannot be taken from 1 pint; so borrow one quart, equals two pints, from the next column:

$$2 \text{ pints} + 1 \text{ pint} = 3 \text{ pints} - 1.5 \text{ pints} = 1.5 \text{ pints}$$

Taking the next column, the 1 quart in brackets in the minus column is the quart we borrowed; so we have no quarts left from which to take the 2 quarts. We must therefore borrow 1 gallon, equals 4 quarts, from the next column:

$$4 \text{ quarts} + 0 \text{ quarts} - 2 \text{ quarts} = 2 \text{ quarts}$$

Now taking the next column, we had 9 gallons, but we borrowed 1 gallon for the quarts column; so we have only 8 gallons left:

$$8 \text{ gallons} - 4 \text{ gallons} = 4 \text{ gallons}$$

Amount left: 4 gal. 2 qt. 1.5 pt.

In practice, the original figures of the minuend may be changed as we borrow; thus, using the same example:

Gallons	Quarts	Pints
~~9~~ 8	4 ~~1~~ ~~0~~	3 ~~1~~
4	2	1.5
4	2	1.5

Rules For Subtracting Mixed Units

1. Commence with the right-hand, smallest unit, column and subtract. If the subtrahend is larger than the minuend, borrow one unit from the next larger unit and convert it into the smaller unit.
2. Continue with each column separately, borrowing from the next larger unit when necessary.

WORKOUT EXERCISE III

Add the following:

1. 1 yd. 2 ft. 3 in., 17 ft. $2\frac{1}{2}$ in., 5 yd. 1 ft. 10 in., 18 ft. $9\frac{1}{2}$ in., 5 yd. 2 ft. $1\frac{1}{4}$ in.
2. 21 ft. $8\frac{1}{4}$ in., 13 ft. $2\frac{1}{8}$ in., 9 yd. 8 in., 10 yd. $4\frac{1}{4}$ in., 11 yd. 2 ft. $1\frac{3}{4}$ in., 4 yd. 1 ft. 4 in.
3. 2 cwt. 74 lb. $10\frac{1}{4}$ oz., 1 cwt. $11\frac{3}{4}$ oz., 95 lb. $13\frac{1}{2}$ oz., 65 lb. $7\frac{1}{8}$ oz., 3 cwt. $14\frac{3}{4}$ oz., 3 cwt. 17 lb. $5\frac{3}{4}$ oz.
4. 6 cwt. $10\frac{3}{4}$ oz., 4 cwt. $89\frac{3}{4}$ lb., 2 cwt. 99 lb. $15\frac{3}{4}$ oz., $328\frac{1}{4}$ lb., 829.25 lb., 7 cwt. $8\frac{1}{4}$ oz., 131 lb. $1\frac{3}{4}$ oz.

Subtract the following:

	<i>Minuend</i>	<i>Subtrahend</i>
5.	9 yd. 1 ft. 3 in.	8 yd. 2 ft. 4 in.
6.	13 yd. 0 ft. 10 in.	5 yd. 1 ft. $11\frac{1}{4}$ in.
7.	5 cwt. 83 lb. $10\frac{1}{4}$ oz.	176 lb. 413 oz.
8.	18 cwt. 68 lb. $11\frac{3}{4}$ oz.	17 cwt. 98 lb. $14\frac{1}{4}$ oz.
9.	8 gal. 2 pt.	6 gal. 1 qt. 3 pt.

10. Nellie claims that her mathematics assignment last night took her 1 hour 25 minutes and 55 seconds. If it took Nickie 2 hours 15 minutes and 45 seconds, how much longer did she spend than Nellie doing her homework? (Table for time at back of book.)

11. Farmer Dell has 812 bushels of potatoes for sale. He takes these to market and sells the following amounts:

Bu.	Pk.	Gal.	Qt.	Pt.
16	3	—	2	1
157	—	1	1	—
201	2	—	3	$1\frac{1}{2}$
189	—	1	2	—
73	3	1	—	1
89	1	—	3	—
91	2	—	1	$1\frac{1}{4}$

How much does he sell? How much has he left?

12. Mr. Johnston boarded a plane in Halifax and flew to Montreal in 3 hr. $3\frac{1}{2}$ min. After conducting his business in Montreal, he travelled to Toronto by plane in 1 hr. $35\frac{1}{4}$ min. From Toronto he travelled by

another plane in 3 hr. 31 min. to Winnipeg, and finally to Calgary from Winnipeg in 3 hr. 21¼ min. Find his total flying time.

Multiplying Mixed Units

Example: Con Tratante requires 17 lengths of pipe, each 5 yd. 1 ft. 3½ in. long, to complete a job for which he has contracted. What is the total length of pipe he requires? Ignore any loss from cutting.

5 yd.	1 ft.	3.5 in.	
		$\times 17$	
		59.5 in.	
		= 4 ft. 11.5 in.	
	17 ft. + 4 ft.		
	= 7 yd. 0 ft.		
85 yd. + 7 yd.			
92 yd.	0 ft.	11.5 in.	

Rules For Multiplying Mixed Units

1. Multiply smallest unit by the multiplier; divide by the number of times this unit is contained in the next higher unit; e.g.

$$3.5 \times 17 = 59.5 \text{ in.}, \frac{59.5}{12} = 4 \text{ ft. } 11.5 \text{ in.}$$

2. Enter the smaller unit and carry the larger units to the next column.

3. Multiply the next unit by the multiplier; add the number carried, if any. Divide this total by the number of times this unit is contained in the next higher unit; e.g.

$$17 \times 1 \text{ ft.} = 17 \text{ ft.} + 4 \text{ ft.} = 21 \text{ ft.}, \frac{21}{3} = 7 \text{ yd. } 0 \text{ ft.}$$

4. Continue on as above until all units have been multiplied.

Dividing Mixed Units

Example: At the Fall fair Bizz and Beau Mark set up a stand to sell grape juice. If they purchased 74 gallons 1 quart 1.5 pints at 20¢ a gallon and divided this into 6 vessels:

- (a) How much grape juice would be in each vessel?
- (b) If each glass contained 2 gills, how many glassfuls did they sell?
- (c) How much money did they make if they charged 5¢ a glass and sold all the grape juice?

17. 28 yd. $11\frac{3}{4}$ in. by 7 18. 11 hr. 40 min. 15 sec. by 5
 19. 3 days 23 hr. 4 min. 37 sec. by 13 20. 3 cwt. 61 lb. 12 oz. by 9
 21. 5 cwt. 48 lb. 8 oz. by 8 22. 4 Imp. gal. 1 qt. 3 pt. by 8
 23. 2 bu. 2 pk. 1 qt. by 12 24. 8 bu. 3 pk. $3\frac{1}{2}$ qt. by 9
 25. If a can of tomato juice when full weighs 1 lb. $14\frac{3}{4}$ oz., how much will the contents of a case containing 24 cans of tomato juice weigh?
 26. How many lengths of tubing, each 1 ft. 3 in. in length, can be cut from a length of tubing 13 ft. $2\frac{1}{4}$ in. long? Each cut wastes $\frac{1}{8}$ inch of tubing. How much will remain?
 27. Numode Dress purchase 13 pieces of cloth each 60 yards long. If one dress requires 3 yd. 9 in., how many dresses can be made up from the material purchased?
 28. Sarah and Sally work in a department store three nights a week from 4.30 p.m. to 9.25 p.m. If they work 13 nights in January, how many minutes and hours will they each have worked?
 29. In the manufacture of one widgett, the Peculiar Products use 1 lb. 6 oz. of nails. If they manufacture 550 widgetts a week and have 2 tons 1,350 lb. of nails on hand, how many full weeks will the supply of nails last?
 30. How many slices of toast $\frac{3}{8}$ in. thick can be made from a loaf of bread 12 in. long?

SECTION 5 Measurement of Area

Plane figures are figures that lie on a flat surface. They have length and breadth. *Perimeter* is the length of the boundary line around the outside of the plane figure. The school yard or the farmer's field form plane figures; and if you walk around the outside, you walk around the perimeter. We now want to know how to measure the area of the plane figure. First we must look at the tables for the measurement of square units.

The British System of Square Measure

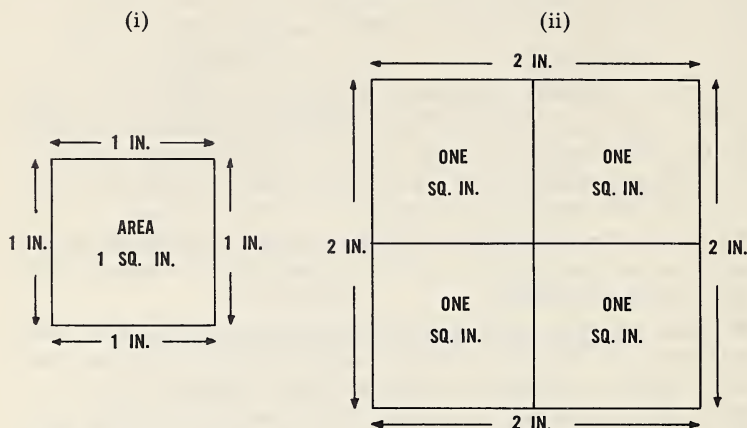
144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (ac.)
640 acres	= 1 square mile (sq. mi.)

The Square

A square, shown in figure 7-1, is a four-sided figure of which all the sides are equal in length and all the angles are right angles.

Figure 7-1

A Square



The perimeter of the square in (i) is 4 in.; the perimeter in (ii) is 4×2 in. = 8 in. The area of the square in (i) is 1 sq. in. The area of the square in (ii) is 4 sq. in. The area is calculated by multiplying the number of units of one side by the number of units of the second side:

$$(i) \text{ Area} = 1 \text{ sq. in.} \times 1 \times 1 = 1 \text{ sq. in.}$$

$$(ii) \text{ Area} = 1 \text{ sq. in.} \times 2 \times 2 = 4 \text{ sq. in.}$$

Thus, the area of a square may be stated as an equation:

$$A = s^2 \quad (1)$$

where A is the area of the square and s is the length of one side of the square.

Example: Farmer Dell has a field 75 yards square.

- How many feet of fencing will he require to completely fence it in?
- What area will he have to plow if he plows the whole field?

$$\begin{aligned} (a) \text{ Perimeter} &= 4 \times 75 \text{ yd.} = 300 \text{ yd.} \\ &= 3 \times 300 \text{ ft.} = 900 \text{ ft.} \\ \text{Fencing required} &= 900 \text{ ft.} \end{aligned}$$

$$\begin{aligned} (b) \text{ Area in square yards} &= 75 \times 75 = 5,625 \\ \text{Area to be plowed} &= 5,625 \text{ sq. yd.} \\ \text{or } \frac{5625}{4840} \text{ ac.} &= 1.16 \text{ ac.} \end{aligned}$$

WORKOUT EXERCISE V

- Find the perimeter and the area of a square one side of which is:
(a) 3 ft., (b) 2 ft. 9 in., (c) $4\frac{1}{4}$ in., (d) $13\frac{1}{2}$ miles, (e) $17\frac{1}{8}$ rods.
- Find the unknown properties of the following squares:

	Side	Perimeter	Area
(a)	9 in.	—	—
(b)	—	—	225 sq. in.
(c)	8 yd. 2 ft.	—	—
(d)	—	10 yd. 2 ft.	—
(e)	—	—	1,681 sq. in.

- Tam McDougall ploughed a 44-rod square to sow wheat. Around the outside of the ploughed area he left a 6-foot strip and then fenced in the total area. If he placed fence posts at 6-foot intervals, how many posts would he need for the field?
- Find the fenced-in area of Tam McDougall's field in acres correct to two decimal places.
- The side of a baseball diamond is 90 feet. What is the area of the diamond in square yards? How many yards must a player run to score a home run?
- A room 16 feet square has the ceiling tiled with 10-inch square sound proof tiles. How many complete tiles will be required? How many cut tiles will be required? If the tiles cost \$1.50 a dozen, what will it cost to tile the ceiling?
- When completing a house for sale, Webuildem Better Builders laid broadloom on some of the floors instead of hardwood. If two bedrooms were 12 feet square and 14 feet square, respectively, and the living room was 20 feet square and the average cost for the broadloom was \$4.25 a square yard, how much would it cost Webuildem Better Builders to purchase broadloom for 22 houses? Assume a portion of a square yard cannot be purchased for any room. How much wastage would there be?

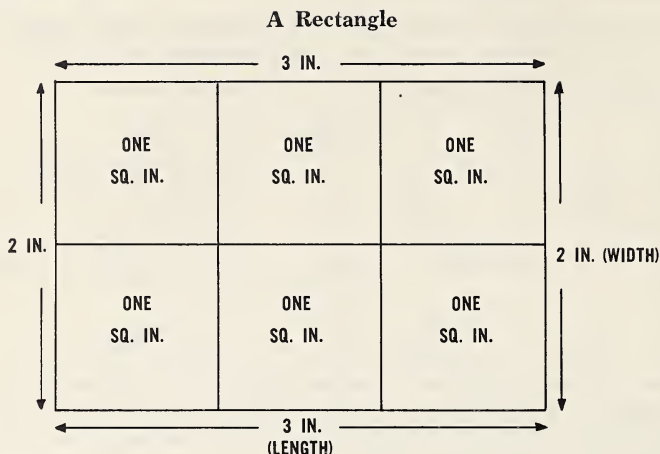
The Rectangle

A rectangle is shown in figure 7-2. The opposite sides of a rectangle are equal to each other, and the angles are all right angles. (A square is a special type of rectangle.)

The perimeter of the rectangle is the sum of the four sides or twice the length plus twice the width:

$$2 \times 3 + 2 \times 2 = 6 + 4 = 10 \text{ in.}$$

Figure 7-2



The area of the rectangle in figure 7-2 is 6 sq. in. It is calculated by multiplying the length by the width. Thus, the equation for the area of a rectangle is:

$$A = l \times w \quad (2)$$

where A is the area, l is the length, and w is the width. The area of the rectangle in figure 7-2 may therefore be stated as:

$$\text{Area} = 1 \text{ sq. in.} \times 3 \times 2 = 6 \text{ sq. in.}$$

Example: Farmer Dell's corn field has a perimeter of 188 rods. If the width of the field is 30 rods, how long is it? What is the area of the field in acres?

$$\begin{aligned} \text{Perimeter} &= 2l + 2w = 188 \text{ rods} \\ &\quad w = 30 \text{ rods} \\ \therefore 2l + 2 \times 30 &= 188 \text{ rods} \\ \therefore 2l &= 188 - 60 \text{ rods} \\ &= 128 \text{ rods} \\ \therefore l &= 64 \text{ rods} \end{aligned}$$

Area:

$$\begin{aligned} A &= l \times w \\ \therefore \text{Area} &= 1 \text{ square rod} \times 64 \times 30 = 1920 \text{ square rods} \\ &\quad 160 \text{ square rods} = 1 \text{ acre} \\ \therefore 1920 \text{ square rods} &= \frac{1920}{160} = 12 \text{ acres} \\ \therefore \text{Area} &= 12 \text{ acres} \end{aligned}$$

WORKOUT EXERCISE VI

1. Find the perimeter of the rectangles with the following dimensions:

Length	Width	Length	Width
(a) 2 ft. 6 in.	1 ft. 3 in.	(b) 3 yd. 1 ft. 2 in.	2 yd. 1 in.
(c) $3\frac{1}{4}$ yd.	$1\frac{1}{2}$ yd.	(d) 15.27 ft.	11.42 ft.
(e) 17 rd.	15 rd.		

2. Find the unknown properties of the following rectangles:

	Length	Width	Perimeter	Area
(a)	2 in.	3 in.	—	—
(b)	18 in.	—	62 in.	—
(c)	—	3.2 yd.	—	16 sq. yd.
(d)	6.8 in.	—	—	23.12 sq. in.
(e)	—	2.7 rd.	21.2 rd.	—

3. Tam McDougall has a 12-acre corn field. If one side of the field is 60 rods, how many yards long is the other side? If fencing cost \$1.75 a foot, how much would it cost to fence the field?

4. Tony Jones decides to tile his recreation room which is 20 ft. by 30 ft. If the tiles are 6 in. by 4 in., how many tiles must he buy? If the tiles cost 6¢ a piece, how much will the material cost him?

5. Tam McDougall uses $2\frac{1}{2}$ bushels of oats to sow one acre. If his field is 44 rods by 40 rods and there are 34 pounds in 1 bushel of oats, how many pounds of oats must he sow?

6. A six-lane highway is built from Skinnyburgh to Mogulton, a distance of 85 miles. If each lane is 11 feet wide, find, to the nearest hundred, the number of square yards of paving needed.

7. One pint of paint is needed to cover 80 sq. ft. of surface. If you wish to paint the walls and ceiling of a room which is 20 ft. by 30 ft. and 9 ft. high, how many pints of paint will it take? There are two doors, each measuring 7 ft. by 3 ft., and one window, measuring 8 ft. by 6 ft., which do not require painting.

8. Farmer Dell purchased additional land adjoining his farm. The land had a frontage of 500 feet and a depth of 900 feet. If he paid \$50 an acre or part of an acre, how much did the land cost him?

9. You have the opportunity to purchase a rectangular lot 40 ft. by 60 ft. or a square lot having the same perimeter, for \$2,000. Which is the better buy? How much per square foot would you pay in each case?

10. The area of a field is $144a^2b^2$ sq. ft. If the width is $6ab$ feet, how long is the field?

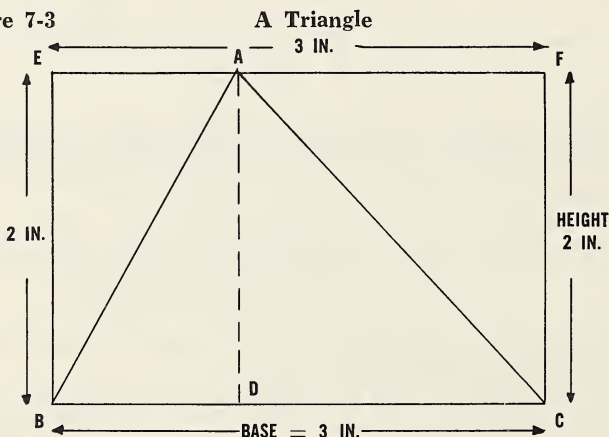
11. Desks are to be placed in an office 47 ft. wide by 53 ft. long. Each desk is 40 in. by 24 in. They are to be placed one behind the other in rows. There are to be 4-foot passages between the rows and between the side walls and the desks; there are to be 3 feet between the end walls and desks and between the desks. How many desks can be placed in the room?

12. The area of four walls of a room is $138x^2y^2$ square feet. If the height of the room is $3xy$ feet, how many feet of baseboard would be required excluding two 3-foot wide doorways?

The Triangle

A triangle is a plane figure enclosed by three sides. The perimeter of a triangle is the sum of the three sides. The area of a triangle is shown in figure 7-3.

Figure 7-3



For convenience, we have lettered the points of intersection. ABC is the triangle.

To find its area we look first at the rectangle $EBDA$ and see that half of this rectangle is in the triangle. If we look at rectangle $ADCF$, we see that half of this rectangle is also in the triangle. Therefore, if we look at the big rectangle $EBCF$, we see that the triangle occupies half of its total area. We know how to find the area of a rectangle; so we can find the area of a triangle because it will always be one half the area of a rectangle formed by the base times the height of the triangle. The formula is:

$$\text{Area} = \frac{1}{2} (\text{base} \times \text{height})$$

$$\text{or} \quad A = \frac{1}{2}bh \quad (3)$$

The area of the triangle in figure 7-3 is therefore:

$$1 \text{ sq. in.} \times \frac{1}{2} (3 \times 2) = \frac{1}{2} \times 6 = 3 \text{ sq. in.}$$

Example: Farmer Dell has a triangular field which he uses to grow potatoes. The base of the field is 40 rods, and the other two sides are both 25 rods long. The distance from the peak (the apex of the triangle) to the base is 15 rods. How many feet of fencing will he require to fence in the field? How many acres of potatoes will he have to plant?

Perimeter: $40 + 25 + 25 = \underline{90 \text{ rods}}$

$1 \text{ rod} = 16\frac{1}{2} \text{ ft.}$

Length of fencing $= 90 \times 16.5 \text{ ft.}$
 $= \underline{1,485 \text{ ft.}}$

Area: $\frac{1}{2} (b \times h) = 1 \text{ sq. rd.} \times \frac{1}{2} (40 \times 15)$
 $= \frac{1}{2} (600) \text{ sq. rd.}$

$\therefore \text{Area} = \underline{300 \text{ square rods}}$

$160 \text{ sq. rd.} = 1 \text{ acre}$

$\therefore 300 \text{ sq. rd.} = 300/160 \text{ acres}$

$= \underline{1\frac{7}{8} \text{ acre}}$

WORKOUT EXERCISE VII

1. A triangle has a perimeter of 48 and a base of 21. If the second side is twice the length of the third side, how long are the other two sides?

2. Find the areas of the following triangles:

	Base	Height
(a)	14.2 in.	8.4 in.
(b)	3 ft. 2 in.	2 ft. 1 in. (in sq. in.)
(c)	8 ft. 4 in.	4 ft. 3 in. (in sq. ft.)

3. Find the cost of fencing in a triangular courtyard if fencing cost \$3.75 a yard and the base is 20 feet and the other two equal sides are $\frac{3}{5}$ as long.

4. If the perimeter of a triangle is 72 in. and the sides are in a 4:3:2 ratio, what is the length of each side?

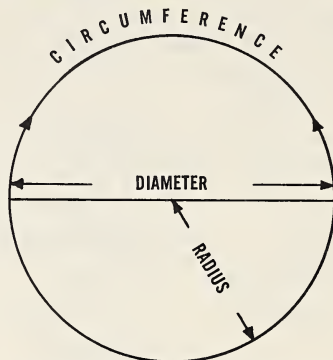
5. What is the area in square yards of any triangle that is contained within the rectangle measuring 64 rd. by 18 rd?

The Circle

The perimeter of a circle is called the *circumference*. A straight line drawn through the centre of the circle to cut the circumference in two places is called the *diameter*. A line drawn from the centre to the circumference is called the *radius* of the circle. The diameter is equal to twice the radius or $d = 2r$.

Figure 7-4

A Circle



In every circle, no matter what size, there exists the same relationship between the circumference and the diameter, i.e.,

$$\frac{\text{circumference}}{\text{diameter}} = \pi \quad (4)$$

Pi, written π , is a Greek letter which is used as a symbol to express this relationship. The value of π is approximately $2\frac{2}{7}$ or 3.1416. If we know either the diameter or the circumference of any circle, we can find the other measurement and the area. Our equation to find the circumference is:

$$\frac{c}{d} = \pi, \text{ or}$$

$$c = \pi d, \text{ or,}$$

$$\text{as } d = 2r,$$

$$c = 2\pi r \quad (5)$$

The equation to find the area of a circle is:

$$a = \pi r^2 \quad (6)$$

Example: Sunny Nager is building a circular swimming pool in his garden. The diameter of the pool is to be 25 feet. How many square feet of tiling will be needed to tile the bottom of the pool? If there is a 2-foot wide pathway around the pool, how many feet and inches of fencing will be required to fence in the pool?

Radius of pool = 12.5 ft.

$\therefore$ Area of bottom = 1 sq. ft. $\times$ 3.1416 $\times$ 12.5 $\times$ 12.5 =
490.875 sq. ft.

$\therefore$ Tiling required = 490.875 sq. ft.

Radius of area enclosed = 12.5 ft. + 2 ft. = 14.5 ft.

Circumference = $2\pi r = 2 \times 3.1416 \times 14.5$ ft.

$\therefore$ Length of fence = 91 ft. 1.3 in.

WORKOUT EXERCISE VIII

Use $\pi = 3.14$ unless otherwise stated.

1. Find the circumference and the area of the following circles:

(a) $r = 20$ in., (b) $r = 2.5$ in., (c) $d = 4.6$ in., (d) $d = 4$ ft. 2 in.,
(e) $r = 2$ yd. 1 ft. (Find areas in square yards, square feet, and square inches to the nearest square inch.)

2. Find the unknown properties in the following table. Give answers to two decimal places using $\pi = 3.1416$.

	Radius	Diameter	Circumference	Area
(a)	10 in.	—	—	— sq. ft.
(b)	—	9 ft.	—	— sq. in.
(c)	—	—	42 ft.	— sq. ft.
(d)	0.25 in.	—	—	— sq. in.
(e)	—	5 yd. 2 ft. $2\frac{1}{2}$ in.	—	— sq. ft.
(f)	—	—	160 rods	— sq. yd.

3. Measured around the equator, the circumference of the earth is 25,000 miles. What is the radius of the earth to the nearest mile?

4. A circular lawn has a radius of 15 yards. How many square yards of sod are required to cover it? If it has a 6-foot walk around the lawn, how many feet of fencing will be required to enclose the total area?

5. Farmer Dell has a field 44 rd. by 40 rd. to plough. The wheels of his tractor have a radius of $12\frac{1}{2}$ in., and one trip down the length of the field ploughs a strip 63 in. wide. How many revolutions, to the nearest revolution, does a wheel make for one trip down the field? How many trips are required? Assuming no loss at the ends of the field when turning, how many revolutions will each wheel make to plough the whole field?

6. In the example above how many square feet of paving will be needed to pave the pathway around the circular swimming pool of Sunny Nager. (Diameter of pool, 25 ft.; width of path, 2 feet.)

7. The inside diameter of the cylindrical gasoline tanks used by Five Star Gas to haul their gasoline is 6 ft. 6 in. What is the area of the cross-section in square feet and square inches to the nearest square inch?

8. An indoor track is formed by a rectangle 150 ft. long by 75 ft. wide. At each end of the rectangle is a semi-circle as shown below. What is the total area of the floor space? What is the length of the guard fence around the track area? If the track is 6 ft. wide, what is the area of the track? If a boy ran once around down the middle of the track, how many yards would he run?

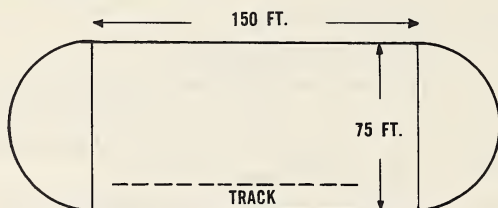


Figure 7-5

9. The doorway leading into Armstrong Hall has an opening composed of a rectangle 7 ft. 6 in. wide and 9 ft. high with a semi-circular arch on top. What is the area in square feet and square inches of the doorway? If a beading is placed up the sides and around the arch, how many feet will be required?

10. In the Greasy Ladle Pancake Inn you are offered two 6 in. diameter pancakes or four 3 in. diameter pancakes for 30¢. How much per square inch would you pay in each case? Which is the better bargain? (Answer to two decimal places.)

11. A circular fountain with a radius of 3 ft. stands in the centre of a courtyard 25 ft. square. If the courtyard is to be asphalted, how many square feet of asphaltting will be required? If the asphaltting cost \$27.50 per square yard or part thereof, how much will this job cost?

Project: To Determine the Value of π .

Collect six tin cans of different size and for each make the following measurements: With a piece of string measure the distance around the curved surface of the can. This is the circumference. With a ruler measure across the largest part of the top of the can. This is the diameter.

Draw up a table and for each can record the circumference, the diameter, the ratio of the circumference to the diameter, and the value of π to 3 decimal places. Take the average value of π for all the values you have calculated and compare your result with the value given.

SECTION 6 Volume and Capacity

A *solid* is any article having length, width, and height. The *volume* or the *cubic content* of a solid is the amount of space it takes up or its bulk. In the case of a hollow solid, its inside dimensions measure its *capacity* or its cubic contents.

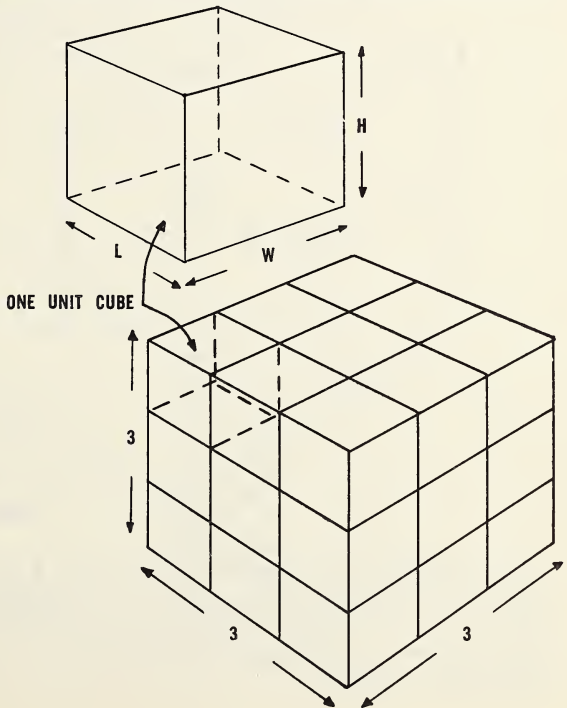
Measurements of volume and cubic contents are derived from units of linear measure as shown in the following table. Note that when feet are multiplied by feet we have square feet and when feet are multiplied by feet and again multiplied by feet we have cubic feet.

Volume	
1,728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (cd.)
1 cubic foot	= 6¼ gallons, approx.

The Cube

A cube is a solid of equal length, width, and height. All its sides are at right angles to one another, and thus its six faces are six squares.

Figure 7-6
A Cube



As can be seen from figure 7-6, the number of small cubes (unit cubes) in the large cube is $1 \text{ cubic unit} \times 3 \times 3 \times 3 = 27 \text{ cubic units}$. The equation is:

$$V = l \times w \times h$$

where V is the volume, l the length, w the width, and h the height. But l , w , and h are all the same length; so we may call them all l ; in which case our formula may be shown as:

$$V = l^3$$

Build figure 7-6 with sugar cubes. Measure the length and find the volume.

The Rectangular Solid

A rectangular solid has six faces, and all its sides are at right angles to one another. The opposite sides of each face are equal. When all the sides are equal, they form a cube. The cube is, therefore, a special case of a rectangular solid. So we can use the same general formula we first obtained for the cube:

$$V = l \times w \times h \quad (7)$$

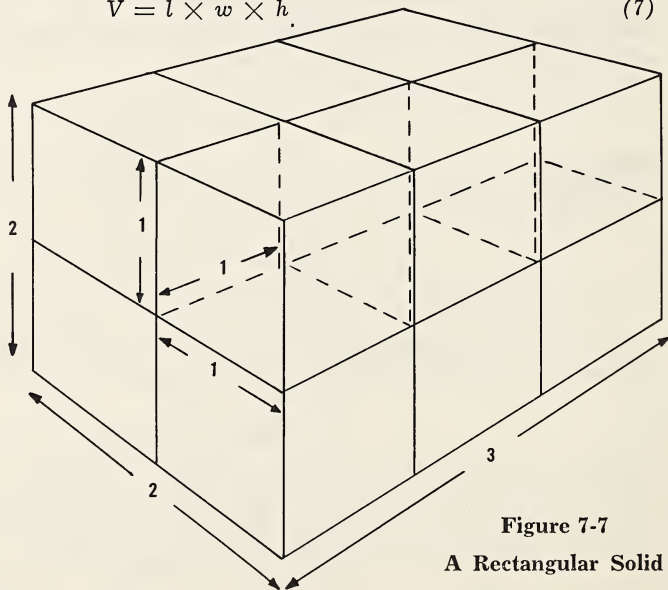


Figure 7-7
A Rectangular Solid

The rectangular solid shown in figure 7-7 is 2 units wide, 2 units high, and 3 units long. The number of unit cubes contained in the solid can be seen to be 12. Try this with sugar cubes; use either three banks of four, or two banks of six cubes, or $2 \times 2 \times 3$. Or, if we use the above formula:

$$\begin{aligned}\text{Volume} &= 1 \text{ cubic unit} \times 3 \times 2 \times 2 \\ \therefore \text{Volume} &= 12 \text{ cubic units}\end{aligned}$$

Example: Donald Dingus is building a house with the dimensions 21 feet by 35 feet. If the ground is excavated to a depth of 9 feet, with an additional foot on the outside of the walls, how many cubic yards and cubic feet must be excavated?

Dimensions of hole: 23 feet $\times$ 37 feet $\times$ 9 feet

$$\begin{aligned}\therefore \text{Volume of hole:} &= 1 \text{ cu. ft.} \times 23 \times 37 \times 9 \\ &= 7,659 \text{ cubic feet.}\end{aligned}$$

There are 27 cubic feet in 1 cubic yard.

$$\therefore \text{Volume of hole} = 283 \text{ cu. yd. } 18 \text{ cu. ft.}$$

Example: The Gizmo Mfg. Co. pack their Hootenannies in cartons having a 36 cu. ft. volume. If the cartons are 2 ft. wide and 3 ft. high, how long are they? How many square yards of floor space will be taken up to store 6 dozen cases piled three high? How many cubic yards of space will be used?

From formula (7), $V = lwh$.

Divide each side by w and h :

$$\frac{V}{wh} = \frac{lwh}{wh}$$

$$\therefore \frac{V}{wh} = l$$

$$\therefore l = \frac{V}{wh} = \frac{36}{2 \times 3} = \frac{36}{6} = 6$$

$$\therefore \text{Length} = 6 \text{ feet}$$

Floor space of one carton = length $\times$ width

$$= 6 \times 2 \times 1 \text{ sq. ft.}$$

$$= 12 \text{ sq. ft.}$$

6 dozen cases

$$= 72 \text{ cases}$$

If piled 3 high

$$= 24 \text{ piles}$$

Each pile occupies

$$= 12 \text{ sq. ft.}$$

$\therefore$ Area needed

$$= 12 \times 24 \times 1 \text{ sq. ft.}$$

$$= 288 \text{ sq. ft.}$$

$$= 32 \text{ sq. yd.}$$

$\therefore$ Space needed

$$= 288 \times 9 \times 1 \text{ cu. ft.}$$

$$= 2,592 \text{ cu. ft.}$$

$$= 96 \text{ cu. yd.}$$

WORKOUT EXERCISE IX

1. Give the length, width, and height of the following cubes: (a) 27 cubic inches, (b) 8 cubic yards, (c) 12 cubic yards 19 cubic feet.

Find the missing dimensions of the rectangular solids in the following table:

	Length	Width	Height	Volume
2.	3 ft. 6 in.	2 ft. 6 in.	5 ft. 3 in.	— cu. in.
3.	6 ft. 0 in.	8 ft. 0 in.	—	528 cu. ft.
4.	—	18 ft. 0 in.	15 ft. 0 in.	290 cu. yd.
5.	7 ft. 0 in.	—	5 ft. 6 in.	173¼ cu. ft.
6.	100 ft.	3 ft. 3 in.	—	1,056 cu. ft. 432 cu. in.
7.	—	4 ft. 6 in.	6 ft. 6 in.	9 cu. yd. 20 cu. ft. 432 cu. in.
8.	8 ft. 6 in.	1 ft. 3 in.	2 ft. 6 in.	— cu. ft. — cu. in.
9.	29 ft.	—	14 ft.	34 cd. 4 cu. yd. 6 cu. ft.
10.	3 ft. 9 in.	2 ft. 6 in.	4 ft. 4 in.	—

11. Paul Sodden built a concrete boathouse with outside dimensions of 12 feet wide by 18 feet long by 11 feet high. The floor was 12 inches thick. The walls, which were built up flush with the outer edges of the floor, were 9 inches thick. The ceiling, to the outside edges of the walls, was 6 inches thick. If a hole 2 feet by 1½ feet was left for a window and a hole 6 feet by 6 feet was left for the door, how many cubic feet of concrete were needed? If the concrete cost \$40.50 per cubic yard, how much did the boathouse cost?

12. The Doohickey Tidbit Corporation pack their tidbits in cans having a diameter of 3 inches and a height of 5 inches. The cans are shipped 6 dozen to a crate. What are the cubic contents of the crate in cubic feet and cubic inches? If they were packed four in a row and three high, what would the inside dimensions of the crate measure?

The Cylindrical Solid

See figure 7-8.

Area of a Cylinder

The total surface area of the cylinder is the sum of the area of the circle at the top, the area of the circle at the bottom, both called bases, and the area around (the wall of) the cylinder. We already know how to find the area of the circle from formula (6).

The area of the sides can be checked in the following way. Take a can of fruit or vegetables and, with a sharp knife, slit the label from top to bottom and then lay it out flat. You will see that you now

Figure 7-8
A Cylinder

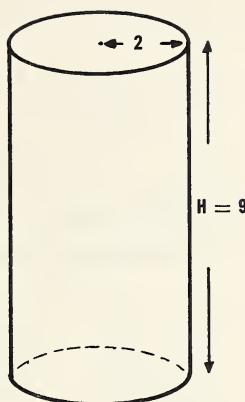
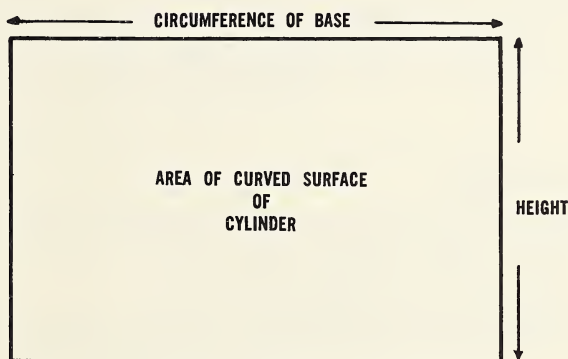


Figure 7-9
Area of the Curved Surface of a Cylinder



have a rectangle which is formed by the height of the cylinder and by the circumference of the circle of the base.

If the height of the cylinder in figure 7-8 is 9 units and the radius of the base is 2 units, the area of the circle, from formula (6), is:

$$\begin{aligned}
 A &= \pi r^2 \\
 &= \frac{22}{7} \times \left(\frac{2}{1}\right)^2 \times 1 \text{ square unit} = \frac{88}{7} \text{ square units} \\
 &= 12\frac{4}{7} \text{ square units}
 \end{aligned}$$

$$\therefore \text{Area} = 12\frac{4}{7} \text{ square units}$$

The circumference of the circle is found from formula (4):

$$\begin{aligned} C &= \pi d \\ &= \frac{22}{7} \times \frac{4}{1} = \frac{88}{7} \\ &= 12\frac{4}{7} \end{aligned}$$

$$\therefore \text{Circumference} = 12\frac{4}{7} \text{ units}$$

The area of the rectangle forming the curved surface is then, from formula (2):

$$\begin{aligned} A &= lh \\ &= \frac{88}{7} \times \frac{9}{1} \\ &= 113\frac{1}{7} \end{aligned}$$

$$\therefore \text{Area} = 113\frac{1}{7} \text{ square units}$$

The total surface area of the cylinder is:

Bases:	$2 \times 12\frac{4}{7} = 25\frac{1}{7}$	square units
Sides:	$113\frac{1}{7}$	square units
	<u><u>$138\frac{2}{7}$</u></u>	square units

Volume of a Cylinder

When finding the volume of a cube or rectangular solid, we first found the square measure of one side and then multiplied it by the third dimension to get the cubic measurement.

This procedure can be followed for other solid figures including a cylinder. First find the area of the base, and then multiply it by the height.

The volume of a cylinder is:

$$V = \pi r^2 h \quad (8)$$

Example: The Glace Creamery collect cream from farms, to manufacture ice cream, in a refrigerated cylindrical truck. If the inside of the cylinder is 25 feet long with a diameter of 6 feet and each cubic foot holds $6\frac{1}{4}$ gallons of cream, how many gallons of cream does the cylinder hold? If the average pick-up per farm is 25 gallons, how many farms must be called on to fill the cylinder?

From formula (8):

$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ &= 3.1416 \times 3 \times 3 \times 25 \\ &= 706.86 \text{ cu. ft.}\end{aligned}$$

Each cubic foot holds 6.25 gallons.

$$\begin{aligned}\therefore \text{Cylinder holds} & 706.86 \times 6.25 \text{ gallons} \\ &= 4,417.875 \text{ gallons} \\ &= 4,417\frac{7}{8} \text{ gallons}\end{aligned}$$

$$\text{Average pickup} = 25 \text{ gallons}$$

$$\begin{aligned}\text{Number of farms called on} &= \frac{4,418}{25} \\ &= 177 \text{ farms}\end{aligned}$$

Example: The steel of which the cylindrical trucks of the Glace Creamery are made is $\frac{1}{2}$ inch thick. If the cylinder is painted on the outside, what is the total area to be painted? Aluminum paint costs \$11.95 a gallon, and a quart covers an average of 180 square feet; how much will it cost to paint the outside if one gallon is the smallest quantity that can be purchased at this price?

Area of cylinder = 2 $\times$ area of base + area of curved surface

$$\text{or, } A = 2\pi r^2 + 2\pi rh$$

$$\begin{aligned}&= 2 \left(\frac{22}{7} \times 3 \frac{1}{24} \times 3 \frac{1}{24} \right) + 2 \left(\frac{22}{7} \times 3 \frac{1}{24} \times 25 \frac{1}{12} \right) \\ &= \left(\frac{2}{1} \times \frac{22}{7} \times \frac{73}{24} \times \frac{73}{24} \right) + \left(\frac{2}{1} \times \frac{22}{7} \times \frac{73}{24} \times \frac{301}{12} \right) \\ &= 58.15 + 479.57 \\ &= 537.72 \text{ sq. ft.}\end{aligned}$$

One quart covers 180 square feet.

$\therefore$ 3 quarts are required to cover cylinder.

$$\text{Smallest quantity required} = 1 \text{ gallon}$$

$$\text{Cost of painting cylinder} = \$11.95$$

WORKOUT EXERCISE X

1. State the formulas for: the area of the base of a cylinder, the area of the curved surface of a cylinder, the area of the cylinder, the volume of the cylinder.

Find the missing measurements of the following cylinders to the nearest inch using $\pi = 2\frac{2}{7}$.

	Radius	Diameter	Circumference	Length	Area	Volume
2.	6 ft. 0 in.	—	—	6 ft. 0 in.	—	—
3.	—	—	154 ft.	5 ft. 6 in.	—	—
4.	—	2 ft. 0 in.	—	3 ft. 3 in.	—	—
5.	1 ft. 9 in.	—	—	8 ft. 0 in.	—	—
6.	—	—	208 ft.	10 ft. 6 in.	—	—
7.	—	18 ft. 6 in.	—	15 ft. 9 in.	—	—
8.	—	—	49 ft.	7 ft. 6 in.	—	—
9.	5 ft. 3 in.	—	—	11 ft. 0 in.	—	—
10.	—	7 ft. 9 in.	—	9 ft. 3 in.	—	—

11. Tot Rod Tricycle Manufacturers erect a water tank on their premises to supply water in case of fire. The tank is cylindrical in shape and is 23 feet high with a diameter of 10 feet. How many gallons of water will it hold if it is filled to within 1 foot of the top? If a standard hose delivers 250 gallons per minute and there are 3 hoses connected to the supply, how long will it take before the tank is half empty? (1 cu. ft. holds $6\frac{1}{4}$ gal., $\pi = 3.1416$)

12. The gasoline storage tanks of Blue Star Supereth are 40 feet in diameter and 25 feet in height. If one of these tanks is three-quarters full, how many gasoline haulage trucks will it completely fill if the haulage tanks are 20 feet long with a diameter of 8 feet?

SECTION 7 Figuring for Fun

1. If Sam Davis were to walk around the earth at the equator and the top of his head was exactly 6 ft. above the earth, how much farther would his hat travel than his feet? (Circumference of the earth at the equator is 25,000 miles.)

2. At your favorite restaurant you are offered two pancakes 10 inches in circumference or four pancakes 5 inches in circumference for the same price. If you are very hungry, which will be the better bargain?

3. If you had to cut a piece of string to go around the circumference at the equator (25,000 miles) and you cut the string 1 yard too long, how far above the earth, all the way round, would it be?

A GROCER	
Statement of Profit and Loss for the grocery year	
REVENUE from:	
Sale of groceries	\$15,600
EXPENSES are deducted:	
Grocery purchases	\$9,000
Clerk's wages	4,000
Supplies	1,000
Light, heat, telephone	500
Total expenses	<u>14,500</u>
REMAINDER IS:	
PROFIT FOR YEAR	<u>\$1,100</u>

Profit and Loss

8

A Business Problem

Peculiar Products Limited manufacture budgets. However, their machinery is worn-out, and this month the repair bill amounted to \$4,000. The fuel, heat and light bills amounted to \$1,500. The labour paid amounted to \$2,500. The revenue from sales amounted to \$11,000.

More efficient new machinery can be rented for \$6,000 a month, and, if this machinery is installed, it is estimated the following changes will occur:

Increase in work manufactured and sold: 40%;

Increase in fuel, heat and light bills: 18%;

Increase in labour costs: 25%;

Decrease in repair bills: 100%.

By means of two comparative statements of profit and loss, discuss the new proposition with a view to advising **Peculiar Products Limited** as to their best course of action.

When a person, a group of persons, or a company operate a business, they normally do so to make a *profit*. In order to find out how much profit is made, or *loss* is suffered, in any period of time, all sales and all expenses are gathered together in a *profit and loss statement*. The result of this statement will show the profit or the loss for the period under discussion.

SECTION 1 Equations — Gross Profit

The profit and loss statement is divided into two sections showing (a) the gross profit and (b) the net profit.

The first section can be represented by the equation:

$$\text{Net sales} - \text{Cost of goods sold} = \text{Gross profit} \quad (1)$$

Example: Net sales of the Soggy Log Lumber Company for the month of January amounted to \$45,160; the cost of the goods sold amounted to \$27,000. What was the gross profit?

$$\begin{aligned}\text{Gross profit} &= \text{Net sales} - \text{Cost of goods sold} \\ &= \$45,160 - \$27,000 \\ &= \$18,160\end{aligned}$$

Net Sales

When goods are sold to customers they are listed as 'sales', and the total of these 'sales' is the *gross sales*, generally referred to simply as *sales*. However, in many businesses, some of these goods are returned to the seller as being unsatisfactory, unwanted or damaged. An allowance must then be made for the returned goods and the allowances collected together and listed as *sales returns*. This amount must be deducted from the amount of the gross sales to give the figure for the *net sales*. The equation is:

$$\text{Net sales} = (\text{Gross}) \text{ Sales} - \text{Sales returns} \quad (2)$$

Example: The Soggy Log Lumber Company reported their sales for the month of January as \$53,130. Sales returns for the same period amounted to \$7,970. What were the net sales for January?

$$\begin{aligned}\text{Net sales} &= (\text{Gross}) \text{ Sales} - \text{Sales returns} \\ &= \$53,130 - \$7,970 \\ &= \$45,160\end{aligned}$$

Cost of Goods Sold

John buys a penknife for 50¢ and sells it for 75¢, making a profit of 25¢. The cost of the penknife to John was 50¢. The cost of the merchandise a company buys is, in the same way, the cost of the goods it sells. Suppose that, during the month of January, John decides to buy one dozen penknives to sell. At the end of the month he has five of these penknives left. When we calculate the cost of the penknives sold, we can count only the seven penknives he actually sold; so we say:

$$\text{Cost of goods sold} = \text{Merchandise purchased} - \text{Merchandise left over at end} \quad (3)$$

$$\begin{aligned}\text{Cost of penknives sold} &= (\text{Cost of 12 penknives purchased}) - (\text{Cost of 5 penknives left over at end}) \\ &= \text{Cost of 7 penknives}\end{aligned}$$

During February, John sells the five penknives he had at the beginning of the month, buys another dozen, and sells nine of them. What is the cost of the penknives he sold in February? Putting this in the form of an equation, we have:

$$\text{Cost of goods sold} = (\text{Merchandise on hand at the beginning}) + (\text{Merchandise purchased}) - (\text{Merchandise left over at the end}) \quad (4)$$

$$\begin{aligned} \text{Cost of penknives sold} &= (\text{Cost of 5 penknives on hand at the beginning}) + (\text{Cost of 12 penknives purchased}) \\ &\quad - (\text{Cost of 3 penknives left over at end}) \\ &= \text{Cost of 14 penknives} \end{aligned}$$

We see that the cost of the penknives John sold in February would be $14 \times 50¢ = \$7.00$. His profit would be $14 \times 25¢ = \$3.50$.

Example: The Soggy Log Lumber Company had merchandise on hand at the beginning of January worth \$18,000. During the month they purchased merchandise worth \$35,000. At the end of the month there was \$26,000 of merchandise on hand. What was the cost of goods sold?

$$\begin{aligned} \text{Cost of goods sold} &= (\text{Merchandise on hand at the beginning}) + (\text{Purchases}) - (\text{Merchandise left over at the end}) \\ &= \$18,000 + \$35,000 - \$26,000 \\ &= \$27,000 \end{aligned}$$

Merchandise that a business has on hand is called *merchandise inventory* or simply *inventory*.

WORKOUT EXERCISE I

Find the cost of goods sold in each of the following cases using equation (4):

	Inventory at beginning	Pur- chases	Inventory at end
• 1. The Red Bowes Bootery Company	\$ 10,000	\$ 8,000	\$ 7,000
• 2. The Shiny Slats Company	17,000	7,000	5,000
3. The Chubby Chappie Company	110,000	75,000	85,000
4. The Baldy, Slim & Fatso Factory	1,002,000	325,000	342,000
5. Company XYZ	594,326	98,423	101,479

Find the gross profit for each of the above if the sales and sales returns are as follows:

6. The Red Bowes Bootery Company	Sales \$	17,000
	Sales returns	2,000

7. The Shiney Slats Company	Sales	26,500
	Sales returns	2,225
8. The Chubby Chappie Company	Sales	150,000
	Sales returns	31,000
9. The Baldy, Slim & Fatso Factory	Sales	1,506,600
	Sales returns	163,400
10. Company XYZ	Sales	752,000
	Sales returns	78,424

11. During the past year the Soggy Log Lumber Company sold \$475,824.75 worth of lumber, but \$3,420.18 worth was returned to them as unsatisfactory. They had lumber worth \$18,000 on hand at the beginning of the year and \$15,425.50 worth on hand at the end of the year. If they purchased \$365,000 worth of lumber during the year, what was their gross profit for the year?

12. Queenie King and Lillie Bett sold \$542,482 worth of dresses during the year. If dresses worth \$3,472 were returned as unsatisfactory, and they had a gross profit of \$56,684, how much did the cost of goods sold amount to? If they had an inventory of \$75,000 at the beginning of the year and a closing inventory of \$64,950, what was the value of their purchases during the year?

13. Ham Negg and Pigsel Eatit are proprietors of the Greasy Griddle and, during January, made a gross profit of \$5,492.73. They purchased food worth \$19,723.24. At the beginning of the month, their inventory was worth \$672.43, and at the end of the month was \$27.85 less than at the beginning of the month. What was the value of the meals they served?

14. Ima Taylor and Ura Seamstress run the Astitchin Time Toggery and had sales totalling \$54,983.75 for the first three months of the year. Their purchases for the same period amounted to \$31,429.76. The inventory at the end of the third month was twice as large as at the beginning of the first month. If the cost of goods sold for the three months amounted to \$21,067.56, what was the value of the inventory at the beginning of the first month and at the end of the third month? How much was the gross profit for the three months?

15. Fire & Brimstone Ltd., of Down Below, reported their gross profit for the year to be \$486,920. Their purchases amounted to \$1,345,900. The opening inventory was equal to 5% of the purchases and the closing inventory to 4% of the purchases. The cost of goods sold amounted to how much? What was the value of their sales for the year?

(Keep these solutions for future use.)

SECTION 2 The Profit and Loss Statement — Gross Profit

We can now bring the information of the two examples in Section 1 together in the form of a statement of gross profit.

Figure 8-1

THE SOGGY LOG LUMBER COMPANY	
Statement of Gross Profit	
For the Month of January, 19-1	
Sales	\$53,130
Less sales returns	<u>7,970</u>
Net sales	\$45,160
Cost of goods sold	
Merchandise on hand, January 1	\$18,000
Purchases during January	<u>35,000</u>
Merchandise available for sale	\$53,000
Less merchandise on hand, Jan. 31	<u>26,000</u>
Cost of goods sold	<u>27,000</u>
Gross profit	<u><u>\$18,160</u></u>

This statement forms the first half of the Profit and Loss Statement.

The form of presentation should be noted:

(a) The Heading. The heading consists of three lines; the first line states the name of the company, the second line the name of the statement, and the third line the *period* of time the statement covers. It is particularly important to note that income statements report the operations over a period of time and the whole period must be stated. The above statement could also read "For January 19-1" or "From January 1 to January 31, 19-1." It is incorrect to write "January 31, 19-1", as this would indicate it was the profit for that day alone.

(b) The Columns. The right-hand column contains figures for the net sales, the total of the cost of goods sold, and the gross profit; i.e., the figures of equation (1). The left-hand column contains the calculation of the cost of goods sold, with only the final figure extended to the right-hand column.

(c) The Dollar Signs. A good rule to follow is to place a dollar sign before the top figure of any column of figures and before any figure above which a line has been drawn and which is the result of a calculation. Therefore, in any one column the dollar sign is not repeated unless a line is drawn to perform an arithmetical operation; then it is repeated immediately below the line.

WORKOUT EXERCISE II

The information given in Workout Exercise I is used for the following problems.

1. The figures in numbers 1 and 6 in exercise I should be combined to prepare the Statement of Gross Profit for The Red Bowes Bootery for last year.
 2. The figures in numbers 2 and 7 in exercise I should be combined to prepare the Statement of Gross Profit for The Shiny Slats Company for the past three months.
 3. The figures in numbers 3 and 8 in exercise I should be combined to prepare the Statement of Gross Profit for The Chubby Chappie Company for the last month.
 4. The figures in numbers 4 and 9 in exercise I should be combined to prepare the Statement of Gross Profit for The Baldy, Slim & Fatso Factory for the six months that ended on the last day of last month.
 5. The figures in numbers 5 and 10 in exercise I should be combined to prepare the Statement of Gross Profit for the X Y Z Company for the year ending on the last day of last month.
 6. Prepare a Statement of Gross Profit for number 11, exercise I.
 7. Prepare a Statement of Gross Profit for number 12, exercise I.
 8. Prepare a Statement of Gross Profit for number 13, exercise I.
 9. Prepare a Statement of Gross Profit for number 14, exercise I.
 10. Prepare a Statement of Gross Profit for number 15, exercise I.
- (Keep these solutions for future use.)

SECTION 3 Calculation of Gross Profit

Pricing merchandise for sale is one of the important decisions that the owners of a business must make. The amount by which a company increases the cost price of an article to establish a selling price is called the *markup*, the gross margin, or the gross profit. The markup is usually based on either a percentage of the cost of the goods or a percentage of the selling price, and an equation can be set up:

$$\text{Selling price} = \text{Cost price} + \text{Calculated markup} \quad (5)$$

To find the required percentage markup, it is necessary to estimate all expenses; for instance, wages, heat and light; anticipate possible price reductions resulting from competition; make allowances for shortages through loss or breakage; and provide for a margin of profit.

To help business owners make such a decision, government departments, like the Dominion Bureau of Statistics, together with trade organizations publish lists of normal markup percentages applying in various types of industry. These serve as a guide to the business man and warn him if his markup is more than it should be for his particular industry. He may not be able to compete if his selling price

is higher than that of his competitors, and he will be well advised to investigate all costs very thoroughly to see if they can be reduced, in order to reduce his percentage markup.

Markup Based on a Percentage of the Cost Price

Example: A television set costs a dealer \$150. At what price must he sell the set if he desires a 40% markup on the cost?

$$\text{Markup} = 0.40 \times \$150 = \$60$$

$$\begin{aligned}\text{Selling Price} &= \$150 + \$60 \\ &= \$210\end{aligned}$$

Rate of Markup (on cost) When Markup Is a Percentage of Cost

The above problem can also be set up as:

$$\text{Cost Price} \quad 100\% = \$150$$

$$+ \text{Markup} \quad 40\% = \underline{60}$$

$$\text{Selling Price} \quad 140\% = \underline{\underline{\$210}}$$

As can be seen from the above, the selling price is 140% of the cost price, or

$$1.40 \times \$150 = \$210$$

We can therefore compute our answer by finding

$$140\% \text{ of } \$150 = \$210$$

This figure 1.40 represents a 40% markup on cost and is called the rate of markup (on cost). It is the ratio of the selling price to the cost (1.40:1).

This equation would be:

$$\text{Selling price} = \text{Cost price} \times \text{Rate of markup (on cost)} \quad (6)$$

Example: Mr. Brimley Brown purchases bedroom furniture at a cost of \$290 a suite. If he desires a 30% markup on the cost, what would be his rate of markup on cost? His selling price per suite?

$$\text{Selling price} = 130\% \text{ of cost}$$

$$\therefore \text{Rate of markup} = 1.30$$

$$\begin{aligned}\therefore \text{Selling price} &= \$290 \times 1.30 \\ &= \$377 \text{ per suite}\end{aligned}$$

Note: The first method requires two steps: (i) to find the markup and (ii) to add it to the cost price. The second method requires only one step because the ratio is used as the multiplier.

WORKOUT EXERCISE III

Find the selling price using the first method. (Estimate your answers first.)

1. Cost, \$78; Markup on cost, 30%.
2. Cost, \$329.80; Markup on cost, 63%.
3. Cost, \$1,429.68; Markup on cost, 36%.
4. Cost, \$7,283.33; Markup on cost, 55%.
5. Cost, \$7.95; Markup on cost, 58%.
6. Cost, \$1.75; Markup on cost, 48%.
7. Cost, \$329.50; Markup on cost, 42%.
8. Cost, \$97.75; Markup on cost, 61%.
9. Cost, \$9.85; Markup on cost, 59%.
10. Cost, \$17.25; Markup on cost, 66 $\frac{2}{3}$ %.

11-20. Find the rate of markup on cost in each of the above cases.

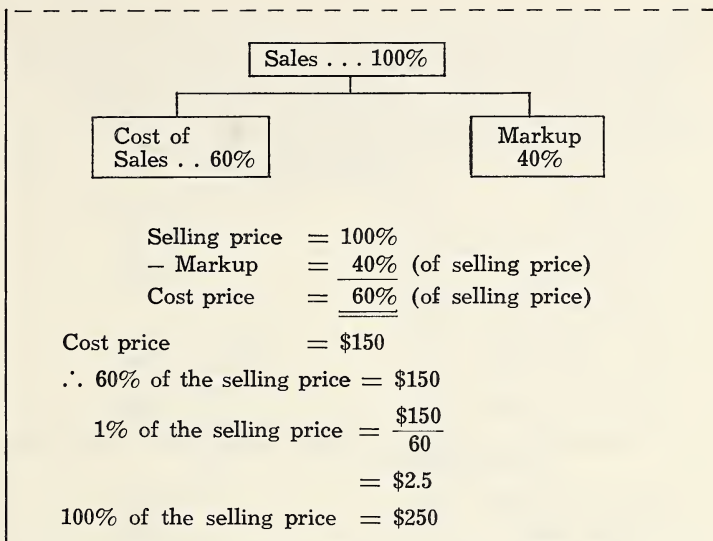
Find the selling price using the second method:

Cost	Markup on Cost	Cost	Markup on Cost
21. \$8.95	33%	26. \$395	24 $\frac{1}{2}$ %
22. \$1.68	65%	27. \$18.18	33 $\frac{1}{3}$ %
23. \$4.98	45%	28. \$598.87	45 $\frac{1}{4}$ %
24. \$5.75	55%	29. \$432.14	51 $\frac{1}{8}$ %
25. \$1,794.37	73%	30. \$10.79	62 $\frac{1}{4}$ %

Markup Based on a Percentage of the Selling Price

The selling price is more often used as a basis for calculating the markup percentage. One reason is that owners frequently desire to compute all items in the profit and loss statement in terms of sales. That is, gross profit is expressed as a per cent of sales, an expense item is expressed as a per cent of sales, and net profit is expressed as a per cent of sales. These percentages can then be compared from year to year to see how business is progressing and to help with future planning which, in most cases, will be based on expected sales. The sales figure, then, is a more important and meaningful figure on which to base the markup than is the cost figure.

Example: A television set cost a dealer \$150. At what price must he sell the set if he desires a 40% markup on the selling price?



Rate of Markup (on cost) When Markup Is a Percentage of Sales

It may also be desirable to find the rate of markup (on cost). Here the ratio of the selling price to the cost price is 100:60, which reduces to 1.6667:1. This figure 1.6667 is the rate of markup and represents a 66.67% markup on cost. If we know the cost of any article, we can find the required selling price by multiplying the cost by this rate of markup as we did in equation (6).

$$\begin{aligned}\text{Selling price} &= \$150 \times 1.6667 \\ &= \$250\end{aligned}$$

Example: Mr. Brimley Brown purchases bedroom suites at a cost of \$290 a suite. If he desires a 20% markup on the selling price, what would be his rate of markup on cost? His selling price?

$$\begin{aligned}\text{Cost price} &= 80\% \text{ of the selling price} \\ \therefore \text{The rate of markup on cost} &= 100:80 \\ &= 1.25 \\ \therefore \text{The selling price} &= \$290 \times 1.25 \\ &= \$362.50\end{aligned}$$

Example: Rockers Limited purchased a shipment of furniture for \$350, plus \$10 freight charges. They sold the furniture for \$510, but later made an allowance of \$15 for damages.

- (a) What was the amount of the gross profit realized?
 (b) What per cent of net sales was the gross profit?
 (c) What was the rate of markup on cost?

(a) Sales	\$510
Less allowances	<u>15</u>
Net sales	\$495
Cost of goods sold	
Purchases	\$350
Add freight	<u>10</u>
	<u>360</u>
Gross profit	<u>\$135</u>

Note: The payment of freight increases the cost of the goods purchased.

(b) Gross profit	\$135
Net sales	495

We wish to find the ratio of gross profit to sales:

$$= \frac{135}{495} = 0.2727:1$$

Gross profit = 27.27% of net sales

(c)	Sales	100%
	Cost	72.73%

We wish to find the ratio of sales to cost:

$$= \frac{100}{72.73} = 1.375:1$$

Rate of
markup on cost = 1.375

WORKOUT EXERCISE IV

When markup is based on a percentage of the selling price, using the first method, find to the nearest cent the selling price of the following (estimate your answers first):

	Cost	Markup on selling price		Cost	Markup on selling price
1.	\$ 15.00	45%	6.	1,384.24	41.5%
2.	1.98	40%	7.	3,428.81	37.8%
3.	2.98	55%	8.	4.76	31.75%
4.	10.75	42%	9.	384.17	43.2%
5.	175.79	39%	10.	24.50	47.35%

11-20. Find the rate of markup on cost, to two decimal places, and the selling price in the above questions 1-10 by the second method outlined above. How do you account for the discrepancies in the answers?

For each of the following find (a) the gross profit, (b) the gross profit as a per cent of net sales, to two decimal places, (c) the rate of markup on cost, to three decimal places:

	Cost of Goods Sold	Sales	Sales Returns
21.	\$ 510.00	\$ 850.00	\$ 40.00
22.	10.50	15.00	1.00
23.	155.60	250.75	49.50
24.	79.50	126.40	10.90
25.	10.75	16.95	1.35
26.	49.80	65.75	5.00
27.	5.98	10.73	1.08
28.	1.98	2.70	—
29.	12.75	18.45	—
30.	1,320.80	2,310.60	476.40

SECTION 4 Net Profit — Equations

In business organizations, the initial expense is the cost of the goods or services sold. This amount, deducted from the net sales, gives the gross profit. However, there are additional expenses, such as salaries to employees, rent, heat, light, advertising, and many other charges incurred in the operation of a business.

These expenses, which are called *operating expenses*, are deducted from the gross profit. The remainder is the actual profit or the *net profit*. The operating expenses and the net profit are contained in the second half of the profit and loss statement. The equation is:

$$\text{Net profit} = \text{Gross profit} - \text{Operating expenses} \quad (7)$$

This is the amount the owner may regard as his income, and on which he will be required to pay income tax.

Example: The Go-Ahead Products report their net sales for the year to be \$149,100, their cost of sales as \$100,129, and their operating expenses as \$25,132. What is their gross profit for the year? What is their net profit for the year?

$$\begin{aligned}
 \text{Gross profit} &= \text{Net sales} - \text{Cost of goods sold} \\
 &= \$149,100 - \$100,129 \\
 &= \$48,971
 \end{aligned}$$

$$\begin{aligned}
 \text{Net profit} &= \text{Gross profit} - \text{Operating expenses} \\
 &= \$48,971 - \$25,132 \\
 &= \underline{\$23,839}
 \end{aligned}$$

If operating expenses are greater than the gross profit, then the business has suffered a loss. The equation becomes:

$$\text{Net loss} = \text{Operating expenses} - \text{Gross profit} \quad (8)$$

Example: The records of the Lag and Sag Company for the year show net sales to be \$73,125; cost of sales, \$48,101; and operating expenses, \$31,016.

What was the gross profit for the year?

What was the net profit or loss for the year?

$$\begin{aligned}
 \text{Gross profit} &= \text{Net sales} - \text{Cost of goods sold} \\
 &= \$73,125 - \$48,101 \\
 &= \underline{\$25,024}
 \end{aligned}$$

$$\begin{aligned}
 \text{Net loss} &= \text{Operating expenses} - \text{Gross profit} \\
 &= \$31,016 - \$25,024 \\
 &= \underline{\$5,992}
 \end{aligned}$$

WORKOUT EXERCISE V

The following operating expenses were incurred by the companies in Workout Exercise 1. Using equation (7) or equation (8), find the net profit or net loss for each company:

1. The Red Bowes Bootery, numbers 1 and 6,
total operating expenses, \$ 2,500
2. The Shiney Slats Company, numbers 2 and 7,
total operating expenses, \$ 3,426
3. The Chubby Chappie Company, numbers 3 and 8,
total operating expenses, \$ 19,750
4. The Baldy, Slim & Fatso Factory, numbers 4 and 9,
total operating expenses, \$410,060
5. Company XYZ, numbers 5 and 10,
total operating expenses, \$47,849
6. The Soggy Log Lumber Company, number 11, paid out the following operating expenses: Wages, \$50,429.60; Light, heat and power, \$12,430.18; Repairs to machinery, \$1,328.13; Sundry logging expenses, \$9,470.29.
7. Queenie King and Lillie Bett, number 12, paid the following operating expenses: Salaries, \$12,420; Wages to clerks, \$13,724; Rent of store, \$2,400; Supplies, \$1,540; General expenses, \$3,182.

8. The operating expenses for the Greasy Griddle, number 13, for January were: Rent, \$1,200; Heat and light, \$760; Taxes, \$412.35; Wages, \$2,500.75; Incidental expenses, \$433.28.

9. The operating expenses of the Astitchin Time Toggery, number 14, for January, February and March were: Salaries, \$17,820; Clerks' wages, \$4,520.50; Rent, \$900; Supplies, \$750; Cleaning, \$410; Light and heat, \$750.84; Advertising, \$950.75.

10. Fire & Brimstone Ltd., number 15, report their operating expenses for the year to be: Rent of factory, \$117,000; Rent of office building, \$2,400; Salaries, \$98,428; Wages to factory workers, \$201,429; Supplies, \$21,731; Repairs to machinery, \$27,820; Sundry factory expenses, \$17,316; Insurance, \$1,428.

SECTION 5 Profit and Loss Statement — Complete

The list of operating expenses and their deduction from the gross profit, in order to calculate the net profit or loss for the period under discussion, forms the second half of the Profit and Loss Statement.

Example: Sartin Trains Ltd. report the following expenses for the year: Salaries, \$35,100; Rent, \$12,000; Taxes, \$3,510; Delivery expenses, \$5,420; Advertising, \$1,250; Insurance, \$1,000; Gross sales amount to \$275,450; Return sales total \$4,690; The cost of the sales is \$101,316. What is the gross profit for the year? What is the net profit for the year?

Figure 8-2 **SARTIN TRAINS LTD.**
Statement of Profit and Loss
For the Year 19 - 1

Sales	\$275,450
Less sales returns	<u>4,690</u>
Net sales	\$270,760
Cost of goods sold	<u>101,316</u>
Gross profit	\$169,444
Operating expenses:	
Salaries	\$35,100
Rent	12,000
Taxes	3,510
Delivery expenses	5,420
Advertising	1,250
Insurance	<u>1,000</u>
Total operating expenses	<u>58,280</u>
Net profit	<u>\$111,164</u>

Example: The net sales for The Slow and Slopsy Company for the year ended December 31 were \$175,000. Their inventory on December 31 amounted to \$18,320 which was \$3,500 less than the inventory they had on hand at the beginning of the year. Purchases during the year amounted to \$118,175. Other operating expenses were: salesmen's salaries and commissions, \$15,000; wages, \$11,000; rent, \$20,000; advertising, \$2,500; delivery expenses, \$6,970. What was the gross profit for the year? What was the net profit or loss?

THE SLOW AND SLOPSY COMPANY

Statement of Profit and Loss

For the Year Ended December 31, 19-1

Sales	\$175,000
Inventory January 1	\$ 21,820
Purchases	118,175
	<u>139,995</u>
Inventory December 31	<u>18,320</u>
Cost of sales	121,675
Gross profit	\$ 53,325
Operating expenses:	
Salesmen's salaries and commissions	\$15,000
Wages	11,000
Rent	20,000
Advertising	2,500
Delivery expenses	<u>6,970</u>
Total operating expenses	55,470
Net loss	<u>\$ 2,145</u>

WORKOUT EXERCISE VI

1-10. Prepare profit and loss statements for the companies in Workout Exercise V.

SECTION 6 Percentage of Sales

As pointed out in Section 3, all items in the profit and loss statement are frequently expressed as percentages of net sales. When the owners or managers of a business wish to analyze the profit and loss statement and compare it with statements of previous years, or with standards of operation existing in businesses of a like nature, the use of percentages, instead of the actual figures, makes the comparison much easier. For instance, if Big Deal Company has net sales of

\$1,000,000 and a net profit of \$100,000 in the same year that Small Deal Company has net sales of \$45,000 and a net profit of \$5,000, it would be difficult to compare the two companies unless the net profit in each case is brought to a common base. The policy followed is to express all items as a percentage of sales. In this case the net profit of:

$$\text{Big Deal Co. is } \frac{100,000}{1,000,000} \times \frac{100}{1} = 10\% \text{ of sales.}$$

$$\text{Small Deal Co. is } \frac{5,000}{45,000} \times \frac{100}{1} = 11.11\% \text{ of sales.}$$

Example: Express the items in the profit and loss statement of the Sartin Trains Ltd. given in Section 5 as a per cent of sales.

SARTIN TRAINS LTD. Statement of Profit and Loss For the Year 19-1			Per cent of sales
Sales	\$275,450		101.7
Less sales returns	4,690		1.7
Net sales	\$270,760		100.0
Cost of goods sold	101,316		37.4
Gross profit	\$169,444		62.6
Operating expenses:			
Salaries	\$35,100	12.9	
Rent	12,000	4.4	
Taxes	3,510	1.3	
Delivery expenses	5,420	2.0	
Advertising	1,250	.5	
Insurance	1,000	.4	
Total operating expenses	58,280		21.5
Net profit	\$111,164		41.1

Example: The Eleventh Dial Company sell washing machines, which they purchase at 35% of the selling price, for \$450 each. If the net profit is 15% of the selling price, find the expenses of selling a washing machine as a percentage of the selling price.

By formula:

$$\begin{aligned} \text{Gross profit} &= 100\% - 35\% \\ &= 65\% \end{aligned}$$

$$\begin{aligned}\text{Net profit} &= \text{Gross profit} - \text{Operating expenses} \\ \text{or } \text{Operating expenses} &= \text{Gross profit} - \text{Net profit} \\ &= 65\% - 15\% \\ &= 50\% \text{ of sales}\end{aligned}$$

By statement:

THE ELEVENTH DIAL CO.
Statement of Profit and Loss
For the Period --

		Per cent of sales
Sales	\$450.00	100%
Cost of goods sold	157.50	35%
Gross profit	\$292.50	65%
Operating expenses	225.00	50%
Net profit	<u>\$ 67.50</u>	<u>15%</u>

Therefore the operating expenses are 50% of sales.

WORKOUT EXERCISE VII

1-5. By formula, find the cost of goods sold as a percentage of net sales for the five companies in Workout Exercise I, numbers 1-10.

6. By formula, find the gross profit as a percentage of sales for the Soggy Log Lumber Company, Workout Exercise I, number 11.

7. By formula, find the cost of goods sold and the purchases as percentages of sales for the Queenie King and Lillie Bett establishment.

8. By formula, find the gross profit of the Astitchin Time Toggery as a percentage of their sales.

9-18. Prepare a statement of profit and loss for each of the companies in Workout Exercise VI showing the per cent of sales of each item.

19. Net sales and net profits were reported by the following companies for last year's operations. List these in ascending order of percentage of sales.

Company	Net Sales	Net Profit
W	\$ 176,429	\$ 18,306
X	25,381	5,462
Y	1,473,817	90,327
Z	536,814	74,814

20. List the operating expenses of the above companies in ascending order of percentage of sales:

Company W, \$25,410; Company X, \$13,829;
Company Y, \$310,020; Company Z, \$135,428.

Calculate the gross profit in each case and list the gross profit in ascending order of percentage of sales.

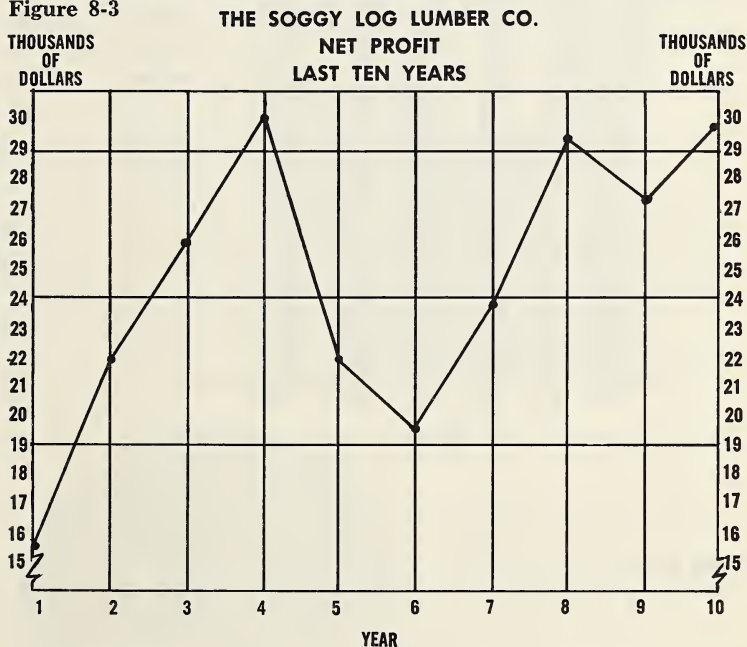
SECTION 7 The Use of Graphic Presentation*

An extensive use of charts and graphs is made by business enterprises to present a visual picture of their operations. It has been said that "one picture is worth a thousand words", and we could add "or a thousand figures." Company reports, government reports, advertisements, and the financial papers all make good use of this form of reporting.

Example: The Soggy Log Lumber Company net profits for the last ten years were: \$15,429, \$21,832, \$25,763, \$30,028, \$21,728, \$19,492, \$23,667, \$29,345, \$27,224, \$29,884.

Draw a line diagram to illustrate this information.

Figure 8-3



Example: The Astitchin Time Toggery report the following monthly sales: Ladies' wear, January, \$17,158; February, \$15,487; March, \$20,028; April, \$18,032; May, \$14,346; June, \$16,234; Children's wear, January, \$10,238; February, \$8,261; March, \$7,238; April, \$11,415; May, \$9,626; June, \$7,237.

Draw a vertical bar chart to show these sales. See figure 8-4.

*See Chapter 4.

Figure 8-4

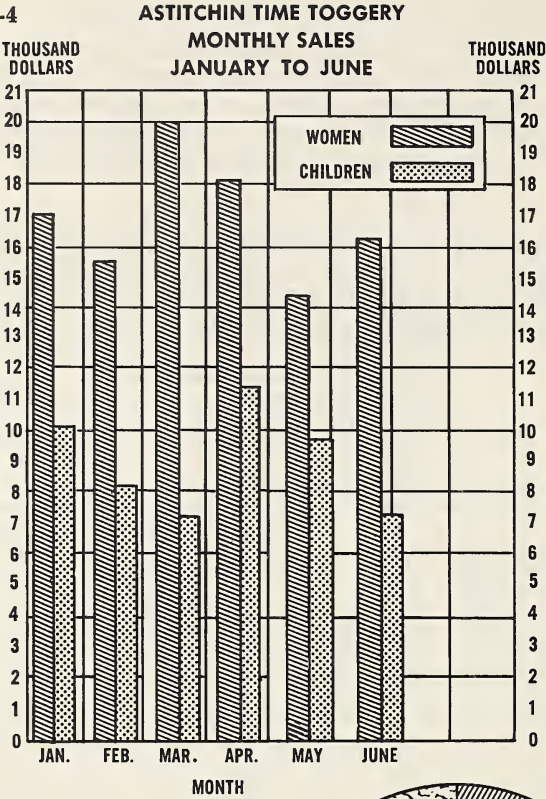
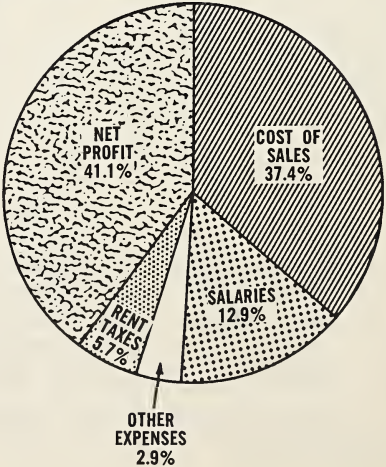


Figure 8-5

The SARTIN TRAINS LTD.
Percentage Breakdown of the
Sales Dollar for the Year 19-1



Example: The net sales of the Sartin Trains Ltd., are \$270,760. The cost of goods sold amounts to \$101,316. The operating expenses are: Salaries, \$35,100; Rent and taxes, \$15,510; Other expenses are \$7,670.

Illustrate the percentage breakdown of the sales dollar by means of a pie chart. Percentages given in first example of Section 6.

See figure 8-5.

WORKOUT EXERCISE VIII

1. Class assignment and discussion: Bring to class a graph published by a company, the government, or from the financial section of a newspaper with a brief written explanation of the information which the graph conveys to you.

2. Draw percentage pie charts for three of the profit and loss statements drawn up in Workout Exercise VI.

3. Draw a horizontal bar chart to compare the per cent of net profit of the ten companies in Workout Exercise VI.

4. Draw a line chart to show the monthly sales of the Glory Bee Honey Company for last year. The sales were: January, \$132,072; February, \$125,725; March, \$100,100; April, \$90,726; May, \$115,320; June, \$95,775; July, \$105,348; August, \$80,320; September, \$90,700; October, \$103,420; November, \$125,360; December, \$150,480.

5. Draw a vertical bar chart to show and compare sales of Widgets and sales of Pudgets manufactured by the Odd Bobs Products during the last ten years:

Widgets: \$43,790; \$45,070; \$48,225; \$49,170; \$42,043; \$39,284; \$41,480; \$44,810; \$46,120; \$44,820;

Pudgets: \$25,381; \$23,417; \$22,418; \$27,311; \$27,500; \$29,890; \$30,570; \$25,310; \$24,029; \$26,660.

6. Draw two percentage pie charts to illustrate the effect of new machinery for the Peculiar Products Limited; the business problem at the beginning of the chapter.

SECTION 8 Figuring for Fun

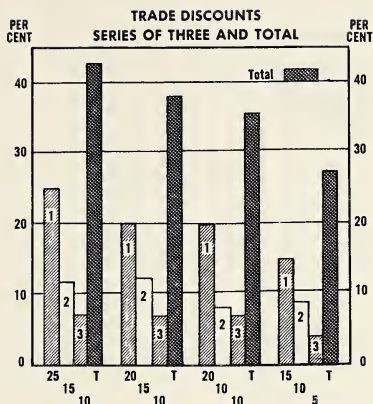
1. This is a very old puzzle and is found in a number of forms. A man comes to the bank of a river and has with him a wolf, a goat and a cabbage. There is only one very small boat in which to row his possessions across the river, and he is only able to take one at a time. However he cannot leave the wolf and the goat together as the wolf

will eat the goat. Neither can he leave the goat and the cabbage together as the goat will eat the cabbage. How is he to cross the river and keep his three possessions safe?

2. Take two glasses and half fill one with water and the other with orange juice. Remove one teaspoonful of the orange juice and pour it into the glass with the water. Then from the mixture remove 1 teaspoonful and pour this into the orange juice. Is the percentage of orange juice in the water now greater or less than the percentage of water in the orange juice?

(Hint: Assign a certain number of teaspoonfuls of liquid to the two glasses, or try to solve graphically.)

3. A quotation from Mark Twain: "Work consists of whatever a body is obliged to do, and play consists of whatever a body is not obliged to do."



Invoices, Cash and Trade Discounts

9

A Business Problem

The Huzzentop Company have been in the habit of purchasing appliances from Werontop Corporation at trade discounts of 45%, 20% and 10%. Huzzentop use the same list price to resell these goods to retailers allowing trade discounts of 40%, 10% and 5%.

The Werontop Corporation have just informed the Huzzentop Company that they are changing their trade discount allowances to 40%, 10% and 5%.

The Huzzentop Company have recently received price lists from Werintoo Limited for the same grade of article. These list prices are 10% higher than those of Werontop Corporation, but the trade discounts they offer are 50%, 15% and 10%.

The Huzzentop Company wish to maintain the same margin of profit as they are at present making. Advise the Huzzentop Company with which company they should deal and what single trade discount they can offer to their customers, if they deal with the company offering them the best price.

SECTION 1 The Invoice

When a sale or a purchase is made, it is recorded in the form of an invoice, or itemized bill, which is rendered by the seller to the buyer, usually through the mail. The seller will make the invoice out in duplicate or triplicate; the original copy is sent to the purchaser, a copy is sent to the accounting department to be recorded, and a copy sent to the shipping department if goods are shipped to the customer.

The invoice should contain the following minimum information: the names and addresses of the seller and of the buyer; the date of the sale; the credit terms of the sale; the invoice number; shipping instructions and details of the merchandise sold. This latter should be itemized to show the quantity and price of each item. The quantity of each item is then multiplied by the price of each item, and the product extended to the total column. The total column is then added.

Figure 9-1

A Form of Invoice

TRANQUIL TIDBITS LTD. 100 Centre St. HAMVILLE			
SOLD TO: Snowdin Caterers, Picleton.		SHIP TO: Same	
YOUR ORDER NO. 64.		VIA Truck	
INVOICE NO. 326. DATE: 20 September, 19-1 TERMS. 2/10, a/30.			

Quantity	Description	Unit Price	Amount
30 doz.	Meat pies, frozen	\$.10	\$ 36.00
24 doz.	Sandwiches, per doz.	.50	12.00
10 lb.	Vegetable salad	.75	7.50
15 lb.	Potato salad	.60	9.00
36 doz.	Assorted tarts	.03	12.96
			\$ 77.46

The Terms of the Sale

The terms inform the buyer when the bill is to be paid. A list of some of the more common terms of sale follows:

Cash: indicates the bill must be paid immediately.

n/30: read, 'net 30 days'; that is, the bill must be paid in full within 30 days from the date of the invoice.

2/10: read, '2 ten'; this means that if the bill is paid within 10 days of the date of the invoice there is a reduction of 2% of the invoiced price. The 2 indicates the rate of cash discount, and the 10 indicates the number of days in the discount period.

2/10, n/30: read, '2 ten, net 30'. This means there is a choice of paying the bill within 10 days of the date of the invoice and receiving a cash reduction of 2%, or paying the full amount 30 days from the date of the invoice.

2/10, n/30, E.O.M.: E.O.M. is an abbreviation for 'end of month'. This means that the discount period starts from the end of the month of the date of the invoice and extends ten days into the following month.

2/10, n/30, R.O.G.: R.O.G. is an abbreviation for 'receipt of goods'. The discount period in this case starts when the buyer receives the goods. It is often used when goods are shipped a long distance.

Finding the Due Date

The days are counted ahead from the date of the invoice.

Example: Plumstone & Company receive an invoice dated September 24th, with credit terms quoted as 2/10, n/30. On what date must they pay the account to take advantage of the discount? On what date must it be paid if they do not take advantage of the discount?

If the 2% discount is taken:

Invoice must be paid in	10 days
Days left in September	<u>6</u>
Invoice is due October	<u>4</u>

If the discount is not taken:

Invoice must be paid in	30 days
Days left in September	<u>6</u>
Invoice is due October	<u>24</u>

Checking the Invoice

When the invoice is received, it should be checked with a copy of the purchase order to make sure the goods charged for on the invoice are the same as the goods ordered. When the goods arrive, they should be examined as to quality and checked against the purchase order to ensure the correct quantity has been received. Extensions should be recalculated and totals checked. Any errors should be noted. Check marks should be placed against each entry as it is checked for accuracy. Many firms use a rubber stamp which is initialed by each person responsible for the various phases of checking.

WORKOUT EXERCISE I

Class discussion and assignment: Visit one or two business organizations and obtain invoice forms, preferably ones that have been used. List and discuss the information obtainable from the invoices.

Find the date on which each of the following invoices would fall due:

1. April 18, 2/15.
2. March 25, 2/10.
3. September 25, 2/10, n/30, E.O.M.
4. December 31, 3/10, n/30, E.O.M.
5. June 15, 2/10, n/30, R.O.G.
6. July 25, n/30.
7. August 24, n/10.
8. October 17, 1/15, n/30.
9. November 12, 1/30, n/60.
10. January 30, 3/5, 2/10, n/30.

Check the following extensions; then find the correct total for problems 11-20.

11. 5 articles at \$1.75 each, \$8.65;
12. 17 articles at \$1.25 each, \$21.25;

13. 6 dozen articles at 49¢ per dozen, \$2.95;
14. 5 dozen articles at \$1.49 per dozen, \$7.55;
15. 7 dozen articles at \$1.15 each, \$8.05;
16. 3 gross articles at 79¢ per dozen, \$28.14;
17. 12 gross articles at \$18 per dozen, \$216.00;
18. 18½ dozen articles at \$10.95 per dozen, \$205.58;
19. 6½ gross articles at \$1.95 per dozen, \$145.60;
20. 178 articles at \$178 each, \$3,258.40.
- 21. The above articles 11-20 were sold by Simplex Limited to Mr. Arthur Jones on November 27th., terms 2/10, n/30, E.O.M. Draw up invoice number 306 to send to Mr. Arthur Jones and name each article.
22. Make out invoice number 37 for the following goods sold by Acme Discount House to Harry Good, of Up Above, on June 26th; terms are 1/5, 2/10, n/30. Extend and total. What are the due dates? 7 ladies' proportioned slacks at \$4.99 each; 15 Artron petticoats at \$1.99 each; 13 washable dusters at \$6.99 each; 18 flannelette sleepwear at \$2.99 each; 23 corduroy surcoats at \$15.95 each; 26 corduroy slacks at \$6.95 each; 17 pairs of athletic boots at \$3.49 a pair; 19 pairs of Sahara boots at \$5.99 a pair.
23. Homey Furnishings Company invoice Solly Samuels of 10 Snowdin Court, Winterville, with the following items. Invoice number 2316 is dated September 30, and the terms are 2/10, n/60.
3, five-piece dinette suites at \$89.95 each; 7 record cabinets at \$19.95 each; 6 wrought-iron dining sets at \$78.95 each; 9 metal bridge sets at \$19.95 each; 5 reclining chairs at \$39.95 each; 4 old colony-corner cupboards at \$35.95 each.

Questions for Discussion:

1. What is an invoice?
2. What minimum information should an invoice contain?
3. What are the terms of an invoice?
4. Bring to class an invoice from a business firm for discussion.
5. What is meant by 2/10, n/30; E.O.M.; R.O.G.?

SECTION 2 Cash Discounts

In order to encourage the prompt payment of an invoice, the seller will offer cash reductions in the form of discounts. If payment is made within a specified time, frequently 10 days from the date of the invoice, a discount, expressed as a percentage of the invoiced cost, is allowed the purchaser. This period of time is known as the *discount period*. This practice enables the seller to collect his money quickly and have ready cash available to meet his obligations. From the purchaser's view point it is important to take advantage of all discounts

offered as they constitute a reduction of the cost of his merchandise purchases and so increase his margin of profit.

To Find the Cash Discount and the Net Invoice Price

If the purchaser pays for his goods within the discount period, the amount he is required to pay is the invoice price less the cash discount. The discount allowed is always calculated on the invoice price; therefore the invoice price is the base:

$$\text{Cash discount} = \text{Rate of discount} \times \text{Invoice price} \quad (1)$$

The cash discount is then deducted from the invoice price.

$$\text{Cash price} = \text{Invoice price} - \text{Cash discount} \quad (2)$$

An alternative method. In the above equation we can substitute the percentage values using the invoice price as 100%. Example: if cash discount is 2%, then cash price = 100% - 2% = 98% = 0.98 of invoice price.

Example: Flamingo Frames Ltd. received an invoice for a shipment of merchandise dated March 20. The amount of the invoice was \$1,500, and the terms were 3/10, 2/30, n/60.

What would be the date and the amount of the cheque sent out in payment:

- (a) on the last day of the 3/10 discount period?
- (b) on the last day of the 2/30 discount period?
- (c) on the due date?

- (a) To obtain 3/10:

Last date available

$$\text{March 20} + 10 = \text{March 30}$$

$$\begin{aligned} \text{Cash discount} &= 0.03 \times \$1,500 \\ &= \$45 \end{aligned}$$

$$\begin{aligned} \text{Cash price} &= \$1,500 - 45 \\ &= \$1,455 \end{aligned}$$

$$(\text{or } 0.97 \times 1,500 = \$1,455)$$

$\therefore$ \$1,455 is due on March 30.

- (b) To obtain 2/30:

Invoice must be paid in 30 days

Days left in March 11

Invoice is due on April 19

$$\begin{aligned} \text{Cash discount} &= 0.02 \times \$1,500 \\ &= \$30 \end{aligned}$$

$$\begin{aligned} \text{Cash price} &= \$1,500 - 30 \\ &= \$1,470 \end{aligned}$$

(or $0.98 \times 1,500 = \$1,470$)

$\therefore$ \$1,470 is due on April 19.

(c) Invoice due:

Invoice must be paid in	60 days
Days left in March	<u>11</u>
	49
Days in April	<u>30</u>
Invoice due on May	<u>19</u>

$\therefore$ \$1,500 is due on May 19.

To Find the Cash Discount When Some of the Merchandise Has Been Returned

If any of the merchandise has been returned, the cash discount will not apply to the portion returned.

Example: On August 21st Tom Grinn purchased 16 crates of oranges at \$2.50 a crate. The terms of the sale were 2/10, n/30, E.O.M. On receipt of the crates it was discovered that 5 of the crates were damaged, and these were returned to the vendor. If Tom Grinn paid his bill on September 8th, for how much did he make out the cheque?

Invoice:

16 crates of oranges at \$2.50 a crate	\$40.00
Less returns; 5 crates damaged	<u>12.50</u>
Amount of invoice	<u>\$27.50</u>

The discount period starts at the E.O.M. Therefore September 8, the date on which payment was made, was within the 2/10 discount period.

Cash discount = $0.02 \times \$27.50$
= \$0.55

Amount of payment = $\$27.50 - \$0.55$
= \$26.95

(or $0.98 \times 27.50 = \$26.95$)

To Find the Cash Discount If Partial Payment Is Made

If a partial payment is made within the discount period, the cash discount may be deducted from this portion at the same rate as expressed in the terms of the invoice.

Example: Gertie Goodchild purchased a bedroom suite on December 20 for \$1,250. The terms of the sale were 3/10, 2/30, n/60. She paid 40% of the invoice on the last day of the 10-day discount period, another 40% on the last day of the 30-day discount period, and the remaining 20% at the end of the 60-day credit period. On what date did she issue the three cheques, and for how much was each cheque made out?

1st cheque

Issued December 20 + 10	=	December 30
40% of invoice price	=	$0.40 \times \$1,250$
	=	\$500
Cash discount on \$500	=	$0.03 \times \$500$
	=	\$15
Cash payment	=	$\$500 - 15$
	=	\$485
(or $0.97 \times \$500$)	=	\$485)
Issued December 30th, for \$485.		

2nd cheque

Issued: Discount period	30 days
Left in December	<u>11 days</u>
Last discount day Jan.	<u>19</u>
40% of invoice price	= \$500
Cash discount on \$500	= $0.02 \times \$500$
	= \$10
Cash payment	= $\$500 - \$10$
	= \$490
(or 0.98×500)	= \$490)
Issued January 19th, for \$490.	

3rd cheque

Issued: Credit period	60 days
Left in December	<u>11 days</u>
	49
Days in January	<u>31</u>
Last payment due Feb.	<u>18</u>
20% of invoice price	= \$250
Issued February 18th, for \$250.	

WORKOUT EXERCISE II

Give the alternative dates, assuming it is not leap year, on which the following invoices can be paid, together with the amount due on each date. (For rounding off figures, see Appendix A.)

<i>Date of Invoice</i>	<i>Amount of Invoice</i>	<i>Terms</i>
1. November 20	\$1,200.00	2/10, n/30
2. March 17	17.50; returns, \$5.80	1/10, n/60
3. January 31	119.75	1½/15, 1/20, n/30
4. December 10	12,760.25	2/5, 1/10, n/20, E.O.M.
5. February 28	1,020,200.00	2½/10, 2/15, 1/20, n/30
6. October 18	30,890.90	2/15, 1½/25, 1/30, n/60
7. April 27	59.98	1/15, ½/20, n/30
8. September 24	543.35; returns, \$98.05	2/20, 1/30, n/60
9. July 29	810.15	1½/20, 1/25, n/30
10. May 22	76.60	2/10, 1/20, n/30, E.O.M.

If the above invoices were paid as follows, calculate the amount of each cheque sent in payment:

11. Number 1, 50% paid in 10 days, 50% in 30 days.
12. Number 2, 60% paid in 10 days, 40% in 60 days.
13. Number 3, 55% paid in 15 days, 30% in 20 days,
balance in 30 days.
14. Number 4, 30% paid to receive 2/5, 20% to receive 1/10,
balance in 20 days.
15. Number 5, 35% paid in 10 days, 25% in 15 days, 20% in 20 days,
balance in 30 days.
16. Number 6, 40% paid in 15 days, 25% in 25 days, 15% in 30 days,
balance in 60 days.
17. Number 7, 25% paid in 15 days, 30% in 20 days,
balance in 30 days.
18. Number 8, 45% paid in 20 days, 30% in 30 days,
balance in 60 days.
19. Number 9, 65% paid in 20 days, 20% in 25 days,
balance in 30 days.
20. Number 10, 70% paid to receive 2%, 15% to receive 1%,
balance when due.

Questions for Discussion:

1. What is a cash discount?

2. Why are cash discounts offered?
3. What is the advantage of a cash discount from the buyer's viewpoint?
4. Why is the cash discount deducted only from the portion of the bill paid?

SECTION 3 Trade Discounts

Manufacturers and wholesalers often issue catalogues to firms *in the trade*. This is an expression used to indicate that the firm is in the same line of business. The catalogue lists all the items of merchandise for sale, with specifications as to size, colour and quality, and quotes a *list price* for each item. This quoted list price may also be the suggested consumer price; and, in order to sell the article and make a profit on the sale, the retailer will have to buy from the manufacturer or wholesaler at less than list price.

This is done by issuing a separate *discount sheet* with each catalogue. The discount sheet lists the rates of *trade discounts* (to firms in the trade) that apply to various items in the catalogue. The trade discount is always based on the list price; so the list price is the base, and the equation is:

$$\text{Trade discount} = \text{Rate of discount} \times \text{List price} \quad (3)$$

The net price to the retailer is therefore the list price reduced by the amount of the trade discount allowed. The equation is:

$$\text{Invoice price} = \text{List price} - \text{Trade discount} \quad (4)$$

Example: The catalogue of the Took Car Accessory Company lists the price of electric horns at \$65 each. The trade discount allowed to Harry Hotrod, a retailer, is 45%. How much does he have to pay for the horn?

$$\begin{aligned}
 \text{Trade discount} &= 0.45 \times \$65 \\
 &= \$29.25 \\
 \text{Invoice price} &= \$65 - \$29.25 \\
 &= \$35.75 \\
 \text{or } 0.55 \times 65 &= \$35.75 \text{ (see below)} \\
 \therefore \text{Harry Hotrod pays } &\$35.75.
 \end{aligned}$$

There is an alternative method. In the above example we can substitute the percentage values in equation (4) using the list price as 100%.

$$\begin{aligned}
 \text{Invoice price} &= 100\% - 45\% \\
 &= 55\% \\
 \text{Invoice price} &= 55\% \text{ of the list price}
 \end{aligned}$$

$$\begin{aligned} &= 0.55 \times 65 \\ &= \$35.75 \end{aligned}$$

This equation would be:

$$\text{Invoice price} = (100\% - \text{Rate of discount}) \times \text{List price} \quad (5)$$

Advantage of Allowing Trade Discounts

Catalogues are expensive to compile and issue. Trade discounts allow the manufacturer to issue one catalogue periodically which contains only suggested consumer, or list, prices. Buyers other than consumers receive a discount sheet with their catalogue which gives the discount they are allowed off the list prices. Where there are frequent changes in production costs which necessitate consequent changes in prices, instead of reprinting the catalogue, the manufacturer will issue another discount sheet which will nullify previous sheets. Thus, prices can be raised or lowered without changing the list prices. This is both faster and cheaper than issuing a new catalogue.

By offering *quantity* discounts, manufacturers can use the method as an incentive to buyers to purchase goods in larger quantities. The seller knows that it is less expensive, per unit, to handle large orders than small orders. The same amount of clerical time, handling time and shipping time may be spent on each order. Assume, for instance, that two orders leave the sales floor; one for \$10 and one for \$100. The overall costs of handling orders from the time they leave the sales floor to the time they are delivered to the customer is calculated at 50¢ per order. In the case of the \$10 order, 5% of each sales dollar is used up for handling and delivery expenses; whereas only 0.5% of each sales dollar of the \$100 order is used up for handling and delivery expenses.

The manufacturer may sell to different classes of buyers and offer different rates of discounts to each class, according to the buyer's function. For instance, wholesalers may buy large quantities, repackage the items, store them, insure them, and then redistribute them. They must therefore be recompensed by a larger trade discount than could be offered to a retailer or to a consumer. These are called *functional* discounts.

WORKOUT EXERCISE III

Questions for Class Discussion:

1. Describe the items listed in a manufacturer's catalogue.
2. What is a discount sheet?
3. What does the term 'in the trade' mean?
4. List and discuss four advantages of issuing separate catalogues and discount sheets. Can you think of any other advantage? Any disadvantage?

5. Describe: cash discount, trade discount, quantity discount, and functional discount.

How much is the price to the retailer in each of the following cases:

	<i>List Price</i>	<i>Trade Discount</i>		<i>List Price</i>	<i>Trade Discount</i>
6.	\$ 105.00	40%	7.	\$ 37.50	45%
8.	82.95	42½%	9.	1,076.76	33⅓%
10.	39.91	38½%	11.	18.60	41¼%
12.	156.42	30%	13.	543.80	47½%
14.	715.20	49%	15.	1,340.60	45½%
16.	48,290.00	41¼%	17.	681,342.25	39⅛%
18.	17.95	43⅜%	19.	8.20	42¼%

SECTION 4 Rate of Trade Discount

The trade discount is the actual amount of the discount in dollars and cents. The rate of trade discount is the per cent value of the discount allowed; thus a 25% discount rate would mean a reduction of 25¢ on every dollar of the invoice price. It should be remembered that the trade discount is always based on the list price, so that the rate of the trade discount may be determined by dividing the amount of the trade discount by the list price.

Or, from equation (3).

$$\text{Trade discount} = \text{Rate of discount} \times \text{List price}$$

Divide each side by list price.

$$\frac{\text{Trade discount}}{\text{List price}} = \text{Rate of discount}$$

That is:

$$\text{Rate of discount} = \frac{\text{Trade discount}}{\text{List Price}} \quad (6)$$

Example: Bag and Bog Ltd. listed their Baggy chafing dishes at \$11.50 each. They sold these chafing dishes to Solomon Sag for \$8.00 each. What was the rate of discount allowed Solomon Sag?

$$\text{Amount of discount} = \$11.50 - \$8 = \$3.50$$

$$\text{Rate of discount} = \frac{3.50}{11.50}$$

$$= 0.3044$$

$$\therefore \text{Rate of discount} = 30.44\%$$

WORKOUT EXERCISE IV

What is the rate of discount in each of the following cases:

	List Price	Cash Price		List Price	Cash Price
1.	\$ 10.00	\$ 8.00	2.	\$ 12.00	\$ 9.00
3.	15.52	11.64	4.	117.75	78.52
5.	892.71	595.14	6.	1,024.72	896.63
7.	37,042.64	32,412.31	8.	159.65	135.70
9.	138,980.00	20,217.70	10.	3,496.00	2,901.68
11.	43,949.00	35,598.69	12.	17,624.00	13,834.84
13.	36,580.00	26,703.40	14.	432,786.00	360,655.00
15.	7,842.72	6,536.60	16.	472.80	295.50
17.	812,620.88	507,888.05	18.	2,010,000.00	1,321,575.00
19.	478.00	394.35	20.	287.00	203.77

SECTION 5 Successive Trade Discounts

The raising or lowering of the list price by means of trade discounts offered to various types of buyers is further facilitated by means of a series of discounts. When a series of discounts is quoted, the first discount is always taken on the list price, this discount is then deducted from the list price and the second discount taken on the remainder. The third discount will be based on the remainder after the first two discounts have been deducted from the list price. For example, a series of discounts would be indicated as 25%, 20%, and 10%.

Figure 9-2

1st Discount 25%	75% Remainder after first discount		
	2nd Discount 20% of 75%	80% of 75% Remainder after second discount	
		3rd 10% of 80% of 75%	90% of 80% of 75% Remainder after third discount

Example: Pine Pomits Ltd. sell their merchandise to a wholesale house at the list price, less a series of trade discounts of 25%, 20% and 10%. If Pine Pomits Ltd. purchase \$750 of merchandise, for how much would they be invoiced?

1st Method: Compute all the discounts.

1st discount	=	25% of \$750
	=	$0.25 \times \$750$
	=	<u>\$187.50</u>
2nd discount	=	20% of (\$750 - \$187.50)
	=	$0.20 \times \$562.50$
	=	<u>\$112.50</u>
3rd discount	=	10% of (\$562.50 - \$112.50)
	=	$0.10 \times \$450$
	=	<u>\$45</u>
Total discount	=	\$187.50
		112.50
		<u>45.00</u>
		<u>\$345.00</u>
Invoice price	=	\$750.00 - \$345.00
	=	<u>\$405.00</u>

2nd Method: Compute each successive balance using equation (5).

Balance after 1st discount

$$\begin{aligned}
 &= (100\% - 25\%) \times \$750 \\
 &= 0.75 \times \$750 \\
 &= \underline{\underline{\$562.50}}
 \end{aligned}$$

Balance after 2nd discount

$$\begin{aligned}
 &= (100\% - 20\%) \times \$562.50 \\
 &= 0.80 \times \$562.50 \\
 &= \underline{\underline{\$450.00}}
 \end{aligned}$$

Balance after 3rd discount

$$\begin{aligned}
 &= (100\% - 10\%) \times \$450.00 \\
 &= 0.90 \times \$450
 \end{aligned}$$

$$\therefore \text{Invoice price} = \underline{\underline{\$405.00}}$$

Notes:

(1) It must be clearly understood that these successive discounts *cannot be added together*. As can be seen from figure 9-2, each successive discount is a percentage of what remains, or a percentage of a part of the whole. Figure 9-3 shows the result of adding the discount which, in this instance, would be the same as a single rate of 55%.

Figure 9-3

Incorrect	25% of whole	20% of whole	10% of whole	Left, 45% of whole
Correct	1st Discount	2nd Discount	3rd Discount	Left, 90% of 80% of 75% of whole

From figure 9-2

(2) As can be seen from figures 9-2 and 9-3, the amount left, or the invoice price, is $90\% \times 80\% \times 75\%$; this is the same as $75\% \times 80\% \times 90\%$. In other words, the order in which these percentages are taken makes no difference to the ultimate result. The same is true for any series of discounts. The discounts are usually quoted with the largest one first, as in the above example: 25%, 20%, and 10%. It is left for the reader to prove for himself that the same discount will result by calculating 20%, 25%, and 10%; or 10%, 20%, and 25%; or 10%, 25%, and 20%.

Example: Would you prefer to be allowed a discount of 40% on a radio your purchase or a series of 25% and 15%?

We can use any figure we wish as the list price, say \$100.

Then a discount of 40% = \$40.00

The series 25% and 15%:

First discount = 0.25×100 = \$25.00

Balance left = $\$100 - 25 = \$75$

Second discount = 0.15×75 = \$11.25

Total discount = \$36.25

$\therefore$ A straight discount of 40% would be preferable.

The above example illustrates again the fact that a single discount rate is always greater than any two (or more) rates which *add up* to the single rate. This was illustrated in figure 9-3.

WORKOUT EXERCISE V

By computing each discount separately, find the invoice price of the following merchandise:

- \$1,000, less 30%, 20%, 10%.
- \$795, less 25%, 10%, 10%.
- \$842.10, less 10%, $7\frac{1}{2}\%$, 5%.
- \$3,894.75, less 20%, 15%, 10%.
- \$11,110, less 40%, 25%, 10%.
- \$113,113, less 20%, 15%, 10%.
- \$8,429.92, less $12\frac{1}{2}\%$, 10%, $7\frac{1}{2}\%$.
- \$348.30, less 15%, 13%, 11%.
- \$138.30, less 37%, $21\frac{1}{2}\%$, 15%.
- \$819.90, less 9%, 8%, 7%.

11-20. Prove the answers to the above by using the second method of computing the invoice price.

21. Roland Furniture Company have obtained prices from two manufacturers for dining-room suites. The Beyo Manufacturing quote a price of \$450, less 20% and 10%. The Cayo Manufacturing quote \$510, less 25% and 15%. If the quality is comparable, which is the better buy? How much would the Roland Furniture Company save by taking the better offer?

22. The Crandall Couch Company offer their chesterfields to Duckworth Home Furnishings at \$295, less 25%, 20% and 10%. They find they can obtain the same quality chesterfield from Delta Manufacturing Company for \$225, less 20%, 15% and 10%. From which company will they obtain the lowest price? How much will they save on each chesterfield by accepting the lower quotation?

23. Henry Caldwell wishes to purchase new office furniture. Sturdy Products list this furniture at \$3,900, less 25% and 10%, terms 2/10, n/30. Stalwart Mfg. Co. list the same furniture at \$3,700, less 30%, terms n/30. If he is able to pay cash, which is the better offer? By how much?

24. The Doggie Boutique were invoiced for supplies on March 28. The amount of the invoice was \$3,720.25, less 12%, 10% and 10%, terms 2/10, n/30. If they paid the invoice on April 5, for how much was the cheque made out?

25. The Dollie Drake Company paid an invoice on June 23 for \$760.30, less 10%, 10% and 5%, with terms 3/10, 2/20, n/30. If the invoice was dated June 3, how much would they have saved if they had paid it on or before June 13?

26. Draw a multiple bar graph similar to the one heading this chapter to show the effect of the trade discounts in problems 1-10.

Questions for Discussion:

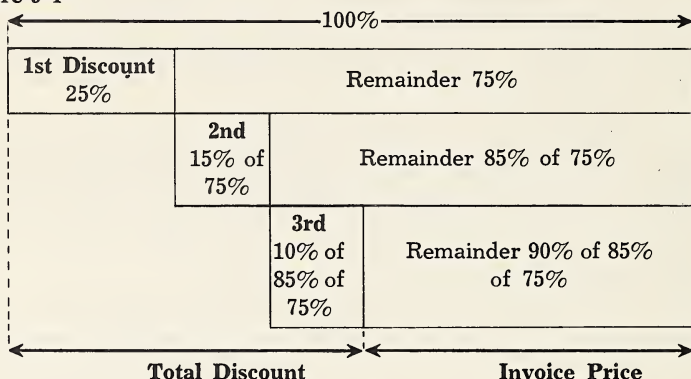
1. Why is a single discount rate always greater than a series of rates that add up to the single rate?
2. What purpose is served by offering successive discount rates?

SECTION 6 Equivalent Single Discount

If a merchant is buying from various sources with numerous price quotations, it may be advantageous for him to know which quotations offer him the best value. If otherwise identical merchandise is offered from two sources at the same price, one allowing trade discounts of 25% and 15% and the other 20% and 20%, from which source should he buy? Also, where a certain discount series is used continuously, time can be saved when computing invoice prices by using a single rate of discount which would equal the series.

If the discount rates offered by a manufacturer are 25%, 15% and 10% and the merchant buys regularly from this manufacturer, he will want to know to what single discount he is entitled.

Figure 9-4



The total discount = 100% – Final remainder.

To find the final remainder subtract each separate discount rate from 100% and multiply the result:

$$100\% - 25\% = 75\%$$

$$100\% - 15\% = 85\%$$

$$100\% - 10\% = 90\%$$

$$\text{Remainder} = \text{Net invoice price} = 75\% \times 85\% \times 90\%$$

$$\text{as can be seen from figure 9-4} = 50.375\%$$

$$\therefore \text{Single discount rate} = 100\% - 50.375$$

$$= 49.625\%$$

Example: The Simple Sewing Machine Company list their model 2X machine at \$250 subject to a discount of 30%, 20% and 10%.

(a) What net price would a customer pay?

(b) What single discount rate is equal to the series offered?

$$(a) \quad 100\% - 30\% = 70\%$$

$$100\% - 20\% = 80\%$$

$$100\% - 10\% = 90\%$$

$$\text{Invoice price} = 70\% \times 80\% \times 90\% \text{ of } \$250$$

$$= 50.4\% \text{ of } \$250$$

$$= 0.504 \times 250$$

$$= \underline{\underline{\$126}}$$

$$\begin{aligned}
 \text{(b) Single discount rate} &= 100\% - 50.4\% \\
 &= \underline{\underline{49.6\%}}
 \end{aligned}$$

Note: Once again it should be pointed out that the single discount rate equivalent to a series of discounts is *always less than* the sum of the series. In the above example the single rate is 49.6%, whereas the sum of the series is 60%.

Example: The Simple Sewing Machine Company list their 4E model at \$200 subject to a 20% and 10% discount. Due to competition, they find they will have to reduce the invoice price of this machine to \$108.

- (a) What additional trade discount must they offer to reduce the list price to the required \$108?
 (b) Name the new discount series.

$$\begin{aligned}
 \text{(a) Present invoice price} &= 80\% \times 90\% \times \$200 \\
 &= 72\% \times \$200 \\
 &= 0.72 \times 200 \\
 &= \underline{\underline{\$144}}
 \end{aligned}$$

$$\text{Required invoice price} = \$108$$

$\therefore$ An additional discount of \$36 must be granted.

The base in this case is the last balance on the present invoice price.

$$\begin{aligned}
 \therefore \text{Additional discount} &= \frac{36}{144} \times \frac{100}{1} \\
 &= \underline{\underline{25\%}}
 \end{aligned}$$

- (b) The new discount series will be: 25%, 20% and 10%.

WORKOUT EXERCISE VI

Find the single equivalent discount to three decimal places for each of the series:

1. 20%, 10%, and 10%.
2. 25%, 10%, and 10%.
3. 30%, 20%, and 10%.
4. 30%, 20%, and 20%.
5. 25%, 20%, and 10%.
6. 40%, 20%, and 10%.
7. 35%, 10%, and 5%.
8. 30%, 10%, and 5%.
9. 15%, 15%, and 10%.
10. 20%, 15%, and 15%.
11. The East Wind Trading Company list their batten boxes at \$75 each with a trade discount of 20%. If they wish to reduce the invoice price to \$51, what additional discount must they offer?

12. The Simplex Gadget Company have a series of discounts on their kitchen gadget of 25%, 20%, and 10%. If the gadget is listed at \$2.00 and they wish to invoice it to a large department store at 99¢, what additional discount must they offer them? What is the new series?

13. Due to the introduction of new machinery, the Goahead Mfg. Co. are able to reduce the price of their product. If the list price is now \$60 with trade discounts of 30% and 10%, what additional discount should they allow to bring the invoice price to \$34.02?

14. Salvo Radio Corp. offer a transistor radio to the trade at \$90, less 20%, 10%, and 10%. The Primers Radio Company offer a comparable set at \$100, less 20%, 20%, and 10%. Which is the more expensive? What additional discount must be given to reduce this higher price to meet the competition of the lower priced set? (Answer to two decimal places.)

15. At what price should the Secundus Corporation list a product which costs them \$30.00 to manufacture if they wish to make 20% profit on the cost and also offer discounts of 20% and 10%?

16. Fred Manners, a salesman, sold \$975 worth of men's wear. If he allowed a discount of 20%, instead of a discount series of 10% and 5%, how much did his error cost his company?

SECTION 7 Equivalent Single Discount Table

Tables have been prepared which give a single equivalent discount factor for various series of discounts. Or a table may be prepared for the series of rates most frequently used by a business. The table shows the invoice price for each dollar of the list price. An example is given in figure 9-5.

Figure 9-5

Equivalent Single Discount Factor Table

Rate Per Cent	5	10	15	20
5	0.9025	0.8550	0.8075	0.7600
5-2½	0.8799	0.8336	0.7873	0.7410
10	0.8550	0.8100	0.7650	0.7200
10-10	0.7695	0.7290	0.6885	0.6480
25	0.7125	0.6750	0.6375	0.6000

To read the table: If an article is listed at \$100 subject to a discount of 15%, 10% and 10%, what is the invoice price?

If the discount series consists of three rates, go down the rate per cent column until two of the three rates are located; follow this row

along to the right until the column containing the third rate is reached. This factor is used to find the invoice price. In this case, an article listed at \$100 with a discount series of 15%, 10% and 10% would be invoiced at $\$100 \times 0.6885 = \$68.85$.

If the discount series consists of two rates only, locate the first by going down the rate per cent column until one rate is found, then follow this row to the right until the column headed by the second rate is reached. This is the required figure.

Example: Find the invoice price per dollar for a discount of 25% and 20%.

25% and 20% gives factor 0.6000
= 60¢ per dollar of the list price.

WORKOUT EXERCISE VII

1. Complete the following table to four decimal places.

Rate Per Cent	2	5	12½	25
5-3	—	.8754	—	—
12½	—	—	—	—
15-10	—	—	—	—
20	—	—	—	—

Find the net prices for the following using the table in figure 9-5 and problem 1 above.

2. \$25.00 less 5%, 5%, and 3%.
3. \$179.00 less 20%, 25%, and 10%.
4. \$428.40 less 10%, 10%, and 5%.
5. \$792.20 less 25%, 15%, and 10%.
6. \$7,840.50 less 15%, 10%, and 2%.
7. \$49.75 less 10%, 5%, and 2½%.
8. \$10.20 less 15%, 12½%, and 10%.
9. \$19.85 less 5%, 3%, 2%.
10. \$315.28 less 15%, 5%, and 2½%.
11. From the table on page 353 check the answers to problems 1-10, Workout Exercise VI.
12. From the table on page 353 check the answers to problems 1-10, Workout Exercise V.
13. Draw a multiple bar chart to show the series of discounts and the total discount for each of problems 2-10 above.

SECTION 8 Sales Tax

In 1921, the Federal Government of Canada imposed a *sales tax* on goods manufactured or produced in Canada and on certain classes of goods imported into Canada. This sales tax is collected by the manufacturer or importer when goods are sold to the wholesaler or retailer. The money collected does not belong to the manufacturer and cannot be used by him but represents a debt to the Government. At the end of each month, the total sales tax collected for that month must be remitted to the Government.

In addition to the Federal sales tax, most of the Provinces in Canada levy a sales tax on the retail price of goods when sold to the consumer.

Example: The invoice submitted by Mortimer Egan to Fran Mark amounted to \$1,274. What amount of sales tax must Mortimer Egan collect if the tax is 11%? How much would Fran Mark pay?

$$\begin{aligned}\text{Sales tax} &= 11\% \text{ of } \$1,274 \\ &= 0.11 \times 1274 \\ &= \underline{\underline{\$140.14}}\end{aligned}$$

Invoice	\$1,274.00
Tax	<u>140.14</u>
Fran Mark pays	<u><u>\$1,414.14</u></u>

Example: Tillie Toots paid an invoice amounting to \$1,040.29 submitted by the Velour Hat Forms Ltd., her supplier. The price paid included an 11% sales tax. What was the quoted price of the hat forms? What was the amount of the tax?

The price paid was 111% of the quoted price.

$$100\% \text{ is } \frac{1040.29}{111} \times \frac{100}{1} = \$937.20$$

$$\therefore \text{Quoted price} = \$937.20$$

$$11\% \text{ is } \frac{1040.29}{111} \times \frac{11}{1} = \$103.09 \text{ or } \frac{11}{100} \times \frac{937.20}{1}$$

$$\therefore \text{Sales tax} = \$103.09$$

WORKOUT EXERCISE VIII

1-10. Calculate the amount of sales tax that would be collected in each of the problems 2-11 in Workout Exercise VII, assuming a sales tax of 11%.

Find the quoted price of the following merchandise if the amounts paid, after collection of 11% sales tax, are:

11. \$150.00

14. \$7,360.25

17. \$1,342,820.75

20. \$0.69

23. \$11.25
12. \$369.50

15. \$7.95

18. \$49.20

21. \$555.55

24. \$13.75
13. \$1,428.70

16. \$55.40

19. \$19.95

22. \$74.25

25. \$38,420.19

Questions for Discussion:

1. What is a sales tax? By whom is it collected? By whom is it levied?
2. What must the person or company collecting the tax do with the amount collected?

SECTION 9 Cash and Trade Discount Deductions

Where both cash and trade discounts are allowed, it is customary to deduct the trade discounts first, then the cash discounts; then add the sales tax if it applies.

Example: Karmel Kream, of Brittle town, manufactures television antennae which he lists at \$60, less 20% and 10% discount. He offers an extra quantity discount of 5% on orders over 1 dozen. Terms are 2/n, n/30. Sales tax of 11% applies. On May 2 Marsh Mallow, of Hardcentre, purchases 15 of these antennae. Make up the invoice that Marsh Mallow will pay on May 10.

INVOICE KARMEL KREAM BRITTLETOWN			
SOLD TO: Marsh Mallow, Hardcentre.		DATE: May 2, 19-1 Terms: 2/10, n/30	
Quantity	Description	Unit Price	Amount
15	Television antennae @	\$60	\$900.00
	Less 20%		180.00
			\$720.00
	Less 10%		72.00
			\$648.00
	Less 5%		32.40
			\$615.60
	Less 2% (cash)		12.31
			\$603.29
	Add sales tax 11%		66.36
			<u>\$669.65</u>

WORKOUT EXERCISE IX

At the end of last month Karmel Kream, of Brittletown, mailed out invoices to his customers. In addition to the trade discounts, a 5% quantity discount was offered on orders over one dozen. Draw up the necessary invoices assuming all cash discounts are taken when offered. Terms are 2/10, n/30. An 11% sales tax is in effect.

	<i>Name and Address</i>	<i>Articles</i>	<i>List Price</i>	<i>Trade Discounts</i>
• 1.	John Adams, Evetown,	18 antennae,	\$60	20%, 10%.
• 2.	Abie Christie Sodacrack,	48 antennae,	\$65	15%, 10%, 10%.
3.	Devon Mfg., Creamton,	35 antennae,	\$45	15%, 10%.
4.	Flavour Bros., Foodville,	14 antennae,	\$50	15%, 10%.
• 5.	Hardy Products, Beefburg,	11 antennae,	\$65	15%, 10%, 10%.
6.	Jones & Kelly, Snowdin,	28 antennae,	\$75	25%, 20%.
7.	Layman & Co., Ableton,	13 antennae,	\$55	15%, 10%.
8.	Albert Proctor, Pebble Creek,	23 antennae,	\$63	15%, 10%, 10%.
9.	Sandy Sieve, Colander,	75 antennae,	\$58	10%, 10%, 10%.
10.	Tilly & Toots Ltd., Tootsville,	6 antennae,	\$60	20%, 10%.
11.	Fran Creeper, Creepchester,	18 antennae,	\$50	15%, 10%.
12.	Jill Jive, Swingham,	15 antennae,	\$65	15%, 10%, 10%.

SECTION 10 Figuring for Fun

- Ask a friend to think of a number, (say 9).
Multiply it by the number of months in a year, ($9 \times 12 = 108$).
Divide the answer by the number in a trio, ($108 \div 3 = 36$).
Multiply this result by the number of days in a week,
($36 \times 7 = 252$).

Divide this result by the number of whole weeks in a month,
 $(252 \div 4 = 63)$.

Now divide the result by the number originally thought of,
 $(63 \div 9 = 7)$.

Add to this result the number originally thought of $(7 + 9 = 16)$.
 Ask for this result.

From this result subtract 7, and you will have the answer first
 thought of.

2. Take any number you please, say 13,942. Rearrange the digits in
 any manner and find the difference between the two numbers.

31924

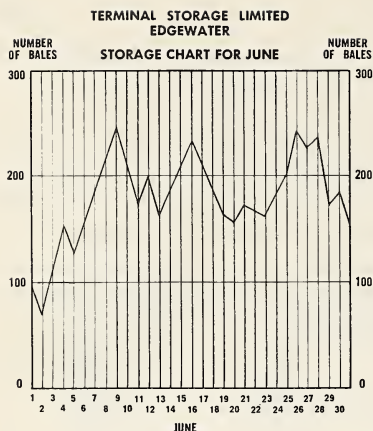
23419

93421

49321

139421394213942139421798294777947935379

The difference is always divisible by 9.



Buying and Selling on Commission and Storage

10

A Business Problem

John Peel, a recent junior college graduate, is considering a career as a salesman. Duyuken Products have offered him a job as sales clerk in the men's haberdashery department at a salary of \$75.00 a week. Dominion Rolette have offered him a job as a travelling salesman at a salary of \$30 a week plus $2\frac{1}{2}\%$ commission on sales he makes over \$6,500 a week. In addition he will be provided with a car and allowed 9¢ per mile operating expenses. Normal sales for this territory have been \$8,000-\$8,500 a week. The third offer is to sell engines on a straight 1 per cent commission basis for Wolfe Machinery Limited. Normal sales have been \$6,000-\$8,000 a week. Which appears the most attractive offer? What advice can you give John Peel? What other considerations besides remuneration would you think important?

Commission is compensation paid to a salesman, or an agent, for services rendered, and usually takes the form of a certain per cent of the money involved in the transaction. Salesmen in many lines of business earn their salary wholly or partially as commission according to the value or the quantity of goods they sell.

According to the 1960 "Taxation Statistics" prepared by the Tax Division of the Department of National Revenue almost 51,000 tax payers claimed to be salesmen. It can be seen, therefore, that this is a career which will be of interest to many people.

SECTION 1 Straight Commission

If a salesman is paid a straight commission on his sales, he will receive as wages a percentage of his total sales for each given period. This plan has the advantage of providing a strong incentive for the salesman to sell as much as possible because the more he sells, the more he earns. On the other hand, he may be more interested in

making quick sales without rendering any special services to the customers, and, in this way, the company's reputation may suffer.

Example: Tom Jones, a high school student, sells boxed Christmas cards in his spare time. He receives a commission of 15¢ a box for each box he sells. During October, November, and December he sells 125 boxes. How much commission does he earn?

Commission on one box	=	15¢
Commission on 125 boxes	=	0.15×125
	=	<u>\$18.75</u>

Example: Paul Pritchard, a salesman for Twinkle Light Display Company, receives a straight commission of 12% on his total sales. His sales during September amounted to \$4,760. How much commission did he earn?

Commission earned	=	12% of \$4,760
	=	0.12×4760
	=	<u>\$571.20</u>

WORKOUT EXERCISE I

Calculate the following commissions:

- 2% of \$742.00
- 1% of \$923.24
- 1½% of \$25.95
- 2½% of \$1,324.18
- 3% of \$79.95
- 3½% of \$118.39
- 4% of \$798.21
- 4½% of \$543.32
- 5% of \$199.98
- 8½% of \$328.42

What rate of commission would a salesman earn in each of the following cases:

- \$25 on \$125
- \$4.29 on \$429
- \$10.23 on \$682
- \$6.94 on \$347
- \$77.13 on \$1,028.40
- \$11.685 on \$934.80
- \$136.395 on \$389.70
- \$642,939.75 on \$1,428,755
- \$22.45 on \$898
- \$8.865 on \$197

SECTION 2 Graduated Commissions

An alternative method of recompensing a salesman is to set up a sliding scale of commissions payable. The rate of commission may vary with the price-range or quality of the merchandise, or may increase with increases in total sales beyond predetermined levels.

Example: Harry Jamieson sells two makes of radios. The Omega sells for \$58.00, and he receives 8% commission on all sales. The Alpha sells for \$85.00, and he receives 10% commission on all sales.

During one month he sold 45 Omegas and 21 Alphas. How much did he earn in the month?

Omegas sold: 45 at \$58 = \$2,610

Commission earned = 8% of \$2,610

$$= 0.08 \times 2610 = \$208.80$$

Alphas sold: 21 at \$85 = \$1,785

Commission earned = 10% of \$1,785

$$= 0.10 \times 1785 = \$178.50$$

Total earned:	on Omegas	\$208.80
	on Alphas	178.50
		<u>\$387.30</u>

Example: The Warder Corporation pay their salesmen each month on the basis of 5% on sales up to and including \$12,000; 7½% on sales over \$12,000, up to and including \$15,000; and 10% on sales over \$15,000. If Tex Riley sold goods amounting to \$15,575 in March, how much did he earn?

Commission earned:

$$5\% \text{ of } \$12,000 = \$600.00$$

$$7\frac{1}{2}\% \text{ of } \$3,000 = 225.00$$

$$10\% \text{ of } \$575 = 57.50$$

$$\text{Total earned} = \underline{\underline{\$882.50}}$$

WORKOUT EXERCISE II

1. Toddy Riley receives 7½% commission on the sale of transistors selling for \$49.50, and 12½% commission on the sale of transistors selling for \$85.40. If he sells 29 at \$49.50 and 17 at \$85.40, how much does he earn in commission?

2. Budge Canary sells men's suits. He receives 2% commission on tailored suits selling at \$49.95; 1½% commission on sports suits selling at \$39.75; and ½% commission on sports jackets selling at \$24.89. If he sells 47 tailored suits, 31 sports suits and 38 sports jackets in one week, how much commission does he earn?

3. Albert Archibald sells three grades of widgetts for the Dennis Arrowsmith Products. Grade 1 sell for \$15.95 each, grade 2 for \$12.50, and grade 3 for \$7.95. He earns 5%, 4% and 2½% commission respectively, on each grade. If he sells 54 grade 1, 78 grade 2, and 69 grade 3 in one week, how much does he earn?
4. James Austin of the Baketwist and Barry Bakery receives a 12% commission on the sale of bread and rolls, 15% commission on the sale of pies, and 7½% commission on the sale of cakes. He sells \$259.78 of bread, \$137.45 of pies, and \$318.24 of cakes in one week. How much does he earn?
5. Pete Bartosh, a salesman for the Batt & Beatty Bedsprings, receives a commission of 5% on the sale of bedsprings priced at \$49.70; 7½% on the sale of mattresses priced at \$57.95; and 6% on the sale of bedsteads priced at \$109.54. If he sells 21 bedsprings, 17 mattresses, and 19 bedsteads in one week, how much commission does he earn?
6. John Beaudro and David Bechtel were salesmen for Beckett & Berman Brollies and were paid commissions of 4% on sales up to and including \$1,500; 7½% on sales over \$1,500 up to and including \$2,000; and 9% on sales over \$2,000. If Beaudro's sales amounted to \$2,430 and Bechtel's sales amounted to \$2,745, how much did each man earn?
7. Robertson Boak sells farm machinery for the Bonus & Boothe Plough Co. on a sliding scale of commissions. He receives 2% on the first \$10,000, 2½% on the next \$15,000 and 3½% on sales over \$25,000. If he sells \$35,000 worth of farm machinery in April, how much commission does he earn?
8. Donald Bootland receives a 1½% commission on all sales of haberdashery up to and including \$4,000, 2½% on sales over \$4,000 up to and including \$6,000 and 3½% on sales over \$6,000. If he sold \$5,480, \$6,270, \$4,320, \$7,206, \$6,120 and \$5,216 in six consecutive weeks, how much commission did he earn?
9. The Bornstein and Boyko Bookstore pay their salesmen a sliding scale of commissions as follows: Books: 4½% on sales up to and including \$1,700, 6% over \$1,700; Magazines and periodicals: 2½% on sales up to and including \$500, 3% over \$500; Stationery supplies: 1½% on sales up to and including \$750, 2½% over \$750. If Alan Brodrick sold \$2,100 worth of books, \$625 worth of magazines and \$810 worth of stationery supplies in one week, how much did he earn?
10. Donald Brown, James Bruce and John Burgess also work for the Bornstein and Boyko Bookstore on the same terms. In one week Donald Brown sold \$1,650 worth of books, \$920 worth of magazines and \$615 worth of stationery. James Bruce sold \$1,295 worth of books, \$475 worth of magazines and \$595 worth of stationery. John Burgess sold \$1,940 worth of books. How much did each man earn?

SECTION 3 Salary and Commission

Sometimes the most satisfactory arrangement for both the company and the salesman is a combination of salary and commission. The salesman receives a basic salary which will probably cover his living expenses, and then receives either a commission on the sales he makes over and above a certain stated value, called a quota, or a smaller percentage on all his sales. The commission serves as a stimulus to increase his sales, which stimulus might be lacking if he were paid a salary only.

Commission Based On Sales Over and Above a Quota

Example: Sam Craig, a salesman for the Plyton Tire Company, receives a weekly salary of \$85, plus a 10% commission on all sales made above his quota of \$1,250. Last week Sam's total sales amounted to \$1,475.

(a) How much commission did he earn last week?

(b) What were his total earnings for the week?

(a)	Sales made	\$1,475
	Required quota	<u>1,250</u>
	Sales above quota	<u>\$ 225</u>
	Commission	= 10% of \$225
		= 0.10×225
		= \$22.50

(b) Total earnings:

Salary	\$85.00
Commission	<u>22.50</u>
	<u>\$107.50</u>

Commission Based On Total Sales

Example: Sally Cook, a sales clerk in the Fur Department of the Teeton Department Store, is paid a salary of \$42.50 a week plus 1% commission on all her sales. During the last week of December, Miss Cook sold furs amounting to \$3,500.

How much did she earn the last week in December?

Salary, per week	\$42.50
Commission: 1% of \$3,500	<u>35.00</u>
Total earnings	<u>\$77.50</u>

A Graphical Solution

If two alternatives were offered, the salesman would be interested in knowing not only how much he would earn under both plans, but at what point the two plans would coincide and by how much they would differ for high and low sales.

Example: The Solar Hardware Ltd. offer Sam Hall, an outside salesman, two methods of payment:

- He would receive a basic salary of \$20 a week plus 5% commission on all sales, or,
- He would receive a straight commission of 10% on all sales. Sam estimates that he should be able to sell \$400 of hardware a week.

Which offer should Sam Hall accept?

*See graph, figure 10-1, below.

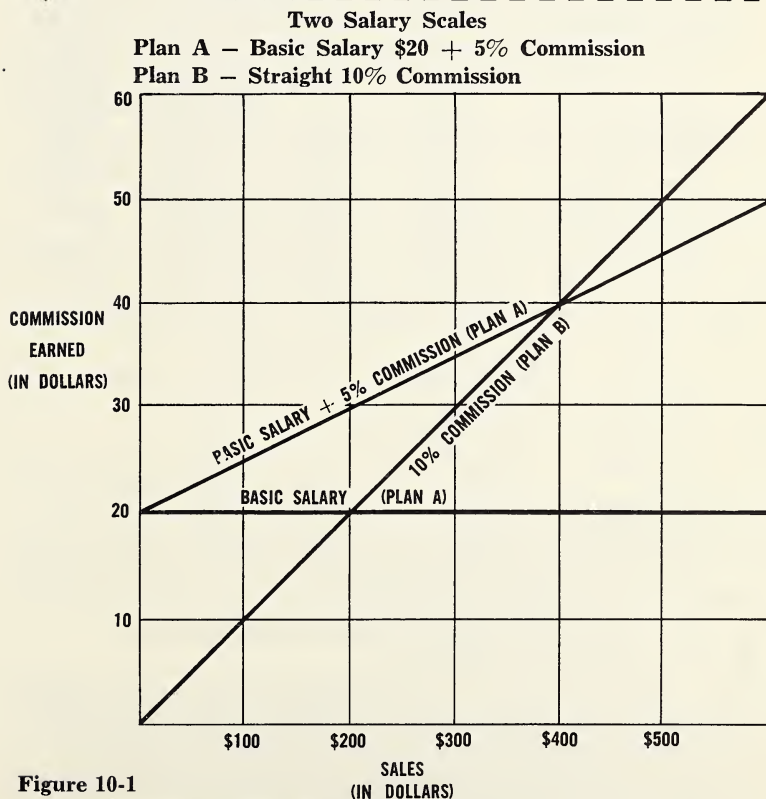


Figure 10-1

* For graphs see Chapter 4.

From the graph we see that, if sales fall below \$400 a week, plan A would be preferable. Sales of \$400 a week would yield a net pay of \$40 a week under either plan. If sales are over \$400 a week, more money will be earned under plan B.

Before making a decision Sam Hall should find out what sales may normally be expected and also what he estimates he will be able to sell.

WORKOUT EXERCISE III

1. Tom Caldwell, a salesman for Campbell Cartons, receives a weekly salary of \$45 plus a 15% commission on all sales above a quota of \$750. If he sold \$975 worth of cartons in one week, how much did he earn?

How much do the following salesmen earn?

<i>Name</i>	<i>Basic Salary</i>	<i>Commission Over Quota</i>	<i>Sales</i>
2. Van Campbell	\$40.00	1½% over \$3,000	\$6,280
3. Bob Cardinali	\$25.00	5% over \$750	\$1,620
4. Jack Carpenter	\$55.00	2½% over \$5,000	\$7,380
5. Neil Carver	\$35.00	4% over \$2,500	\$3,920
6. Dick Cavers	\$41.50	7½% over \$2,000	\$5,815
7. Don Chalmers	\$49.75	5½% over \$3,500	\$4,900
8. Dartnell Clark	\$38.40	6¼% over \$4,600	\$5,870
9. Norm Clegg	\$46.20	8¾% over \$4,000	\$4,750
10. Gary Cochrane	\$51.30	12⅛% over \$7,800	\$8,340

How much do the following salesmen earn if they each sell \$4,000 worth of goods?

<i>Name</i>	<i>Basic Salary</i>	<i>Commission On All Sales</i>
11. Mike Cody	\$25.00	1½%
12. Al Crowley	\$30.00	1¼%
13. Gary Lind	\$35.00	1%
14. Paul Linton	\$40.00	¾%
15. James Lusby	\$45.00	½%

16. Macfarlane & MacLeod Limited offer Thomas Magee a job as salesman on either of the following bases:

- (a) Basic weekly salary \$25, 4½% commission on all sales, or
- (b) 8% commission on all sales.

Draw a graph to show the possible earnings under the two salary schemes. If sales have averaged \$900 a week in the past, which scheme should Magee accept? What other factors would influence his decision?

17. Draw a vertical bar graph of the weekly salaries of the salesmen in problems 2-15 above. Arrange with highest salary at the top.

SECTION 4 Agencies

Many persons, in business for themselves, function as third parties acting between buyers and sellers. They negotiate the sale and purchase of goods that belong to other people. Manufacturers of goods and producers of goods or natural products often find it difficult to sell directly to the consumer. On the other hand, stores and manufacturers may find it difficult to purchase the produce or goods they need to conduct their business. Both the persons or firms wishing to sell goods and the persons or firms wishing to buy goods may employ the services of a middleman, called an *agent*, a *broker*, or a *factor*, to buy, sell, and transact business for them. The person for whom the agent conducts business is called the *principal*.

Payment for the services of an agent is in the form of a *commission* or *brokerage* and is usually calculated either as a percentage of the gross receipts or as a stated amount on each article sold. The agent will be re-imbursed for any expenses incurred in transacting the business. The net amount the principal receives will be the proceeds of the transaction less the agent's commission and expenses.

The Agent's Commission

When an agent sells a commodity, his commission will be based on the net sales price.

$$\text{Commission} = \text{Rate of commission} \times \text{Sales price} \quad (1)$$

Example: The Nu-Land Realty sold a house and lot for John Newton for \$45,000. Commission was charged at 7½%. How much commission did Nu-Land Realty receive?

$$\begin{aligned} \text{Commission} &= 7\frac{1}{2}\% \text{ of } \$45,000 \\ &= 0.75 \times 45,000 \\ &= \underline{\underline{\$3,375}} \end{aligned}$$

The Principal's Net Proceeds

The amount of the net proceeds received by the principal will be the gross proceeds of the sale (that is, the total amount received from the buyer) less the agent's commission and less any expenses incurred by the agent.

$$\begin{aligned} \text{Net proceeds} &= \text{Gross proceeds from sale} \\ &\quad - \text{Commission} - \text{Expenses} \end{aligned} \quad (2)$$

Example: The Paynow Collection Agency collect 80% of an outstanding bill of \$1,200 for Harry Bleus. The commission charged was 12%, and the expenses of collecting the bill amounted to \$75.70. How much did Harry Bleus receive?

$$\begin{aligned}
 \text{Net Proceeds} &= 80\% \text{ of } \$1,200 - \\
 &\quad 12\% \text{ of } 80\% \text{ of } \$1,200 - \$75.70 \\
 &= \$960 - \$115.20 - \$75.70 \\
 &= \underline{\underline{\$769.10}}
 \end{aligned}$$

The Sales Price

If the amount of the net proceeds received by the principal and the rate of commission paid to the agent are known, the gross proceeds from the sale can be found by finding the number on which the given rate per cent is based:

From equation (2):

$$\text{Net proceeds} = \text{Gross proceeds} - \text{Commission.}$$

The rate of commission is always based on the amount collected, or the gross proceeds, and therefore the commission is a percentage of the amount collected. If we let the amount collected = 100%, then:

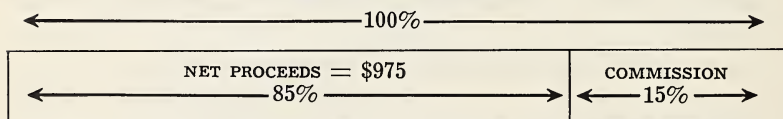
$$\text{Net proceeds} = 100\% - a \% \text{ of } 100\% \quad (3)$$

Example: John Witter, an auctioneer, remitted to Jonathan Smart, his principal, \$975 after deducting a commission of 15%. What amount did John Witter collect from the sale?

$$\begin{aligned}
 &\text{Let the amount collected} = 100\% \\
 &\quad (\text{See figure 10-2}) \\
 &\text{Then net proceeds} = 100\% - 15\% \\
 &\quad = 85\% \\
 &\therefore 85\% = \$975 \\
 &\quad 100\% = \frac{975}{85} \times \frac{100}{1} \\
 &\quad = \underline{\underline{\$1,147.06}}
 \end{aligned}$$

Figure 10-2

Gross Proceeds of Sale



Brokers and Brokerage

Brokers are agents who arrange for the purchase or sale of goods without having possession of them. Typical examples are: real estate brokers, who buy and sell real estate for their principals and may be

either buyers or sellers; insurance agents or brokers; stocks and bonds brokers; auctioneers; and collection agencies, who collect debts which the principals have found difficult to collect. They each concentrate on one specialized area of buying or selling and bring clients into touch with principals. In some cases, orders are taken by the brokers and forwarded in writing to the principal who then fills the order direct. The commission in the above cases is called *brokerage*.

WORKOUT EXERCISE IV

Find the commission earned by the agent for each of the following sales:

	<i>Amount of Sale</i>	<i>Commission</i>		<i>Amount of Sale</i>	<i>Commission</i>
1.	\$7,500	5%	2.	\$25,100	7½%
3.	\$320.95	3½%	4.	\$17,800	4¼%
5.	\$100,100	5¾%	6.	\$1,006,700	6¼%
7.	\$494,800	6¾%	8.	\$320,760	7¼%
9.	\$64,000	7¾%	10.	\$42,200	2½%

Find the net proceeds received by the principal in each of the following cases:

	<i>Amount of Sale</i>	<i>Expenses</i>	<i>Commis- sion</i>		<i>Amount of Sale</i>	<i>Expenses</i>	<i>Commis- sion</i>
11.	\$350	\$23	10%	12.	\$11,240	\$1,479	7%
13.	\$9,680	\$1,782	9%	14.	\$759.50	\$37.25	7½%
15.	\$14,960	\$5,470	3¼%	16.	\$1,300,200	\$27,480	12½%
17.	\$743,880	\$7,830	11%	18.	\$49,100	\$4,320	8¾%
19.	\$68,320	\$3,840	7¼%	20.	\$11,110,110	\$740,000	6¾%

Find the amount of the gross sales in each of the following cases:

	<i>Received by Principal</i>	<i>Commission Charged</i>
21.	\$686.40	12%
22.	\$723.84	13%
23.	\$437.75	15%
24.	\$64,294.09	12½%
25.	\$40,222.94	8½%

Questions for Discussion:

- Define an agent and a principal.
- What is commission or brokerage?
- On what amount is the commission always calculated?
- What is a broker?
- List and describe five different types of broker.

SECTION 5 Commission Merchant

A commission merchant may act as an agent for the purchaser or for the buyer. Unlike the broker, he obtains actual possession and control of the goods, although the principal retains ownership. A farmer, for instance, who is the principal (or consignor), ships goods *on consignment* to the commission merchant (or consignee). The commission merchant receives the goods, stores them temporarily if necessary, makes the sale, delivers the goods to the customer, pays any necessary expenses, deducts his commission which is based on the *gross* receipt, and remits the balance to the farmer. The farmer is therefore relieved of the necessity of finding his own market. It should be stressed that *ownership* of, or title to, the goods never passes to the commission merchant. The principal owns the goods until they are sold, and then ownership passes directly to the purchaser.

Commission agents deal largely in agricultural products such as fruit and vegetables. This often necessitates quick sale, as these goods are perishable, and the commission merchant is frequently empowered to sell them at the best price available. He personally, therefore, runs no risk from a fall in market price.

Net Proceeds to the Principal

The net proceeds received by the principal will be calculated in the same way as outlined in Section 4 using equation (2).

$$\begin{aligned}\text{Net proceeds} &= \text{Gross proceeds from sale} \\ &\quad - (\text{Commission and expenses})\end{aligned}$$

Example: On July 20 Simon Sloan of Bracebridge shipped to Abe Zellar, a commission merchant in Toronto, a shipload of raspberries. Abe Zellar received the raspberries, paid \$10 in delivery charges, and \$8 for advertising.

On July 22, Zellar sold one-third of the shipment for \$250; on July 23, he sold another third for \$200; and on July 24, he sold the remainder for \$75. Abe Zellar receives a commission of 5% on all sales. How much did Simon Sloan receive for his shipment of raspberries?

$$\begin{aligned}\text{Net proceeds} &= (\$250 + \$200 + \$75) - \$10 \\ &\quad - \$8 - 5\% \text{ of } \$525 \\ &= \$525 - \$18 - \$26.25 \\ &= \underline{\underline{\$480.75}}\end{aligned}$$

WORKOUT EXERCISE V

Find the net proceeds for the sale of the following produce:

1. Farmer Dellar consigned a shipload of strawberries to Ronald

Marshak on June 27. Marshak sold $\frac{1}{4}$ for \$210 on June 29, $\frac{1}{3}$ for \$260 on June 30, $\frac{1}{12}$ for \$80 on July 2, and the remainder for \$125 on July 3. He paid: freight charges, \$17; storage, \$12; advertising, \$25. Marshak's commission was 6%.

2. On September 18 Douglas McCaw sent his crop of potatoes to his commission merchant William McKee to sell on consignment. McKee paid the following expenses: freight, \$11; storage, \$7; handling, \$11; advertising, \$20; commission, $7\frac{1}{2}\%$. Sales were: \$470, \$120, \$75, \$65 on September 21, 25, 29 and October 3, respectively.

3. Robert Metzger shipped, on May 26, a consignment of lettuce to Robert Miller in Hydroburg to sell at the best available prices. Miller paid: freight, \$15; storage, \$10; advertising, \$15; commission 9%. Sales were: \$75, \$55, \$40, \$22.50 on May 29, 30, 31 and June 2, respectively.

4. Metzger also shipped to Miller a consignment of tomatoes, on September 18. The following sales took place on September 20, 22, 23, 24 and 25, respectively: \$49.75; \$55.32; \$36.29; \$10.18; \$15.16. Expenses paid were: freight, \$17.50; storage and handling, \$12.15; commission, $7\frac{1}{2}\%$.

5. Douglas Mitchell shipped his apples, on September 28, on consignment to Monnot & Morrison, commission merchants in Stumpton. They sold $\frac{1}{4}$ for \$195 on September 30; $\frac{1}{2}$ of the remainder for \$220 on October 6; and $\frac{3}{8}$ of the shipment for \$180 on September 13. Their expenses amounted to: freight, \$16.25; storage, \$27.80. Commission, 8%. Was any of the shipment still on hand? If so, what fraction of the shipment?

SECTION 6 The Account Sales

The commission merchant, or consignee, will be required to remit to his principal the money he has collected, less expenses and commission, together with a written report giving the details of all transactions. This document is called an *Account Sales* and should contain the following information:

The names and addresses of both the consignor and the consignee.

The details of the consigned goods.

The details of all sales made from receipt of goods to date of the *Account Sales*.

The details of any expenses paid by the consignee.

The amount of commission earned by the consignee.

The net proceeds of the sale.

A cheque for the net proceeds of the sale should be sent to the principal with this account. An *Account Sales* will be sent by the commission merchant to his principal either at the conclusion of each consignment, at periodic intervals, or on a monthly basis, depending on the nature of the consignment and the consigned goods.

Figure 10-3

An Account Sales

ABE ZELLAR Commission Merchant Fleet St. Toronto ACCOUNT SALES			July 24, 19-1
TO: Simon Sloan, Bracebridge, Ont.			
This statement shows the disposition of the consignment of raspberries received July 22, sold for your account and risk, with a commission of 5%.			
SALES			
July 22	One-third consignment		\$250.00
July 23	One-third consignment		200.00
July 24	One-third consignment		<u>75.00</u>
			\$525.00
CHARGES			
July 22	Delivery charges	\$10.00	
	Advertising	8.00	
	Commission	<u>26.25</u>	
			<u>44.25</u>
	NET PROCEEDS (Cheque herewith)		<u><u>\$480.75</u></u>

Abe Zellar will prepare an Account Sales similar to that shown in Figure 10-3 and send it to Simon Sloan with his remittance of \$480.75.

WORKOUT EXERCISE VI

Draw up Account Sales for the consignment problems in Workout Exercise V.

SECTION 7 The Account Purchase

Commission merchants also purchase merchandise for principals who are not able to do so themselves. The price that the commission merchant pays for the goods is the *prime cost*, or first cost. The expenses paid by the commission merchant and his commission, calculated usually on the prime cost, are then added to the prime cost to give the *gross cost*.

$$\text{Gross cost} = \text{Prime cost} + \text{Expenses} + \text{Commission} \quad (4)$$

Example: The Northton Fruit Company, Centreville, ordered Francis Colton of Fruitland to purchase for them 500 baskets of No. 1 peaches at 85¢ a basket, and 500 baskets of No. 2 peaches at 60¢ a basket. Francis Colton made the purchases on August 20, paid \$13.50 cartage charges, \$10 labour charges, and charged 4% commission.

What was the gross cost to the Northton Fruit Company?

Prime cost = 500 baskets at 85¢	= \$425.00
500 baskets at 60¢	= <u>300.00</u>
	<u>\$725.00</u>
Expenses: \$13.50 + \$10 + 4% of	
	\$725 = \$ 52.50
Gross cost = \$725 + \$52.50	= <u>\$777.50</u>

The commission merchant will then submit a report to his principal giving the details of the purchases, the expenses incurred, and his commission for executing the order. This document is called an *Account Purchase*.

An Account Purchase similar to figure 10-4 would be prepared by Francis Colton to submit to the Northton Fruit Company. On receipt of this Account Purchase, the Northton Fruit Company will reimburse Francis Colton for the gross cost of the order.

Figure 10-4 An Account Purchase

FRANCIS COLTON Commission Merchant Fruitland ACCOUNT PURCHASE			
			August 20, 19-1
BOUGHT for the account and risk of: The Northton Fruit Company, Centreville, Ont.			
PURCHASES			
19-1			
August 20	500 baskets at 85¢	\$425.00	
August 20	500 baskets at 60¢	<u>300.00</u>	\$725.00
CHARGES			
August 20	Cartage	\$ 13.50	
	Labour	10.00	
	Commission 4%	<u>29.00</u>	\$ 52.50
	Total charged to your account		<u>\$777.50</u>

WORKOUT EXERCISE VII

Prepare an Account Purchase for each of the following commission purchases:

1. The Murphy Market of Nilsenville requested the Nimeroski Nodwell Agency of Overbaugh to purchase for them 600 baskets of Northern Spy apples at 98¢ a basket, 450 baskets of Mackintosh apples at 95¢ a basket, and 375 baskets of Newton Pippins at 90¢ a basket.

The agency purchased $\frac{3}{4}$ of the Northern Spies, $\frac{2}{3}$ of the Mackintosh, and all the Newton Pippins on September 15; the remainder of the Spies on September 20, and the remainder of the Mackintosh on September 22. The commission charged was 5%, and expenses paid were: cartage, \$21.75; wages, \$14.50; storage, \$17.00.

2. The above agency also purchased for Harry O'Grady: 750 baskets of tomatoes at 27¢ a basket, on August 20; 500 baskets of peaches at 59¢ a basket, on September 5; and 800 baskets of pears at \$1.03 a basket, on September 10. They paid wages of \$21.50, cartage of \$19.87, and incidental expenses of \$15.95. They charged a commission of $5\frac{1}{4}\%$.

3. Stephen Owens' Fruit Store requested the above agency to purchase for them 300 baskets of peaches, 250 baskets of peaches, and 350 baskets of peaches, one week apart, at the best price possible. The agency purchased 300 baskets on September 7; 250 baskets on September 14; and 350 baskets on September 21, at 49¢, 51¢, and 57¢ a basket, respectively. They charged $6\frac{1}{2}\%$ commission and paid \$17.50 for storage, \$31.60 for cartage, and \$7.90 for handling.

SECTION 8 Storage

A service cost that must often be taken into consideration when goods are to be sold is the warehousing or storage of merchandise in a place other than the seller's premises. This occurs with such commodities as wheat; the storage facilities in this case are the grain elevators. Any type of merchandise may be stored in warehouses until needed for use or until shipped to some other location for sale. Perishable goods will be kept in cold storage.

The Rate for Storage

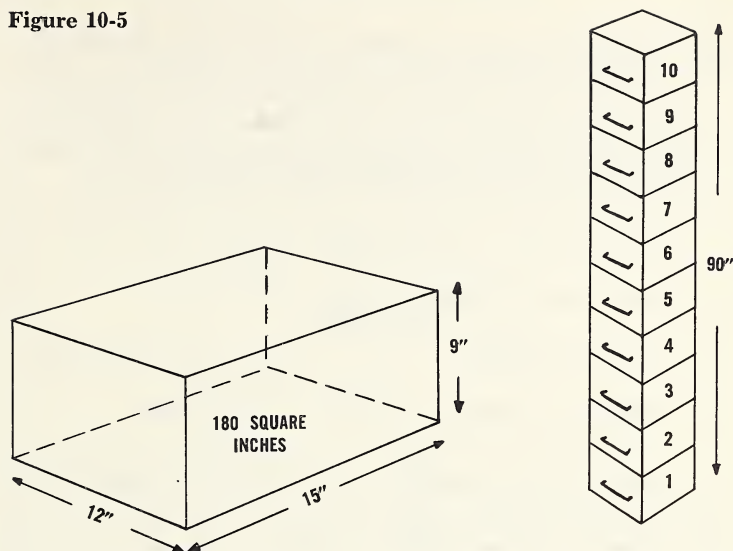
The *rate* that the warehouseman will charge for this service will be made up according to:

- (1) The weight of the goods. The piling of heavy goods will depend on the strength of the floor.
- (2) The cubic contents of the goods. This will determine the amount of space required.
- (3) The nature of the goods. This will determine what type of storage is required (dry, cold, heated, unheated) and what risk is involved in storing the merchandise.
- (4) The handling necessary to load and unload the merchandise.

Calculation of Floor Space Needed

When the warehouse charges are based on the amount of floor space the merchandise will occupy, we must calculate the area of floor space we need.

Figure 10-5



If merchandise is contained in boxes with dimensions of $12'' \times 15''$ and $9''$ tall, and if the boxes cannot be piled higher than $90''$, we will be able to stack ten boxes in each pile. The floor space needed for each pile will be:

$$\begin{aligned} 12'' \times 15'' &= 180 \text{ square inches} \\ &= \frac{180}{12 \times 12} \text{ square feet} = 1\frac{1}{4} \text{ square feet} \end{aligned}$$

There will be a weight restriction per square foot of floor space. If this is fixed, for instance, at 350 pounds per square foot, we must check that the pile of ten boxes will not be too heavy.

1 square foot cannot take more than 350 lb. load.

$\therefore 1\frac{1}{4}$ square feet cannot take more than 437.5 lb. load.

If each box weighs $43\frac{3}{4}$ lb., the pile of ten will be a capacity load. Any weight under $43\frac{3}{4}$ lb. will be within the load limit, but any weight over $43\frac{3}{4}$ lb. per box would be too heavy a load. For instance, if each box weighed 50 lb., the load of ten boxes would be 500 lb. which would not be allowable. To find the number of 50 lb. boxes that may be piled, divide the total allowed load by the box load:

$$\frac{\text{Total load}}{\text{Load per box}} = \frac{437.5}{50} = 8$$

Therefore, only 8 boxes may be piled if the weight of each box is 50 lb.

Example: The Pine Sweets Company use the warehouse facilities of Harbour Storage Limited. They store cartons of a size $12'' \times 18''$ and $10''$ high which weigh 60 lb. each. If the maximum weight allowed is 400 lb. per square foot and the maximum height to which merchandise may be piled in $96''$, how much floor space will be needed to store 250 cartons?

Possible number of cartons in a pile:
by height $= \frac{96''}{10''} = 9$

Possible number of cartons in a pile:
by weight

Area of one carton $= 12 \times 18 = 216$ sq. in.
 $= 1.5$ sq. ft.

Maximum weight per 1.5 sq. ft. $= 1.5 \times 400$ lb.
 $= 600$ lb.

Cartons can be piled $\frac{600}{60} = 10$ carton high

Number of piles $= \frac{250}{9} = 27.78 = 28$ piles

Area required:

Area of 1 carton $12'' \times 18''$

Area of 28 piles $= \frac{12 \times 18 \times 28}{12 \times 12}$ sq. ft.
 $= \underline{\underline{42 \text{ sq. ft.}}}$

WORKOUT EXERCISE VIII

The Coast and Harbour Storage Limited have the following warehousing regulations: maximum weight per square foot, 450 lb.; maximum height, $90''$. What area is required to store the following:

- | | | | | |
|-----|-----|-----------------------------|---------------|---------------|
| 1. | 175 | $12'' \times 15''$ cartons, | weight 48 lb. | height $10''$ |
| 2. | 150 | $12'' \times 12''$ cartons, | weight 80 lb. | height $12''$ |
| 3. | 375 | $12'' \times 18''$ cartons, | weight 52 lb. | height $8''$ |
| 4. | 136 | $10'' \times 12''$ cartons, | weight 35 lb. | height $12''$ |
| 5. | 357 | $10'' \times 10''$ cartons, | weight 54 lb. | height $12''$ |
| 6. | 295 | $11'' \times 11''$ cartons, | weight 71 lb. | height $12''$ |
| 7. | 319 | $11'' \times 12''$ cartons, | weight 43 lb. | height $14''$ |
| 8. | 462 | $14'' \times 12''$ cartons, | weight 39 lb. | height $15''$ |
| 9. | 345 | $16'' \times 14''$ cartons, | weight 40 lb. | height $9''$ |
| 10. | 126 | $13'' \times 12''$ cartons, | weight 90 lb. | height $12''$ |

Calculation of Warehousing Cost

The rate charged is usually based on the square footage needed and the length of time it is used.

Example: The Harbour Storage Limited charge \$4.75 a square foot per year. If the Pine Sweets Company of the previous example left their cartons in storage for 73 days, how much would their storage bill amount to?

$$\text{Area needed} = 42 \text{ sq. ft.}$$

$$\begin{aligned} \text{Cost} &= \frac{42 \times 4.75}{1} \times \frac{73}{365} \\ &= \underline{\underline{\$39.90}} \end{aligned}$$

WORKOUT EXERCISE IX

The Coast and Harbour Storage Limited charge \$3.75 a square foot per year. Calculate the storage charges for Workout Exercise VIII assuming that part of a square foot is charged as a whole square foot. If the cartons were stored for:

- | | | | |
|-------------|-------------|-------------|-------------|
| 1. 75 days | 2. 310 days | 3. 240 days | 4. 79 days |
| 5. 13 days | 6. 98 days | 7. 114 days | 8. 186 days |
| 9. 412 days | 10. 23 days | | |

SECTION 9 The Warehouse Account

Many customers will use the warehouse to store goods temporarily while they are awaiting shipment, and so the amount of the merchandise they are storing will vary from day to day as goods are constantly received and released. An open account will be kept which will show the merchandise received, with date of receipt; the merchandise released, with date of release; and the balance in the warehouse on any particular day.

Example: The Terminal Storage Limited, of Edgewater, have received and released the following bales of merchandise belonging to Whole Fabrics Company, of Textown, during the month of June.

If the charge per bale per day has been established at 2¢ a day, submit the warehouse account showing the storage charges Whole Fabrics Company will have to pay?

Bales Received			Bales Released		
June	2	100	June	3	75
	4	250		6	100
	10	1,000		15	1,100
	21	950		22	750
	28	800		29	1,025

Figure 10-6

The Warehouse Account

WAREHOUSE ACCOUNT The Terminal Storage Limited Edgewater					
In account with: Whole Fabrics Company, Textown.			June 30, 19-1		
Rate per bale: 2¢ per day			Terms n/30		
DATE	Receipts	Deliveries	Balance	No. of Days	Days X Bales
19-1					
June 2	100		100	1	100
3		75	25	1	25
4	250		275	2	550
6		100	175	4	700
10	1,000		1,175	5	5,875
15		1,100	75	6	450
21	950		1,025	1	1,025
22		750	275	6	1,650
28	800		1,075	1	1,075
29		1,025	50	1	50
Total number of day-bales					11,500
<u>Charges</u> = 11,500 x 2¢ = 23,000¢ = <u>\$230.00</u>					

WORKOUT EXERCISE X

The Terminal Storage Limited of Edgewater charge 2¢ per bale per day. Prepare a Warehouse Account for each of the following companies for the month of September:

	Bales Received		Bales Released	
1. Timkins Company,	Sept. 1	100	Sept. 9	700
Timkinville	8	800	17	1,000
	15	900	23	500
	22	600	30	300
	29	500		
2. Godene Corporation,	Sept. 2	400	Sept. 9	800
Springtown	5	700	17	800
	8	800	27	1,200
	12	900		
	24	500		
	28	300		

		<i>Bales Received</i>		<i>Bales Released</i>
3.	Armstrong Assoc. Ltd., Sept. 3	250	Sept. 4	150
	Belverton 11	700	7	50
		17	16	550
		28	26	800
		29		
		30		
4.	Preston Products Ltd., Sept. 5	175	Sept. 6	100
	Armstrongburg 8	325	10	150
		11	18	250
		17	29	550
		21		
		25		
5.	Acme Traders, Ltd., Sept. 3	400	Sept. 5	250
	Alerton 8	350	15	450
		17	26	300
		25	29	400
		30		

Project: The above record 1-5 is the total storage business for the Terminal Storage Limited of Edgewater for the month of September. Prepare a graph similar to the chapter heading showing the number of bales in storage for each day during the month.

SECTION 10 Figuring for Fun

- Can you solve the following problem in addition?

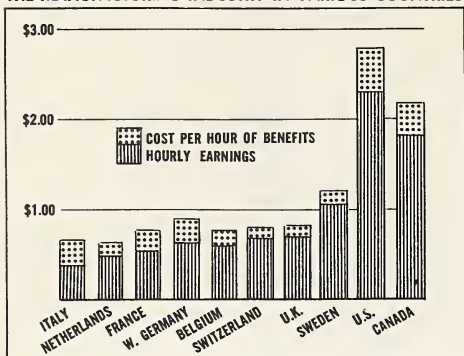
$$\begin{array}{r}
 F \quad I \quad V \quad E \\
 P \quad R \quad I \quad M \\
 \hline
 P \quad R \quad I \quad G \quad S
 \end{array}$$

Hint: $V = 4, E = 5, P = 1, M = 3.$

- Try another magic square. Arrange the first 16 intergers (1-16) using each one once so that every row, every column and every diagonal has the same sum. (The total should be 34!)

A rule for constructing this magic square is given on page 22.

AVERAGE LABOUR EARNINGS AND OTHER BENEFITS IN THE MANUFACTURING INDUSTRY IN VARIOUS COUNTRIES



SOURCE: FINANCIAL POST

Payrolls

11

A Business Problem

Spee-D-X-L Limited are contemplating opening a new sales office in Laertnom. They anticipate their total sales could be increased by 20% from sales in this new territory.

The company have asked for advice on setting up a sales office. They want to know how much they can afford to pay for additional salesmen and how much they can apportion for clerical and administrative salaries for the new branch, assuming these two expenses are to remain in the same ratio to total sales as they are at present.

Upon investigation, you learn that their present net sales total \$2,575,000 and their operating expenses are 30% of sales. Salesmen's salaries are 15% and clerical and administrative salaries are 12% of the present operating expenses.

Submit a report to Spee-D-X-L Limited giving them the information they need.

In any business, one of the large items of expense is the purchase of labour in the form of wages paid to employees. In some industries this item may amount to 75% of the total expenses; it is important, therefore, that great care be taken in computing the payroll. The employees themselves will make sure they do not receive too little recompense, but it is the duty of the employees preparing payroll to make sure, on behalf of the employer, that too much is not paid.

To a great many people the wages they earn represent their whole source of purchasing power; the amount they earn will depend to a large extent on the training and skill required, the prestige of the job, the prevailing wages in the area, the cost of living in the area, and many other things.

Wage Plans

Wages are paid according to some definite wage-payment plan which is usually an adaptation of one of the following:

Straight salary: A definite sum is paid to the employee each day, week, month, or year.

Hourly wage: A stated sum per hour is paid for time worked. Regular time will normally be based on an 8-hour day. If overtime is worked on any day, the employee is usually paid time-and-a-half, or $1\frac{1}{2}$ times the regular hourly rate.

Piece-rate wage: A stated rate is paid for each unit produced or on which a specified amount of work is done by an employee. Many production line and garment making industries operate on this plan.

Differential piece-rate wage: Two rates are established. The lower rate is paid per unit for a stated average output. The higher rate is paid per unit to those employees who produce above average outputs; the faster workers are therefore rewarded.

Commission: Commissions are usually paid to the salesmen, and we learned how to compute commission in the last chapter.

Deductions

If we had worked many years ago, we would have been paid a salary; and when pay-day came, we would have received exactly that amount of money to take home and spend as we wished. This, however, is no longer the case as everyone who is earning money must pay taxes to help run the country, pay insurance against being unemployed or sick, possibly pay union dues, and perhaps save by buying a bond. Instead of giving the employee all his pay and letting him pay these obligations himself, the employer will deduct them before he hands the pay to the employee. What is left is called the "take-home pay" or net pay, and it may differ substantially from the quoted rate of pay.

SECTION 1 Determining Wage Rates

Hourly Wage

Example: Paul Johnstone works in the factory of Holer Corp. and last week worked 45 hours of which 5 hours were overtime. If he is paid \$1.40 an hour and time-and-a-half for overtime, how much did he earn?

Wage earned:

$$\text{Regular time:} = 40 \times \$1.40 = \$56.00$$

$$\text{Overtime} = 5 \times \$2.10 = \underline{10.50}$$

\$66.50

Most companies paying wages on an hourly rate will use time clocks to record the number of hours worked by each employee.

Each employee is furnished with his own individual *time card* which is stored in a rack near the entrance the employee uses. When entering the building in the morning, the employee will insert the time card in the time clock, and the mechanism of the clock will automatically stamp in the appropriate square the hour and the minute he enters. A typical time card is shown in figure 11-1. Again at noon both on leaving the plant and re-entering it the employee will insert the time card and the time of leaving and returning will be stamped upon it. Finally, when leaving the plant at night, he will insert his card to be stamped before departing.

Figure 11-1

TIME CARD									
Week ending:		February 8, 19-1							
Department:		Finishing							
Payroll number:		89							
Name:		Fabian Fitmarsh							
Unemployment insurance number:		98432							
Day	A. M.		P. M.		OVERTIME		TOTAL		O. T.
	In	Out	In	Out	In	Out	Reg.		
Mon	7.59	12.02	1.00	5.00	5.00	6.00	8		1
Tues	7.58	12.01	12.56	5.01			8		
Wed	8.01	12.05	12.59	4.59			7-1/2		
Thurs	7.59	11.59	1.01	5.02			7-1/2		
Fri	8.02	12.00	12.57	4.58			7-1/2		
Sat					8.00	11.30			3-1/2
Sun									
REGULAR TIME					Hours	Rate	Earnings		
					38-1/2	\$1.40	\$53.90		
OVERTIME (1-1/2)					4-1/2	\$2.10	9.45		
GROSS PAY					\$63.35				

As seen above, the time card records the hours worked for one week. Regular hours in the above factory are from 8 A.M. to 12 noon and from 1 P.M. to 5 P.M. Overtime pay is allowed at time-and-a-half for each full quarter hour worked after 5 P.M. from Monday through Friday and all day Saturday. If an employee arrives late or leaves early, these times are frequently printed in heavier type or in red ink. The number of hours worked each day is recorded in hours and quarter hours. Fractional parts of an hour, less than 15 minutes, are not counted when computing either straight time or overtime. If the employee arrives 1 to 15 minutes late, he loses credit for 15 minutes; 16 to 30 minutes late, he loses credit for 30 minutes. The same penalties apply for leaving early, but no offsetting compensation is allowed for early arrival or late leaving. In the above example Fabian Fitmarsh arrives late on Wednesday and leaves early, thereby losing credit for half an hour. On Thursday he leaves early for lunch and returns late, thereby losing another half-hour's credit; and on Friday he is again late arriving in the morning and leaves early at night and again loses

credit for a half-hour. The total time worked each day will be filled in by the time keeper.

At the bottom of the time card the payroll clerk will record the number of hours of straight time and overtime worked, enter the rate of pay, and compute the total wage earned by each employee.

WORKOUT EXERCISE I

Prepare time cards for the following employees of the Soggy Log Lumber Company for the week ending June 20. Regular hours, 8 A.M.-12 noon and 1 P.M. to 5 P.M. daily, 40 hour week. Time-and-a-half paid for overtime for periods of over 15 minutes after 5 P.M. and on Saturdays.

1. Paul Anchor, Planing Department, insurance number 123457, clock number 103, Rate: \$1.50 per hour.

Monday, 7.58 - 12.01; 1.01 - 5.00; 5.00 - 6.15;
Tuesday, 7.59 - 11.55; 1.15 - 5.00;
Wednesday, 8.00 - 12.00; 1.00 - 5.00; 6.00 - 8.00;
Thursday, 8.00 - 12.01; 12.55 - 5.05;
Friday, 8.00 - 12.02; 12.57 - 5.01;

2. Ronald Muir, Planing Department, insurance number 164823, clock number 104, Rate: \$1.90 per hour.

Monday, 8.00 - 12.01; 1.00 - 5.00;
Tuesday, 8.02 - 11.55; 1.01 - 5.01; 5.30 - 8.01;
Wednesday, 8.00 - 11.59; 12.56 - 4.55;
Thursday, 8.05 - 12.00; 12.59 - 5.05;
Friday, 7.53 - 12.01; 12.58 - 4.50;
Saturday, 9.00 - 12.00;

3. Seymour Brown, Sawing Department, insurance number 138720, clock number 105, Rate: \$1.80 per hour.

Monday, 7.50 - 12.00; 1.00 - 7.00;
Tuesday, 7.55 - 12.00; 1.00 - 4.55;
Wednesday, 7.56 - 12.00; 1.00 - 5.05; 5.30 - 7.00;
Thursday, 7.57 - 12.00; 1.00 - 5.01;
Friday, 7.59 - 11.48; 1.00 - 4.38;

4. Terry Jones, Sawing Department, insurance number 158721, clock number 106, Rate: \$1.70 per hour.

Monday, 7.59 - 12.01; 12.55 - 5.00; 6.00 - 8.00;
Tuesday, 8.00 - 12.00; 12.59 - 5.02; 5.30 - 7.00;
Wednesday, 8.03 - 12.00; 1.00 - 4.55;
Thursday, 7.58 - 11.53; 1.02 - 4.59;
Friday, 8.00 - 11.59; 1.01 - 5.01;

5. Gordon Hutton, Sawing Department, insurance number 132148, clock number 107, Rate: \$1.50 per hour.

Monday, 8.01 - 11.55; 1.00 - 5.01;
 Tuesday, 7.59 - 12.00; 1.00 - 4.50;
 Wednesday, 7.50 - 12.01; 1.00 - 6.59;
 Thursday, 7.58 - 12.00; 1.00 - 5.00;
 Friday, 8.00 - 12.01; 1.00 - 5.00;

6. Alfie Proctor, Finishing Department, insurance number 149821, clock number 108, Rate: \$1.60 per hour.

Monday, 8.10 - 12.10; 12.55 - 5.00;
 Tuesday, 7.50 - 12.00; 12.56 - 5.55;
 Wednesday, 7.59 - 12.01; 1.05 - 5.58;
 Thursday, 8.00 - 11.58; 12.59 - 6.50;
 Friday, 8.01 - 11.59; 12.59 - 5.30;
 Saturday, 9.00 - 12.00;

7. Solly Tuck, Finishing Department, insurance number 37840, clock number 109, Rate: \$1.80 per hour.

Monday, 8.00 - 12.00; 1.03 - 3.00;
 Tuesday, 8.01 - 12.01; 1.02 - 5.10;
 Wednesday, 7.58 - 11.55; 1.00 - 4.55;
 Thursday, 7.59 - 11.58; 12.58 - 7.30;
 Friday, 8.01 - 11.59; 12.59 - 5.40;

8. Martin Leacock, Finishing Department, insurance number 142178, clock number 110, Rate: \$1.40 per hour.

Monday, 8.00 - 11.03; 1.00 - 6.00;
 Tuesday, 8.00 - 11.59; 1.00 - 6.00;
 Wednesday, 8.00 - 11.50; 1.00 - 6.00;
 Thursday, 8.00 - 12.10; 1.00 - 6.00;
 Friday, 8.00 - 12.00; 1.00 - 6.00;

9. Benny Price, Finishing Department, insurance number 176839, clock number 111, Rate: \$1.75 per hour.

Monday, 7.55 - 12.00; 1.00 - 6.00;
 Tuesday, 7.56 - 12.00; 1.00 - 6.30;
 Wednesday, 7.59 - 12.00; 12.57 - 6.00;
 Thursday, 7.55 - 12.00; 12.58 - 7.30;
 Friday, 7.58 - 12.00; 12.59 - 5.30;
 Saturday, 7.57 - 12.00;

10. Charles Ardue, Storeroom, insurance number 197827, clock number 112, Rate: \$1.45 per hour.

Monday, 8.00 - 12.00; 1.00 - 6.30;
 Tuesday, 8.10 - 11.55; 12.56 - 5.00;
 Wednesday, 7.55 - 11.57; 12.59 - 5.05;
 Thursday, 7.59 - 12.00; 1.00 - 5.30;
 Friday, 8.00 - 12.00; 12.59 - 4.55;
 Saturday, 9.03 - 12.30;

Keep these time cards for future use.

Piece Rates

Many manufacturing concerns pay their employees according to the number of units or "pieces" they process satisfactorily. A specified rate per piece will be established, and the pieces each employee processes will be counted. Sometimes the count is made by the employee and the pieces and the count examined by an inspector who will pass or reject the pieces counted. A machine operator may have a locked-in device for counting the pieces he processes which will be taken as the basis for his total pieces processed. In this case any processed pieces rejected by the inspector will be deducted from his total count.

Example: The Bonny Belinda Apron Manufacturers operate their power sewing department on a piece-rate basis. The rate for finishing the Workaday model apron is 25¢ a piece. Connie Climps, during the last week of August, finished the following number of pieces:

Monday: 38, Tuesday: 42, Wednesday: 32, Thursday: 35, Friday: 39.

What was the amount of her total earnings?

$$\text{Total pieces finished} = 38 + 42 + 32 + 35 + 39 = 186$$

$$\text{Total earnings} = \$.25 \times 186 = \underline{\$46.50}$$

Differential Piece Rates

As an incentive to produce more units, and so earn a higher wage, some manufacturing firms will establish a minimum number of pieces to be produced in a day called a *quota*. A rate of pay per piece is established. The quota is usually within the reach of the slower workers. For production over the quota the employee will receive a higher rate. This higher rate may be based on the number of pieces produced over the quota or vary according to the total number of pieces processed.

Example: The Dappled Dane Desk Mfg. Company pay differential piece rates in their desk assembly plant. The daily rates are as follows:

Desks Assembled Per Day		Rate Per Desk
Up to 15		50¢
16 to 18	inclusive	55¢
19 to 21	"	60¢
22 to 24	"	65¢
25 to 27	"	70¢
28 and over		75¢

During the week ended January 15, Gordie Graham assembled the following pieces: Monday: 20, Tuesday: 22, Wednesday: 18, Thursday: 25, Friday: 15.

Find the total wages earned by Gordie Graham for the week of January 15.

Monday,	20 pieces	$\times 60¢ =$	\$12.00
Tuesday,	22 pieces	$\times 65¢ =$	14.30
Wednesday,	18 pieces	$\times 55¢ =$	9.90
Thursday,	25 pieces	$\times 70¢ =$	17.50
Friday,	15 pieces	$\times 50¢ =$	<u>7.50</u>
Total wage earned			<u>\$61.20</u>

WORKOUT EXERCISE II

The Bonny Belinda Apron Manufacturers pay 35¢ a piece for finishing the Glamur day model apron. Calculate the earnings of the following employees for the week just ended:

1. Sally Summers: Monday, 31; Tuesday, 35; Wednesday, 34; Thursday, 38; Friday, 37.
2. Evelyn Farmer: Monday, 25; Tuesday, 27; Wednesday, 28; Thursday, 31; Friday, 27.
3. Fanny Fall: Monday, 30; Tuesday, 35; Wednesday, 29; Thursday, 34; Friday, 35.
4. Winnie Winters: Monday, 30; Tuesday, 31; Wednesday, 29; Thursday, 31; Friday, 30.
5. Sarah Spring: Monday, 27; Tuesday, 31; Wednesday, 32; Thursday, 31; Friday, 33.
6. Alice Autumn: Monday, 37; Tuesday, 35; Wednesday, 34; Thursday, 29; Friday, 33.

The Pinto Pant Manufacturers pay differential piece rates for finishing pants as follows:

<i>Pairs Finished Per Day</i>	<i>Rate Per Pair</i>
Up to 20	45¢
21 to 25 inclusive	50¢
26 to 30 "	55¢
31 to 35 "	60¢
36 and over	70¢

Calculate the earnings from the following daily records for the week just ended:

7. Bob Williams: 23, 36, 33, 29, 27. Clock # 76.
8. Tom Jones: 18, 23, 24, 26, 25. " 77.
9. Sam Adams: 23, 32, 37, 28, 35. " 78.

10. Will Burns:	26, 27, 31, 28, 29.	"	79.
11. Vic Carter:	35, 37, 36, 32, 30.	"	80.
12. Carl Yule:	34, 29, 31, 32, 35.	"	81.
13. Dave Welsh:	20, 18, 22, 24, 23.	"	82.
14. Fred Stone:	25, 27, 26, 29, 24.	"	83.
15. George Tallow:	31, 32, 29, 26, 35.	"	84.
16. Ted Edwards:	25, 27, 26, 24, 28.	"	85.
17. Paul Frame:	34, 36, 35, 32, 38.	"	86.
18. Elwood Woods:	25, 24, 29, 31, 35.	"	87.
19. Harry Vale:	34, 35, 31, 39, 41.	"	88.
20. Jack Poole:	25, 27, 24, 29, 30.	"	89.

Fringe Benefits

In addition to the quoted rate of pay, many firms offer further benefits or remuneration to their employees in such forms as: paying all, or part of, their hospitalization insurance; free health service; free, that is, non-contributory, or partially contributory, pension plans; recreation facilities; and many others. For the company, these are an additional expense of employing labour. The graph heading this chapter shows the total cost of labour earnings plus other benefits received in the manufacturing industries in Canada. Whilst Canada provides a little less in the way of fringe benefits than does the United States, Canada is well ahead of most other countries.

WORKOUT EXERCISE III

Draw a single line or bar graph to illustrate the following information:

A large group of companies was surveyed to see how many offered free, or partially free, hospital insurance to their employees. The following results were received:

100% insurance provided by	87% of the companies surveyed
85% insurance provided by	88% of the companies surveyed
75% insurance provided by	92% of the companies surveyed
70% insurance provided by	94% of the companies surveyed
67% insurance provided by	97% of the companies surveyed
65% insurance provided by	99% of the companies surveyed
60% insurance provided by	100% of the companies surveyed

SECTION 2 Deductions From Payroll

As mentioned in the introduction to this chapter, the total wage earned by an employee is not handed to him intact on 'pay day'. In Canada, there are normally a number of deductions from the gross or total pay of the employee which the employer makes on behalf of the Government, or other agencies. After the deductions have been made, the employee receives his net or take-home pay. The employer, then, is responsible for paying the Government or other agencies on behalf of the employee.

Employee's Income Tax

Canadian Income Tax law requires that employers deduct from their employees' pay the amount of the personal income tax the employee must pay on his earnings. At the commencement of his employment, and at the beginning of each year of his employment thereafter, each employee is required to fill out and file with his employer a Tax Deduction Declaration, known as a TD1 form.

Figure 11-2 Tax Deduction Declaration Form TD1

Employee's Tax Deduction Declaration		TD1 Rev. 5-59																									
● COMPLETE AND FILE ONE COPY OF THIS DECLARATION WITH YOUR EMPLOYER: (a) when you commence employment, and (b) within SEVEN DAYS of any change affecting your personal exemptions. If this declaration is not filed with your employer, tax will be deducted as though you were a single person without dependants. ● Income of your wife or dependants includes Old Age Security Pension or Assistance. ● You may not claim a dependant whose income for the year will exceed \$950. ● If this is wife's declaration, read "Husband" for "Wife".																											
<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Family or last name (Print)</td> <td style="width: 30%;">Unemployment Insurance No.</td> </tr> <tr> <td>Christian or first names</td> <td></td> </tr> <tr> <td>Present address (If present address is temporary, give permanent address only)</td> <td>Year of birth if 65 years of age or over.</td> </tr> <tr> <td>Name and address of wife</td> <td></td> </tr> </table>		Family or last name (Print)	Unemployment Insurance No.	Christian or first names		Present address (If present address is temporary, give permanent address only)	Year of birth if 65 years of age or over.	Name and address of wife																			
Family or last name (Print)	Unemployment Insurance No.																										
Christian or first names																											
Present address (If present address is temporary, give permanent address only)	Year of birth if 65 years of age or over.																										
Name and address of wife																											
CLAIM FOR PERSONAL EXEMPTIONS																											
PARTICULARS OF DEPENDANTS EXEMPTIONS WHOLLY DEPENDENT CHILDREN (including grandchildren) Under age 21 or any age if in full-time attendance at a school or university or if infirm. If you claim an exemption of \$1,000 on account of a wholly dependent child (Item 2B) you MAY ALSO claim here if you support the child in a dwelling wherein you employ a full-time servant. If you do, state servant's name here. Servant's name _____ <i>Provide details below and claim \$250 for each child qualified for Family Allowance or \$500 if not qualified.</i> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Name of child (Attach list if space is insufficient)</th> <th>Income in Year</th> <th>Enter age and if over 21 state school attended or whether infirm.</th> <th>Amount of claim for each child</th> </tr> </thead> <tbody> <tr><td> </td><td>\$</td><td></td><td>\$</td></tr> <tr><td> </td><td></td><td></td><td></td></tr> <tr><td> </td><td></td><td></td><td></td></tr> <tr><td> </td><td></td><td></td><td></td></tr> <tr><td> </td><td></td><td></td><td></td></tr> </tbody> </table> ENTER TOTAL IN ITEM 3A AT RIGHT \$ _____		Name of child (Attach list if space is insufficient)	Income in Year	Enter age and if over 21 state school attended or whether infirm.	Amount of claim for each child		\$		\$																	BASIC EXEMPTION FOR EVERYONE—Claim \$1,000 ▶ \$ _____ ADDITIONAL EXEMPTIONS IF APPLICABLE - 1. AGE EXEMPTION—(If age 65 or over, claim \$500) ▶ \$ _____ 2. MARRIED OR EQUIVALENT EXEMPTION - <i>If you are entitled to this exemption, check ☒ the one item on which your claim is based. If wife's income, while married, will exceed \$1,250 this year, you may not claim married or equivalent exemption.</i> A. MARRIED AND ☐ Supporting wife whose income for the year, while married, WILL NOT BE OVER \$250—Claim \$1,000. ☐ Supporting wife whose income for the year, while married, WILL BE OVER \$250 but NOT OVER \$1,250. \$1,250 LESS: wife's income \$ _____ CLAIM \$ _____ B. SINGLE, DIVORCED, SEPARATED OR WIDOW (E.T.). <i>If you claim in this group, provide details of dependant below and claim \$1,000.</i> ☐ Supporting a wholly dependent child under age 21 OR any age if infirm or in full-time attendance at school or university. ▶ \$ _____ ☐ Supporting a wholly dependent person related by blood, marriage or adoption and living in a dwelling maintained by you. ☐ Supporting with one or more other persons, in a dwelling maintained by you and such persons, a wholly dependent relative. (You may claim here ONLY if it has been agreed that no other person will claim for the same dependant or in respect of the same dwelling.) Name of Depend. _____ Address _____ Annual income \$ _____ Relationship to you _____ If a child, state age, and, if over 21, state school attended or whether infirm. _____	
Name of child (Attach list if space is insufficient)	Income in Year	Enter age and if over 21 state school attended or whether infirm.	Amount of claim for each child																								
	\$		\$																								
OTHER DEPENDANTS: Parents, Grandparents, Brothers or Sisters (including in-laws) Claim amount to be spent on each dependant up to \$250 for each dependant qualified for Family Allowance or up to \$500 if not qualified. These amounts must be apportioned if others claim for the same dependant. No claim may be made here for a dependant over age 21 who is not infirm or for one claimed in Item 2B. <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Dependant's Name and Address (Attach list if space is insufficient)</th> <th>Amount of Your Expenses in Support of Depend. _____</th> <th>Dependant's Income _____</th> <th>Age _____</th> <th>Amount of Claim _____</th> </tr> </thead> <tbody> <tr><td> </td><td>\$</td><td>\$</td><td></td><td>\$</td></tr> <tr><td> </td><td></td><td></td><td></td><td></td></tr> <tr><td> </td><td></td><td></td><td></td><td></td></tr> <tr><td> </td><td></td><td></td><td></td><td></td></tr> </tbody> </table> (If over 21, state whether infirm) ENTER TOTAL IN ITEM 3B AT RIGHT \$ _____		Dependant's Name and Address (Attach list if space is insufficient)	Amount of Your Expenses in Support of Depend. _____	Dependant's Income _____	Age _____	Amount of Claim _____		\$	\$		\$																3. DEPENDANTS EXEMPTIONS <i>Provide details at left and enter total claims below</i> ▶ A. Wholly Dependent Children - - - - - ▶ \$ _____ ▶ B. Other Dependents - - - - - ▶ \$ _____ TOTAL PERSONAL EXEMPTIONS - - - - - \$ _____
Dependant's Name and Address (Attach list if space is insufficient)	Amount of Your Expenses in Support of Depend. _____	Dependant's Income _____	Age _____	Amount of Claim _____																							
	\$	\$		\$																							
EXEMPTION CLAIM FOR CASUAL, SEASONAL OR PART-YEAR EMPLOYMENT This portion must be completed by an employee who is receiving pay at a rate that is subject to tax deduction but who claims exemption because total earnings (including free board and lodging) from all sources for the full calendar year will be less than the total personal exemptions claimed. No claim may be made here by a person not resident in Canada for the whole of the year unless his earnings for the year in Canada will be less than his exemptions apportioned to the period of residence in Canada. Earnings to date this year from all sources _____ Estimated earnings for balance of year from all sources _____ Total estimated earnings from all sources _____ N.B. This exemption is valid only until the end of the current calendar year.		CERTIFICATION I HEREBY CERTIFY that the information given in this declaration is true, correct and complete in every respect. Signature _____ Date _____ 19____ IT IS A CRIMINAL OFFENCE TO MAKE A FALSE DECLARATION																									
WARNING: An employer should refer a form TD1 containing doubtful statements to the District Taxation Office. Any person who knowingly accepts a form TD1 containing false or deceptive statements commits a criminal offence. Employers must retain completed forms TD1 for inspection by officers of the Taxation Division. Form authorized and prescribed by the Minister of National Revenue																											

It is necessary that this form be completed because the amount of tax the employer must deduct will vary according to whether the employee is single or married and according to the number of children or others he has to support.

Figure 11-3 Weekly Income Tax Deduction Table

12 WEEKLY TAX DEDUCTIONS — TABLE 12 — DÉDUCTIONS D'IMPÔT PAR SEMAINE
Basis—52 Pay Periods per Year Base—52 périodes de paie par année

WEEKLY PAY Use appropriate bracket PAIE PAR SEMAINE Utilisez le guillemet approprié	IF THE TOTAL OF EXEMPTIONS IS — SI LE TOTAL DES EXEMPTIONS EST DE														
	\$1,000-	\$1,250-	\$1,500-	\$1,750-	\$2,000-	\$2,250-	\$2,500-	\$2,750-	\$3,000-	\$3,250-	\$3,500-	\$3,750-	\$4,000-	\$4,250-	\$4,500-
	\$1,249	\$1,499	\$1,749	\$1,999	\$2,249	\$2,499	\$2,749	\$2,999	\$3,249	\$3,499	\$3,749	\$3,999	\$4,249	\$4,499	\$4,750-
DEDUCT FROM EACH PAY — DÉDUISEZ SUR CHAQUE PAIE															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Under-Moins \$23.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00	\$.00
\$23.00 — 23.99	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25
24.00 — 24.99	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40
25.00 — 25.99	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55
26.00 — 26.99	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70	.70
27.00 — 27.99	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85	.85
28.00 — 28.99	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95	.95
29.00 — 29.99	1.10	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
30.00 — 30.99	1.25	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55
31.00 — 31.99	1.40	.70	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
32.00 — 32.99	1.50	.85	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15
33.00 — 33.99	1.65	1.00	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30	.30
34.00 — 34.99	1.80	1.15	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
35.00 — 35.99	1.95	1.25	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60
36.00 — 36.99	2.10	1.40	.75	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
37.00 — 37.99	2.25	1.55	.85	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20	.20
38.00 — 38.99	2.35	1.70	1.00	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35
39.00 — 39.99	2.50	1.85	1.15	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
40.00 — 40.99	2.65	1.95	1.30	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60	.60
41.00 — 41.99	2.80	2.10	1.45	.75	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10
42.00 — 42.99	2.95	2.25	1.55	.90	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25
43.00 — 43.99	3.15	2.40	1.70	1.05	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35	.35
44.00 — 44.99	3.30	2.55	1.85	1.20	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
45.00 — 45.99	3.50	2.65	2.00	1.30	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65
46.00 — 46.99	3.65	2.85	2.15	1.45	.80	.10	.10	.10	.10	.10	.10	.10	.10	.10	.10
47.00 — 47.99	3.80	3.00	2.35	1.60	.95	.25	.25	.25	.25	.25	.25	.25	.25	.25	.25
48.00 — 48.99	4.00	3.15	2.40	1.75	1.05	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40
49.00 — 49.99	4.15	3.35	2.55	1.90	1.20	.55	.55	.55	.55	.55	.55	.55	.55	.55	.55
50.00 — 50.99	4.35	3.50	2.70	2.00	1.35	.65	.65	.65	.65	.65	.65	.65	.65	.65	.65
51.00 — 51.99	4.50	3.70	2.85	2.15	1.50	.80	.15	.15	.15	.15	.15	.15	.15	.15	.15
52.00 — 52.99	4.65	3.85	3.05	2.30	1.65	.95	.30	.30	.30	.30	.30	.30	.30	.30	.30
53.00 — 53.99	4.85	4.00	3.20	2.45	1.75	1.10	.40	.40	.40	.40	.40	.40	.40	.40	.40
54.00 — 54.99	5.00	4.20	3.35	2.60	1.90	1.25	.55	.55	.55	.55	.55	.55	.55	.55	.55
55.00 — 55.99	5.20	4.35	3.55	2.75	2.05	1.35	.70	.05	.05	.05	.05	.05	.05	.05	.05
56.00 — 56.99	5.35	4.55	3.70	2.90	2.20	1.50	.85	.15	.15	.15	.15	.15	.15	.15	.15
57.00 — 57.99	5.50	4.70	3.90	3.05	2.35	1.65	1.00	.30	.30	.30	.30	.30	.30	.30	.30
58.00 — 58.99	5.70	4.85	4.05	3.25	2.45	1.80	1.10	.45	.45	.45	.45	.45	.45	.45	.45
59.00 — 59.99	5.85	5.05	4.20	3.40	2.60	1.95	1.25	.60	.60	.60	.60	.60	.60	.60	.60
60.00 — 60.99	6.05	5.20	4.40	3.60	2.75	2.05	1.40	.75	.05	.05	.05	.05	.05	.05	.05
61.00 — 61.99	6.25	5.40	4.55	3.75	2.95	2.20	1.55	.85	.20	.20	.20	.20	.20	.20	.20
62.00 — 62.99	6.45	5.55	4.75	3.90	3.10	2.35	1.70	1.00	.45	.45	.45	.45	.45	.45	.45
63.00 — 63.99	6.65	5.70	4.90	4.10	3.25	2.50	1.80	1.15	.45	.45	.45	.45	.45	.45	.45
64.00 — 64.99	6.85	5.90	5.05	4.25	3.45	2.65	1.95	1.30	.60	.60	.60	.60	.60	.60	.60
65.00 — 65.99	7.05	6.10	5.25	4.45	3.60	2.80	2.10	1.45	.75	.10	.10	.10	.10	.10	.10
66.00 — 66.99	7.25	6.30	5.40	4.60	3.80	2.95	2.25	1.55	.90	.20	.20	.20	.20	.20	.20
67.00 — 67.99	7.45	6.50	5.60	4.75	3.95	3.15	2.40	1.70	1.05	.45	.45	.45	.45	.45	.45
68.00 — 68.99	7.65	6.70	5.75	4.95	4.10	3.30	2.50	1.85	1.15	.50	.50	.50	.50	.50	.50
69.00 — 69.99	7.85	6.90	5.90	5.10	4.30	3.45	2.65	2.00	1.30	.65	.65	.65	.65	.65	.65
70.00 — 70.99	8.05	7.10	6.10	5.30	4.45	3.65	2.80	2.15	1.45	.80	.10	.10	.10	.10	.10
71.00 — 71.99	8.25	7.30	6.30	5.45	4.65	3.80	3.00	2.25	1.60	.90	.25	.25	.25	.25	.25
72.00 — 72.99	8.45	7.50	6.50	5.60	4.80	4.00	3.15	2.40	1.75	1.05	.40	.40	.40	.40	.40
73.00 — 73.99	8.65	7.70	6.70	5.80	4.95	4.15	3.35	2.55	1.85	1.20	.55	.55	.55	.55	.55
74.00 — 74.99	8.85	7.90	6.90	6.00	5.15	4.30	3.50	2.70	2.00	1.35	.70	.70	.70	.70	.70
75.00 — 75.99	9.05	8.10	7.10	6.15	5.30	4.50	3.65	2.85	2.15	1.50	.80	.15	.15	.15	.15
76.00 — 76.99	9.25	8.30	7.30	6.35	5.50	4.65	3.85	3.05	2.30	1.60	.95	.25	.25	.25	.25
77.00 — 77.99	9.45	8.50	7.50	6.55	5.65	4.85	4.00	3.20	2.45	1.75	1.10	.40	.40	.40	.40
78.00 — 78.99	9.65	8.70	7.70	6.75	5.80	5.00	4.20	3.35	2.55	1.90	1.25	.55	.55	.55	.55
79.00 — 79.99	9.85	8.90	7.90	6.95	6.00	5.15	4.35	3.55	2.70	2.05	1.35	.70	.70	.70	.70
80.00 — 80.99	10.05	9.10	8.10	7.15	6.20	5.35	4.50	3.70	2.90	2.20	1.50	.85	.15	.15	.15
81.00 — 81.99	10.20	9.30	8.30	7.35	6.40	5.50	4.70	3.90	3.05	2.30	1.65	.95	.30	.30	.30
82.00 — 82.99	10.40	9.50	8.50	7.55	6.60	5.70	4.85	4.05	3.25	2.45	1.80	1.10	.45	.45	.45
83.00 — 83.99	10.60	9.70	8.70	7.75	6.80	5.85	5.05	4.20	3.40	2.60	1.95	1.25	.60	.60	.60
84.00 — 84.99	10.80	9.85	8.90	7.95	7.00	6.05	5.20	4.40	3.55	2.75	2.05	1.40	.70	.70	.70
85.00 — 85.99	11.00	10.05	9.10	8.15	7.20	6.25	5.35	4.55	3.75	2.90	2.20	1.55	.85	.20	.20
86.00 — 86.99	11.15	10.25	9.30	8.35	7.40	6.45	5.55	4.75	3.90	3.10	2.35	1.65	1.00	.35	.35
87.00 — 87.99	11.35	10.45	9.50	8.55	7.60	6.65	5.70	4.90	4.10	3.25	2.50	1.80	1.15	.45	.45
88.00 — 88.99	11.55	10.65	9.70	8.75	7.80	6.85	5.90	5.05	4.25	3.45	2.65	1.95	1.30	.60	.60
89.00 — 89.99	11.75	10.80	9.90	8.95	8.00	7.05	6.05	5.25	4.40	3.60	2.80	2.10	1.40	.75	.10
90.00 — 91.99	11.95	11.00	10.10	9.15	8.20	7.25	6.25	5.40	4.60	3.75	2.95	2.25	1.55	.90	.20
92.00 — 93.99	12.30	11.40	10.50	9.55	8.60	7.65	6.65	5.75	4.95	4.10	3.30	2.50	1.85	1.15	.50
94.00 — 95.99	12.70	11.75	10.85	9.95	9.00	8.05	7.05	6.10	5.25	4.45	3.65	2.80	2.10	1.45	.80
96.00 — 97.99	13.05	12.15	11.25	10.35	9.40	8.45	7.45	6.50	5.60	4.80	3.95	3.15	2.40	1.75	1.05
98.00 — 99.99	13.45	12.55	11.60	10.70	9.80	8.85	7.85	6.90	5.95	5.15	4.30	3.50	2.70	2.00	1.35
100.00 — 101.99	13.90	12.90	12.00	11.10	10.15	9.25	8.25	7.30	6.35	5.45	4.65	3.85	3.00	2.30	1.60
102.00 — 103.99	14.30	13.30	12.40	11.45	10.55	9.65	8.65	7.70	6.75	5.80	5.00	4.20	3.35	2.55	1.90
104.00 — 105.99	14.75	13.70	12.75	11.85	10.95	10.00	9.05	8.10	7.15	6.20	5.35	4.50	3.70	2.90	2.20
106.00 — 107.99	15.20	14.15	13.25	12.35	11.30	10.40	9.45	8.50	7.55	6.60	5.65	4.85	4.00	3.20	2.45
108.00 — 109.99	15.65	14.60	13.65	12.60	11.70	10.80	9.85	8.90	7.95	7.00	6.00	5.20	4.40	3.55	2.75
110.00 — 111.99	16.10	15.05	14.05	13.05	12.05	11.15	10.25	9.30	8.35	7.40	6.40	5.55	4.70	3.90	3.10

and a yearly basis. A portion of the table for weekly income tax deductions is reproduced in figure 11-3.

To read this table, you look in the left-hand column for the total amount of pay you earn; say this is \$52 a week. If you are a single person with no dependants, your total exemption would be \$1,000. This exemption figure is read horizontally along the top of the table. The amount of the income tax you must pay and which is deducted from your total pay is found where the horizontal gross wage line and the vertical exemption column meet, as illustrated by the square in figure 11-4.

Figure 11-4 Extract of Table for Weekly Income Tax Deductions

Weekly Pay	If the total exemptions is			
	\$1,000 -\$1,249	=	—	2,500 2,749
—	—			—
—	—		—	
—	—		—	
—	—		—	
—	—		—	
—	—		—	
—	—		—	
—	—		—	
—	—		—	
—	—		—	
52 -52.99	→ \$4.65 ←	- Read tax payable here		—
—				—
—				—
—				—
60 -60.99	----- →			1.40
		Read tax payable here ↗		

If your gross pay amounts to \$60 per week and you are entitled to an exemption of \$2,500, find your gross pay of \$60 in the left-hand column and follow the horizontal line until it meets the vertical column headed with the allowed deduction of \$2,500 a year. The figure contained in this square is the income tax your employer will deduct from your pay each week.

Other forms are filled in by the employer when he remits the total employee income tax deducted to the Government.

WORKOUT EXERCISE IV

Compute the income tax payable by the employees of the Soggy Log Lumber Company, Workout Exercise I, assuming the following exemptions:

- 1. Anchor \$1,000;
- 2. Muir \$2,400;
- 3. Brown \$2,670;

- | | | |
|--------------------|---------------------|---------------------|
| 4. Jones \$1,750; | 5. Hutton \$1,000; | 6. Proctor \$3,150; |
| 7. Tuck \$2,100; | 8. Leacock \$1,400; | 9. Price \$1,000; |
| 10. Ardue \$2,300. | | |

11-16 Compute the income tax payable by the employees of the Bonny Belinda Apron Manufacturers, Workout Exercise II, problems 1-6, assuming all the girls have \$1,000 exemptions.

Compute the income tax payable by the employees of the Pinto Pant Manufacturers assuming the following exemptions apply for the week ended April 18:

- | | | |
|----------------------|---------------------|---------------------|
| 17. William \$2,600; | 18. Jones \$1,800; | 19. Adams \$2,900; |
| 20. Burns \$1,000; | 21. Carter \$2,200; | 22. Yule \$1,350; |
| 23. Welsh \$2,275; | 24. Stone \$1,000; | 25. Tallow \$1,960; |
| 26. Edwards \$1,420; | 27. Frame \$2,200; | 28. Woods \$1,000; |
| 29. Vale \$1,880; | 30. Poole \$2,400. | |

Unemployment Insurance

The Canadian Unemployment Insurance Act requires that persons while employed pay contributions into a fund to protect themselves in case they should become unemployed. The amount they pay depends on the salary they earn. The Act further requires that the employers contribute an equal amount toward each of their employee's insurance protection. When you become employed for the first time, you will be required to register with the Unemployment Insurance Commission and will obtain a book and an unemployment insurance number. This book, see figure 11-5, you will turn over to your employer.

Each pay day your employer will deduct from your pay, say 75¢, for unemployment insurance; he will add 75¢ himself and then affix a stamp worth \$1.50 in your book. If you change your employment, you must take your insurance book with you to your new job. If you are willing to work and unable to find suitable employment, then you take your book to the Unemployment Insurance Commission to collect the unemployment insurance allowance. There are, of course, minimum requirements. You could not work for two or three weeks and then collect unemployment insurance. Persons engaged in certain types of work and paid on a salary basis do not pay unemployment insurance. For instance, persons working in the Civil Service and in non-profit organizations or persons earning a salary of over \$5,400 a year are exempt. Persons employed by the hour, by the day, or on piece-work always pay unemployment insurance wherever they work.

When you turn your book over to your employer, he is responsible for purchasing the unemployment insurance stamps and affixing them each pay day in your book. The employer must obtain a license to purchase these stamps which are obtainable at the post office. To help him calculate the denomination of the stamps required, the commission

issue tables for various types and rates of pay. A table of weekly deductions is shown in figure 11-6.

For example, if your earnings for the week are \$47.50, you will have \$0.66 deducted from your pay, and your employer will affix a stamp valued at \$1.32 in the designated section of your unemployment insurance book.

Figure 11-5 Unemployment Insurance Book

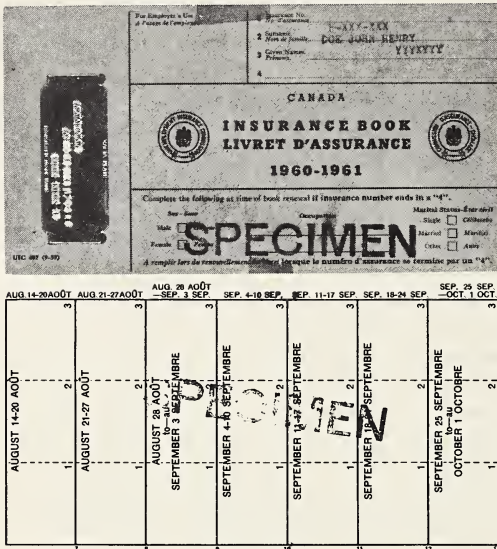


Figure 11-6 Calculation of Contributions Payable

Range of Earnings	Employee's	Employer's	Denomination
	Share	Share	of Stamps
	\$	\$	\$
Up to \$8.99	.10	.10	.20(1)
\$ 9.00 to \$14.99	.20	.20	.40
15.00 to 20.99	.30	.30	.60
21.00 to 26.99	.38	.38	.76
27.00 to 32.99	.46	.46	.92
33.00 to 38.99	.54	.54	1.08
39.00 to 44.99	.60	.60	1.20
45.00 to 50.99	.66	.66	1.32
51.00 to 56.99	.72	.72	1.44
57.00 to 62.99	.78	.78	1.56
63.00 to 68.99	.86	.86	1.72
69.00 and over	.94	.94	1.88

(1) Use one-half of a 40 cent stamp.

WORKOUT EXERCISE V

1-30. Compute the unemployment insurance contributions for the employees of the Soggy Log Lumber Company, the Bonnie Belinda Apron Manufacturers, and the Pinto Pant Manufacturers as outlined in Workout Exercises I, II, and IV.

31. The National Employment Services report the following educational levels of unemployed persons:

	Men	Women	Total
Some primary school or less	63,900	17,600	81,500
Finished primary school	59,500	21,400	80,900
Some high school	66,500	48,700	115,200
Finished high school	35,600	26,100	61,700
Total	<u>225,500</u>	<u>113,800</u>	<u>339,300</u>

Draw a component vertical bar chart to illustrate these figures.

Other Deductions

Some firms offer group insurance plans and group pension plans to their employees. These group plans usually cost the employee less than any plan he could obtain as an individual.

When employees are members of a union, the union stewards usually come to an agreement with the company for the employer to deduct the union dues at source and hand them over to the union.

Some Provinces have a compulsory hospitalization plan, and the employer must deduct a specified amount from each employee's pay and forward it to the Provincial Government.

Many firms offer their employees a pay-as-you-earn method of purchasing Canada Savings Bonds. The bonds are usually offered for sale in this way in November of each year, and the denominations most frequently bought by employees are \$50 and \$100. Once a month the employer deducts the instalment to be paid from the employee's pay at the following rates, regardless of the total value of bonds purchased:

Down payment	5%
10 monthly payments of	9%
Final payment of	7.60%

The amount deducted is sent to the bank which financed the purchase of the bonds. When full payment has been made for the bonds, they are released by the bank and handed over to the purchaser.

Companies also co-operate with United Appeal drives, and if employees pledge to donate a portion of their pay to such a charity fund, it will be deducted by the employer and turned over to the charity at regular intervals.

In all cases the employee must sign a written authorization directing the employer to deduct the required amount from his pay.

WORKOUT EXERCISE VI

Calculate the weekly take-home pay for each of the following employees of the XYZ Company; in addition to deductions for income tax and unemployment insurance, 5% of the gross pay is deducted from all employees for the pension fund.

	Name	Gross Pay	Exemptions	Other Deductions		
				Union Dues	Bond Purchase	United Appeal
1.	Greig, Lawrence	\$60.00	\$1,000	\$0.50	\$ 9.00	\$1.00
2.	Hamilton, Ian	70.00	2,400			
3.	Harker, Paul	65.00	2,840		18.00	
4.	Harrington, Paul	42.50	2,170	.50		.35
5.	Harris, Terry	49.00	1,000		9.00	
6.	Hay, Richard	53.25	2,000			
7.	Hayes, Walter	51.75	1,876			.60
8.	Headley, Douglas	66.80	2,170	.50	4.50	
9.	Hearn, Keith	49.50	1,790			
10.	Hedley, Michael	75.20	1,000			.85
11.	Henderson, Larry	69.80	1,000		4.50	
12.	Hewett, Grant	61.35	1,350	.50		
13.	Hiscock, Joel	56.40	2,000	.50		.25
14.	Hodder, John	65.80	2,540	.50		
15.	Hood, Andrew	72.20	1,000			1.00
16.	Husband, Ken	63.49	2,860			
17.	Jeffers, Allan	43.17	2,740	.50		
18.	Jenkin, Keith	72.40	1,970		4.50	
19.	Kakino, Ed	48.70	1,000	.50		
20.	Kendall, Ron	75.30	2,220	.50		
21.	Killip, John	63.70	1,400			
22.	Kitchin, Jack	76.40	1,000		18.00	.75
23.	Kogut, Ed	53.48	2,830	.50		
24.	Kong, Terry	59.95	1,000	.50		
25.	Kowaluk, Robert	48.36	1,850	.50		.30

26. The following is a table of women employed for the last six years as a percentage of the total labour force:

	% of Total Labour Force	% Single etc.	% Married
5 years ago	21.50	13.50	8.00
4 years ago	22.50	13.50	9.00
3 years ago	22.50	13.25	9.25
2 years ago	23.00	13.50	9.50
Last year	23.50	13.00	10.50
This year	25.00	13.00	12.00

Draw a component line chart similar to Graph 5, Chapter 4, to illustrate this information. Can you draw any conclusion from it?

Figure 11-7 Payroll Register

The Provincial Manufacturing Company																	
Payroll Register					Week Ended					June 15, 19-1							
Deductions																	
Time Clock No.	Name	Hours Worked					Regular Hourly Rate	Total Hours		Total Wages	Income Tax	Unem- ploy- ment Ins.	Hosp. Other	Total	Net Wage		
		M	T	W	T	F		S	Reg.							Over- time	
50	Abe Brown	8	8	8	8	4	2.00	40	4	\$ 92.00	\$ 6.65	\$.94	\$.49	—	\$ 8.08	\$83.92	
51	Chris Dodds	9	8	8	9	8	0	2.40	40	2	103.20	10.55	.94	.98	—	12.47	90.73
52	Ernie Forbes	7	8	8	8	9	0	1.60	39	1	64.80	1.30	.86	.49	—	2.65	62.15
53	George Hall	8	7	8	8	9	2	1.80	39	3	78.30	6.75	.94	.98	—	8.67	69.63
											\$338.30	\$25.25	\$3.68	\$2.94	—	\$31.87	\$306.43

(1) (2) (3) (4) (5) (6) (7)

Columns 2 + 3 + 4 + 5 = Column 6

Columns 6 + 7 = Column 1

SECTION 3 Recording the Payroll

In order to compute the total payroll each pay day, the individual record for each employee is entered from the time cards in a Payroll Register similar to figure 11-7.

When the names of all employees, the calculation of their gross wage, their deductions, and their net wage or take-home pay have been entered, each column is added vertically. Column number 1 shows the total pay earned by all the workers which the company must pay out. The individual deduction columns, after being added vertically, must be summed together to prove the accuracy of the total deduction column. In figure 11-7 the totals of columns 2, 3, 4 and 5 are added and should equal the total of column 6. When this total deduction is subtracted from the gross pay, the amount remaining will be the amount of pay to be paid out to the employees; or, as shown in figure 11-7, the total of column 1 = the total of column 6 + the total of column 7.

The arithmetic accuracy of the payroll register should always be proved in this way.

WORKOUT EXERCISE VII

1. Design a payroll register and record the pay for the employees of the Soggy Log Lumber Company. 5% of gross pay is deducted for a pension plan.
 2. Design a payroll register and record the pay for the employees of the Bonnie Belinda Apron Manufacturers. Assume all employees have a deduction of \$4.50 to buy a Canada Savings Bond.
 3. Design a payroll register and record the pay for the employees of the Pinto Pant Manufacturers. 4% is deducted from all employees for pension, 50¢ a week for union dues.
 4. Design a payroll register and record the pay for the employees of the XYZ Company listed in Workout Exercise VI.
- (Keep these payroll registers for Workout Exercise VIII.)

SECTION 4 Currency Breakdown and Memorandum

Currency Breakdown

Employees receive their pay either in the form of a cheque or in cash. It is still the practice of many firms to pay their employees in cash because many people prefer to receive cash rather than visit a bank to cash a cheque. This means that the paymaster must visit the bank himself, withdraw sufficient cash to cover the net payroll, and then distribute this cash to the employees, usually by "stuffing" a pay envelope for each employee with the amount of net pay due. When the paymaster withdraws this money from the bank, it must be broken down into the proper denominations that will just exactly pay each

person. If he has to pay six people varying sums which total \$500, it will be useless for him to withdraw five one hundred dollar bills!

In order to arrive at the correct breakdown of the currency required to meet a payroll, a *currency breakdown* or *change sheet* is prepared similar to figure 11-8.

Figure 11-8 A Currency Breakdown Sheet

CURRENCY BREAKDOWN											
July 20, 19—											
Name	Net Salary	Bills					Coins				
		\$20	\$10	\$5	\$2	\$1	50¢	25¢	10¢	5¢	1¢
Isaac James	\$75.89	3	1	1			1	1	1		4
Kelly Lowe	99.54	4	1	1	2		1				4
Max Night	61.14	3				1			1		4
Ollie Parks	72.21	3	1		1				2		1
Total	\$308.78	13	3	2	3	1	2	1	4	—	13

When breaking down each person's net pay, bills and coins of the largest denomination are used, so that the pay envelope will not be too bulky and so that the minimum number of bills and coins will be needed from the bank. When stuffing the pay envelopes illustrated in figure 11-8, Isaac James will receive: three \$20 bills, one \$10 bill, one \$5 bill, one 50¢ piece, one quarter, one dime and four coppers, a total of \$75.89.

Currency Memorandum

From the currency breakdown in figure 11-8 a currency memorandum is prepared which is presented to the teller at the bank with the payroll cheque. The teller will then pay the cash in the desired denominations. Figure 11-9 shows the currency memorandum for the breakdown in figure 11-8.

WORKOUT EXERCISE VIII

1. Prepare (a) a currency breakdown and (b) a currency memorandum for the payroll of the Soggy Log Lumber Company as prepared in Workout Exercise VII.
2. Prepare (a) a currency breakdown and (b) a currency memorandum for the payroll of the Bonnie Belinda Apron Manufacturers as prepared in Workout Exercise VII.
3. Prepare (a) a currency breakdown and (b) a currency memorandum for the payroll of the Pinto Pant Manufacturers as prepared in Workout Exercise VII.
4. Prepare (a) a currency breakdown and (b) a currency memorandum for the payroll of the XYZ Company listed in Workout Exercise VII.

Figure 11-9

Currency Memorandum

The Toronto-Dominion Bank Toronto, Ont.				
Currency Memorandum for Smithson Limited July 20, 19—				
Bills				
13	×	\$20	=	\$260
3	×	\$10	=	30
2	×	\$ 5	=	10
3	×	\$ 2	=	6
1	×	\$ 1	=	1
Coins				
2	×	.50	=	1.00
1	×	.25	=	.25
4	×	.10	=	.40
—	×	.05	=	
13	×	.01	=	.13
				<u>\$308.78</u>

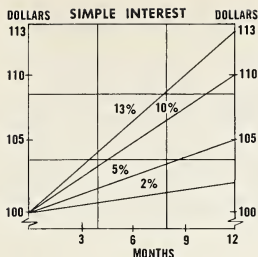
SECTION 5 Figuring for Fun

1. A day's work: Today your heart will beat 103,680 times; you will breathe 22,600 times; your blood will travel nearly 200,000,000 miles; you will inhale and exhale more than 400 cubic feet of air; you will move more than 1,000 major muscles and exercise, I hope, over 5,000,000 brain cells. Quite a day!

2. Here is one for Mom:

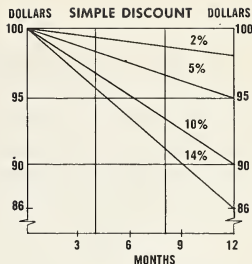
- Ask her to think of a number (say 30).
- Have her multiply the number by the number of days in the week ($7 \times 30 = 210$).
- To this number she is to add the number of days in a leap year ($210 + 366 = 576$).
- She must then divide this number by the number of days in November ($576 \div 30 = 19.2$).
- She now takes $\frac{7}{30}$ of the number originally chosen

$\left(\frac{7}{30} \times \frac{30}{1} = 7\right)$ and subtracts this from the result of the previous operation ($19.2 - 7 = 12.2$). The answer will be $\frac{366}{30} = 12.2$ $\frac{(c)}{(d)}$



Interest and Discount

12



A Business Problem

On March 15, the Mobile Marina Company purchase some additional equipment valued at \$1,900 for use in the manufacture of their scuttle butts. The terms of the sale are 2/10, n/30. The company will have \$1,900 when the bill falls due in thirty days, but on March 25 only \$500 in cash will be available. However, a short-term loan can be obtained from the bank at $5\frac{3}{4}\%$; this would enable the Mobile Marina Company to pay the whole account on March 25.

Would you advise the company to borrow the money? If so, why? If not, why not? How much would the company have to borrow? For how long a period? How much interest would they pay?

Borrowing and Lending

In our personal lives and in business circles, money is continually borrowed and loaned.

When you 'borrow' money you 'rent' it in much the same way you might rent a car or a house. For the privilege of using the money, the car, or the house, you pay a rental fee during the time it is in your possession and at the end of the period return the money, the car, or the house to the owner.

Why do we borrow money? In our personal lives we may need money quickly to pay hospital bills or meet some other emergency for which we do not have cash available. Also, every time we buy a television set, a household appliance, or a car on 'time', or on the instalment plan, we are really borrowing the money from a finance company and paying it back to them in instalments, together with a comparatively high rate of *interest*.

In business a merchant might wish to purchase heavily in September and October in order to have a large merchandise inventory on hand to sell during the Christmas season. However, he may have to pay his creditors for this merchandise before he is able to sell it. If this is the case, he can borrow the money to meet the obligation when it falls due, sell the merchandise during the Christmas season, and, with the proceeds, repay the loan in January.

A business, too, will sometimes need to expand by building an addition, another building, or another store. To finance this expansion,

it is frequently necessary to borrow the money and, it is hoped, repay the loan from the proceeds of the additional business resulting from the expansion. When money is borrowed for these or any other purposes, it should be remembered that the loan must be paid back and that a charge will be made for the use of the money. To the person borrowing money this charge, or interest paid, is an expense. To the person lending money, the interest received is income.

SECTION 1 Simple Interest — Terms

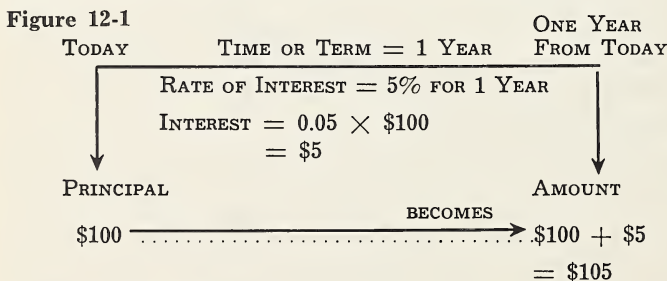
Interest, denoted by (I), is the amount charged, or the rent paid, for the use of money. The amount to be charged will depend on the rate of interest, the amount of money borrowed, and the length of time the borrower has the use of the money.

Principal, denoted by (P), is the sum of money loaned and on which the interest is to be paid.

Amount, denoted by (A), is the sum of the principal and the interest paid on the principal during the period of the loan.

Interest Rate, or simply the *rate* denoted by (r), is the per cent of the principal that must be paid per year for the use of the principal. Unless stated to the contrary, the rate of interest is always understood to be an *annual rate*.

Let us assume that \$100 is borrowed for one year and the *rate of interest* is 5% per annum. This is shown in figure 12-1.



The *principal* today is: \$100

The *interest* paid for the use of \$100 for one year is: \$ 5

The *amount* to be paid back at the end of one year is: \$105

The *interest rate* per annum is: 5%

The *formula* for the amount is:

$$\text{Amount} = \text{Principal} + \text{Interest} \quad (1)$$

Example: Sally Jones borrowed \$500 from the Homey Credit Union, is charged 12% per annum interest, and must repay the loan in one year.

How much will the interest charged amount to?
How much must she pay back?

TIME = 1 year

$$P = \$500$$

$$r = 12\% \text{ p.a.}$$

$$I = 0.12 \times \$500 \\ = \$60$$

$$A = \$500 + 60 \\ = \$560$$

Sally Jones is charged \$60 interest.

She must pay back \$560.

WORKOUT EXERCISE I

If interest is charged at $3\frac{1}{2}\%$ per annum, what *amount* must be repaid at the end of one year on the following sums? Estimate your answers first:

- | | | |
|-----------------|----------------|------------------|
| 1. \$100 | 2. \$250 | 3. \$550 |
| 4. \$6.25 | 5. \$8.30 | 6. \$59.80 |
| 7. \$16.75 | 8. \$528.40 | 9. \$1,125.80 |
| 10. \$33.80 | 11. \$2.95 | 12. \$1,429.80 |
| 13. \$17,483.29 | 14. \$8,500.15 | 15. \$300,700.60 |

If a 6% rate of interest had been charged on the following amounts repaid at the end of one year, how much was the principal?

- | | | |
|------------------|------------------|--------------|
| 16. \$907.36 | 17. \$15.90 | 18. \$677.34 |
| 19. \$1,513.68 | 20. \$20.67 | 21. \$11.13 |
| 22. \$331,491.68 | 23. \$782,784.56 | 24. \$84.27 |
| 25. \$207.23 | | |

Questions for Discussion and Projects:

- For what purpose do you think an individual or a family should borrow money?
- For what purpose should a company borrow money?
- Obtain for class discussion some pamphlets from money lending institutions in your district.
- For a bulletin-board display, obtain a number of forms used by the lending institutions in your district such as banks, credit unions and finance companies.
- Interest charged on a loan is an expense to whom? An income to whom? Explain.
- Name and explain the four terms used in simple interest.
- Write out an example using the amount formula and draw the problem as in figure 12-1.
- Construct a line graph to illustrate problems 4, 5, 7, 11, 17, 20 and 21 above. See graph in chapter heading.

SECTION 2 Simple Interest Formula

To Find the Interest

The rate of interest is always based on the principal. Therefore the principal must be multiplied by the rate to give the amount of interest, as shown in figure 12-1. The rate of interest is quoted *per annum*, and so we have the formula:

$$\text{Interest (for one year)} = \text{Principal} \times \text{rate} \quad (2)$$

However, money will be borrowed and loaned for periods other than one year, and so we must adjust formula (2) to take the time element into consideration, or

$$\text{Interest (total)} = \text{Principal} \times \text{rate} \times \text{time} \quad (3)$$

It is important to remember that the rate is expressed as a *yearly rate*, and therefore the time, denoted by (t), must also be expressed in *years and/or a fraction of a year*. The above basic simple interest formula is usually written:

$$I = P r t \quad (4)$$

This is an extremely important formula and the one with which all our simple interest problems will be solved.

Example: Abe Zeller borrowed \$200 for 73 days to pay a doctor's bill. He was charged interest at 8% per annum. How much interest did he pay on the money he borrowed? How much did he pay back?

From formula (4):

$$\begin{aligned} I &= P \times r \times t \\ &= \frac{200}{1} \times \frac{8}{100} \times \frac{73}{365} \\ &= \$3.20 \end{aligned}$$

From formula (1):

$$\begin{aligned} A &= P + I \\ &= \$200 + \$3.20 \\ &= \$203.20 \end{aligned}$$

Abe Zeller was charged \$3.20 interest.

He paid back \$203.20.

In the above example the term is given in days, and, to bring this to a fraction of a year, we use the number of days in a year (365) as the denominator. The rate can be expressed either as a common fraction, $\frac{8}{100}$, or as a decimal fraction, 0.08, whichever is simpler for the particular problem.

Simple interest problems are very similar to percentage problems with which we have already had some experience, but with the additional factor of time. If you mastered the percentage formulas, you should have very little difficulty with simple interest.

WORKOUT EXERCISE II

Compute the simple interest and the amount for the following periods using the basic simple interest formula (estimate your answers first).

	<i>Principal</i>	<i>Interest Rate</i>	<i>Time</i>
1.	\$1,012.00	3%	54 days
2.	\$725.40	$2\frac{1}{2}\%$	61 days
3.	\$489.00	3%	4 months
4.	\$730.18	12%	1 month
5.	\$837.50	$7\frac{1}{2}\%$	292 days
6.	\$19.20	4%	9 months
7.	\$1,479.15	5%	146 days
8.	\$300,690.75	5%	511 days
9.	\$923.85	4%	3 months
10.	\$4,212.48	$4\frac{1}{2}\%$	17 days
11.	\$540.25	$2\frac{1}{2}\%$	146 days
12.	\$17.84	$1\frac{1}{2}\%$	8 months
13.	\$3,419.18	$1\frac{1}{4}\%$	292 days
14.	\$328.19	$5\frac{1}{2}\%$	$5\frac{1}{2}$ months
15.	\$9,718.28	$3\frac{3}{4}\%$	$3\frac{3}{4}$ months
16.	\$420.17	$1\frac{1}{4}\%$	876 days
17.	\$29.95	$7\frac{1}{2}\%$	292 days
18.	\$128,762.48	$4\frac{7}{8}\%$	73 days
19.	\$28,847.32	$2\frac{5}{8}\%$	146 days
20.	\$198.80	$4\frac{7}{16}\%$	212 days

21. Das Huffenundpuffin Werkes manufacture sails for sail boats, and, as their sales are seasonal, they obtain a loan of \$7,350.00 from the Marine Bank for a period of $3\frac{1}{2}$ months at $6\frac{1}{4}\%$ interest. What does it cost them to borrow this money? How much do they have to repay the bank?

22. The Mythical Marine Loan Company made a short term loan on June 10 of \$1,420.75 to Das Schteerenwerkes, who manufacture rudders for sailing craft. The loan carried an interest rate of $7\frac{1}{4}\%$ and was to be paid back in 95 days. How much interest was paid? What amount was paid back?

23. Das Draginbitt Co. who manufacture marine keels obtained a 146 day loan of \$795.50 on May 6 from the Mythical Marine Loan Company. If they paid $7\frac{1}{2}\%$ interest, how much did they pay in interest charges? What amount was paid back?

To Find the Principal

Our simple interest formula has four components I , P , r and t . If we know any three of these factors, we can find the fourth by applying our rules for equations. From equation (4):

$$I = P r t$$

Divide both sides by $r \times t$, and transfer the principle, P , to the left-hand side of the equation:

$$P = \frac{I}{r \times t} \quad (5)$$

Example: Every 3 months John Lewis receives \$9 interest on money he has loaned to his brother. If he is charging interest at 6% per annum, how much did he lend his brother?

$$I = \$9.00; r = 6\% = 0.06; t = 3 \text{ months} = \frac{1}{4} \text{ year} = 0.25$$

$$P = ?$$

$$\begin{aligned} P &= \frac{I}{rt} \\ &= \frac{9}{0.06 \times 0.25} \\ &= \frac{9}{0.015} \\ &= \$600 \end{aligned}$$

In this example the time was given in months; so, to express the time as a fraction of a year, use the number of months in a year (12) as the denominator. We simplify our equation, in this case, if we express both the rate and the time as decimal fractions.

WORKOUT EXERCISE III

Find the principal in each of the following cases. (Estimate your answers first.)

	<i>Interest</i>	<i>Rate</i>	<i>Time</i>
1.	\$330.30	$7\frac{1}{2}\%$	292 days
2.	\$18.95	$5\frac{1}{2}\%$	61 days
3.	\$506.75	3%	4 months
4.	\$20.20	$6\frac{3}{4}\%$	$8\frac{3}{4}$ months
5.	\$110.30	15%	73 days
6.	\$700.50	$7\frac{1}{8}\%$	$16\frac{1}{4}$ months
7.	\$9,000.90	4%	9 months
8.	\$3.35	$9\frac{5}{8}\%$	$5\frac{1}{2}$ months
9.	\$79.45	5%	657 days
10.	\$23.14	$3\frac{3}{4}\%$	876 days

	<i>Interest</i>	<i>Rate</i>	<i>Time</i>
11.	\$95.45	$11\frac{1}{4}\%$	1 year
12.	\$49.50	$10\frac{3}{4}\%$	18 months
13.	\$18.56	$9\frac{2}{3}\%$	8 months
14.	\$920.71	$6\frac{4}{7}\%$	56 days
15.	\$200.50	4%	3 months
16.	\$35.55	$1\frac{1}{4}\%$	219 days
17.	\$57.50	$7\frac{1}{4}\%$	95 days
18.	\$2.70	$8\frac{3}{8}\%$	7 months
19.	\$9.40	$6\frac{6}{7}\%$	$3\frac{1}{2}$ months
20.	\$57.75	$7\frac{3}{4}\%$	10 months

21. How much did Das Leekinbox Hull Company borrow from the Mythical Marine Loan Company if they paid an interest charge of \$46.80 for a 5-month loan at $6\frac{1}{2}\%$?

22. How much did Das Schnoremwerkes Bunk Company borrow from the Mythical Marine Company at $8\frac{1}{4}\%$ interest if they paid an interest charge of \$157.20 for a 146-day loan?

23. Das Flappenschuff Flag & Ensign Werkes obtained two loans from the Mythical Marine Loan Company during the year on which they paid $6\frac{1}{4}\%$. \$89.20 interest was paid on a 73-day loan and \$117.50 on a 219-day loan. How much was borrowed each time?

To Find the Rate

Again from formula (4) we have:

$$I = P r t$$

Divide both sides by $P \times t$, and transfer the rate to the left side of the equation:

$$r = \frac{I}{P \times t} \quad (6)$$

Example: Bert Yellow borrowed \$511 for his school fees. He had the money from September 8 until November 17, when he paid it back. If he was charged \$8 interest, what rate of interest did he pay on the loan?

Calculation of time:

Sept. 8-30 = 22 days

Oct. 1-31 = 31 days

Nov. 1-17 = 17 days

70 days

$I = \$8$; $P = \$511$; $t = 70$ days.

$$= \frac{70}{365} = \frac{14}{73}$$

$$\begin{aligned}
 r &= \frac{I}{P \times t} \\
 r &= \frac{8}{\frac{511}{1} \times \frac{14}{73}} \\
 &= \frac{8}{98} \\
 &= 0.0816 \\
 &= \underline{\underline{8.16\%}}
 \end{aligned}$$

In this example the time had to be calculated as the number of days between two dates. When the number of days is found, the denominator 365 is used to bring this to a fraction of a year.

In complex fractions it will usually be found simpler to reduce the denominator first. In the above example the denominator is:

$$\frac{511}{1} \times \frac{14}{73}$$

73 cancels into 511, seven times, giving us the denominator 98.

WORKOUT EXERCISE IV

Find the rate in each of the following cases. Estimate your answers first.

	<i>Principal</i>	<i>Interest</i>	<i>Time</i>
1.	\$1,012.00	\$44.92	54 days
2.	\$489.00	\$4.89	4 months
3.	\$837.50	\$50.25	292 days
4.	\$1,479.15	\$29.58	146 days
5.	\$923.85	\$9.24	3 months
6.	\$540.25	\$5.40	146 days
7.	\$3,419.18	\$34.19	292 days
8.	\$9,718.28	\$113.28	3¾ months
9.	\$29.95	\$1.80	292 days
10.	\$28,847.32	\$302.90	146 days
11.	\$5,505.00	\$330.30	292 days
12.	\$50,675.00	\$506.75	4 months
13.	\$33,090.00	\$110.30	73 days
14.	\$2,700,270.00	\$9,000.90	9 months
15.	\$71,505.00	\$6,434.45	657 days
16.	\$848.44	\$95.45	1 year
17.	\$288.00	\$18.56	8 months
18.	\$20,050.00	\$200.50	3 months
19.	\$3,047.19	\$57.50	95 days
20.	\$235.00	\$9.40	3½ months

21. What rate of interest did the Mythical Marine Loan Company charge Das Ropenundholdengrab Buoy Company if the latter paid \$13.30 for a loan of \$760 for $3\frac{1}{2}$ months?

22. Das Schtuffentoburstenbox Locker Mfg. paid interest charges to the Mythical Marine Loan Company of \$307.52 for a loan of \$4,960 which they had for 292 days. What was the rate of interest charged?

To Find the Time

Divide both sides of formula (4) by $P \times r$ and transfer the time, t , to the left-hand side:

$$t = \frac{I}{P \times r} \quad (7)$$

Example: If you have \$500 and can lend it at 7% per annum, how long must you lend it if you wish to earn \$21?

$$I = \$21; P = \$500; r = 7\% = 0.07; t = ?$$

$$t = \frac{I}{P \times r}$$

$$t = \frac{21}{500 \times 0.07}$$

$$= \frac{21}{35}$$

$$= \frac{3}{5} \text{ of a year}$$

$$= \frac{3}{5} \times \frac{365}{1} \text{ days}$$

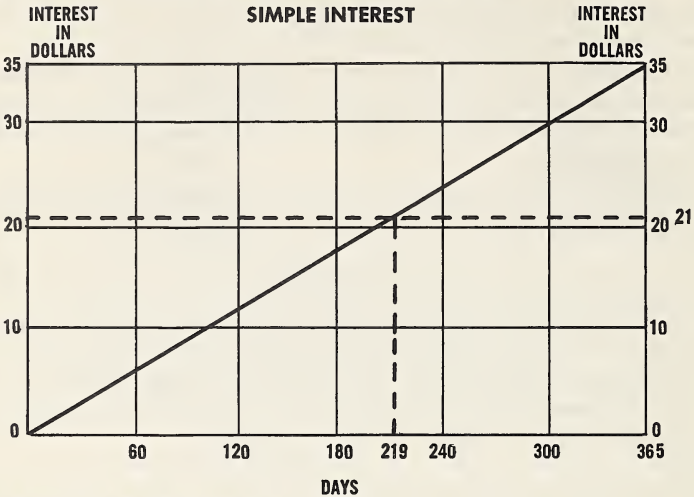
$$= 219 \text{ days}$$

Graphical Solution

Line graphs may be drawn to illustrate and solve simple interest problems. Figure 12-2 illustrates the above example.

The x-axis represents 1 year or 365 days; the y-axis, the dollar value of the simple interest earned. If a line is drawn from the 0 at the beginning of the year to the dollar value of the interest earned at the end of the year, the line can be used to read any desired result. In the above problem we know the final interest earned is \$21. Therefore, draw a horizontal line through the \$21 interest. Where this horizontal line cuts the interest line, drop a perpendicular line to the x-axis and on this axis read the length of time it will take to earn this interest, i.e., 219 days.

Figure 12-2 Simple Interest Graphical Solution



WORKOUT EXERCISE V

Find the time to the nearest day in each of the following instances. Estimate your answers first.

	Interest	Principal	Rate		Interest	Principal	Rate
1.	\$4.10	\$820.00	3½%	2.	\$23.20	\$1,410.00	7¼%
3.	\$11.50	\$328.50	6¼%	4.	\$27.90	\$847.25	5¾%
5.	\$0.50	\$17.60	2⅛%	6.	\$1.75	\$59.20	3¼%
7.	\$8.50	\$520.48	8½%	8.	\$52.14	\$1,573.40	5¼%
9.	\$24.42	\$796.00	9¼%	10.	\$1.75	\$99.20	11¼%
11.	\$3.00	\$896.20	1¼%	12.	\$14.50	\$3,120.40	4⅞%
13.	\$0.75	\$98.56	3¾%	14.	\$12.75	\$3,360.25	7¼%
15.	\$4.60	\$428.70	11½%	16.	\$19.80	\$714.80	6½%
17.	\$6.80	\$1,840.40	4¼%	18.	\$8.20	\$947.30	8¼%
19.	\$105.35	\$6,420.70	10½%	20.	\$41.20	\$1,520.50	9¾%

21. I. Dogood wishes to provide by means of an endowment fund an annual scholarship of \$1,350 for his old university. If the fund can earn 4¼%, how much, to the nearest dollar, must I. Dogood give the university.

22. What principal must John Thomas Jones invest at 3¾% simple interest when John Thomas, Junior, is 1 year old so that \$5,000 will be available for Tommy's university education on his eighteenth birthday?

23. Das Flappenschuff Ensign Co. purchase a new truck for \$5,200. The terms are 2/10, n/30. They will not have cash available to pay for the truck for thirty days but will be able to pay for it then. They can

borrow the money from the bank at 6%, if they wish, and pay the bill in order to obtain the discount. Which is the cheaper method? How much do the company save using the cheaper method?

24. Das Peepenschlot Porthole Manufacturers can borrow \$740 from the bank for 4 months at $6\frac{1}{4}\%$ interest, or the company can borrow from the Mythical Marine Loan Co. and repay the loan in four equal monthly payments, plus $\frac{1}{2}\%$ per month interest charge on the unpaid balance. Which method would have the higher interest charge? How much higher? Would the interest charge be the only consideration on which to base a decision? Explain.

25. Construct line graphs to prove your answers to any five of the problems 1-20 above.

SECTION 3 Simple Interest Tables

When the nature of a business requires frequent calculations of simple interest, it would be too time-consuming to use the simple interest formula for each calculation. To facilitate these calculations, tables are used to determine the interest quickly and accurately. Table number 2 at the back of the book is an example.

This simple interest table is based on a principal of \$1,000, but can be used to find the interest on other amounts by simply multiplying or dividing \$1,000 to obtain the required principal. Tables have also been designed to give the interest on various principals. In Canada these tables are based on a 365-day year. Simple interest tables in the United States are based on a 360-day year, because the base of 360 is a much simpler denominator to work with.

Example: Find the interest on \$100 for 30 days at 4% per annum.

Look at Table 2. In the column headed "Time" find 30 days. Proceed along this row until you reach the column headed 4%. The figure in this square will be \$3.288 and represents the interest on \$1,000 for 30 days at 4%.

∴ Interest earned on \$100 = \$.3288
= 33 cents

TIME	4%
—	—
—	—
—	—
—	—
—	—
30	3.288
—	—

Example: Find the interest on \$450 for 270 days at 5%.

Proceed as in previous example.

Interest on \$100 for 200 days is	\$2.7397
Interest on \$100 for 70 days is	.9589
Interest on \$100 for 270 days is	<u>\$3.6986</u>

∴ Interest on \$450 for 270 days is $\frac{3.6986 \times \$450}{100}$ ∴ \$16.64

TIME	5%
—	—
—	—
→ 70 — — — — — — — — —	9.589
—	—
—	—
→ 200 — — — — — — — — —	27.397
—	—

WORKOUT EXERCISE VI

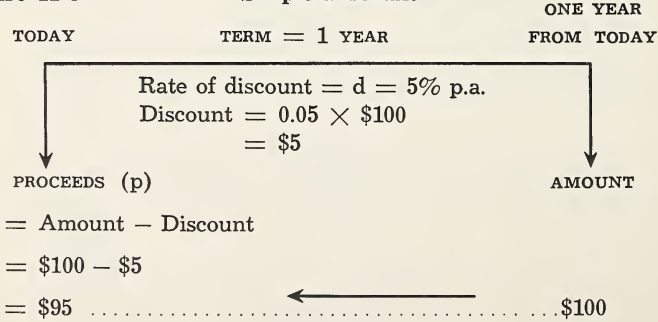
- 1. Solve Workout Exercise II, numbers 1, 3, 6, 7, 8 and 9 again using the tables.
- 2. Construct a line graph from the simple interest table showing the interest earned on \$1,000 at 4%, 5% and 6% from 10 to 100 days.

Find the interest on:

- 3. \$725 for 61 days at 4%
- 4. \$837 for 292 days at 5%
- 5. \$4,212.48 for 17 days at 3%
- 6. \$540.25 for 146 days at 2%
- 7. \$19.32 for 83 days at 1%
- 8. \$3,640.42 for 297 days at 6%
- 9. \$475.25 for 870 days at 3%
- 10. \$50.50 for 301 days at 5%
- 11. \$125.50 for 118 days at 2%
- 12. \$318,428.65 for 323 days at 4%

Figure 12-3

Simple Discount



SECTION 4 Simple Discount

When we borrow money and pay *interest* on it, we obtain the full value of the loan and pay the *interest* when we pay back the loan at the end of the period. We can, however, also borrow money by paying the interest on the loan at the time we borrow the money. In this case the interest charged on the loan is called *discount*. This is illustrated in figure 12-3. Let us assume that \$100 is borrowed for one year and that the rate of discount is 5% per annum.

The rate of discount is *always* based on the amount or sum, and so to find the discount, the formula is:

$$\text{Discount} = \text{Amount} \times \text{Rate of discount} \quad (8)$$

or

$$D = A \times d \quad (9)$$

The *proceeds* we receive are found by subtracting the discount charged from the amount of the loan, or

$$p = A - D \quad (10)$$

If the time involved is other than 1 year, then the formula to find the discount becomes:

$$D = A \times d \times t \quad (11)$$

where 't' is the time in years.

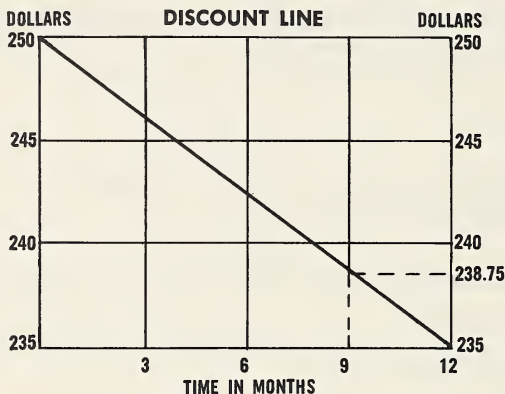
Example: David Wood borrowed \$250 for nine months from Ernie Vole. Vole charged Wood 6% discount on this loan. How much did the discount amount to? What were the proceeds of the loan? How much did Wood have to repay?

TIME = 9 months = $\frac{3}{4}$ year		
	$d = 6\% \text{ p.a.}$ $D = 250 \times 0.06 \times 0.75$ $= \$11.25$	
Proceeds $\$250 - \$11.25$ $\$238.75$	Amount $\$250$	
Discount: $D = A \times d \times t$ $= 250 \times 0.06 \times 0.75$ $= \$11.25$		
Proceeds: $p = A - D$ $= \$250 - \$11.25$ $= \$238.75$		
Repay: The amount of the loan = \$250		

A Graphical Solution

Figure 12-4 solves the above example graphically. The amount charged on \$250 at 6% for 1 year is \$15.00. Therefore, draw the discount line from \$250 to the proceeds, \$235. Draw a vertical line through 9 months, the time of the loan. Where this line cuts the discount line, draw a horizontal line to cut the y-axis and from there read the proceeds if the loan is for 9 months, i.e., \$238.75.

Figure 12-4



WORKOUT EXERCISE VII

Compute the simple discount charged and the proceeds of the following amounts and periods:

	<i>Amount</i>	<i>Discount Rate</i>	<i>Time</i>
1.	\$ 489.00	3%	4 months
2.	198.80	$4\frac{7}{16}\%$	212 days
3.	837.50	$7\frac{1}{2}\%$	292 days
4.	128,762.48	$4\frac{7}{8}\%$	73 days
5.	1,012.00	3%	54 days
6.	420.17	$1\frac{1}{4}\%$	876 days
7.	923.85	4%	3 months
8.	328.19	$5\frac{1}{2}\%$	$5\frac{1}{2}$ months
9.	1,479.15	5%	146 days
10.	17.84	$1\frac{1}{2}\%$	8 months
11.	3,419.18	$1\frac{1}{4}\%$	292 days
12.	725.40	$2\frac{1}{2}\%$	61 days
13.	540.25	$2\frac{1}{2}\%$	146 days
14.	730.18	12%	1 month

	<i>Amount</i>	<i>Discount Rate</i>	<i>Time</i>
15.	29.95	$7\frac{1}{2}\%$	292 days
16.	19.20	4%	9 months
17.	9,718.28	$3\frac{3}{4}\%$	$3\frac{3}{4}$ months
18.	300,690.75	5%	511 days
19.	28,847.32	$2\frac{5}{8}\%$	146 days
20.	4,316.18	$4\frac{3}{4}\%$	19 days

21-26. Draw line graphs to prove your answers to problems 1, 7, 8, 10, 14 and 16 above.

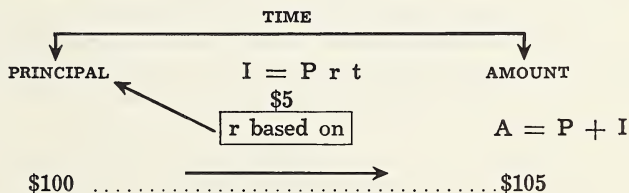
SECTION 5 The Differences Between Simple Interest and Simple Discount

The basic differences between simple interest and simple discount can be seen by bringing together and comparing figure 12-1 with figure 12-3. \$100 is borrowed in both cases; in the first instance, the loan is repayed with interest at the end; in the second case, discount is paid on the loan at the beginning and the amount of the loan repayed at the end.

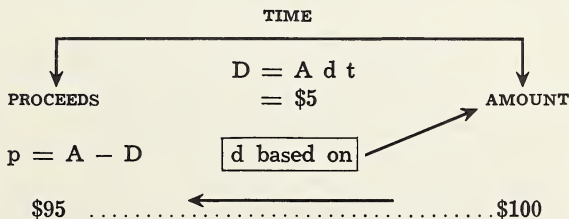
Figure 12-5

Comparison of Simple Interest and Discount

Simple Interest



Simple Discount



In the first case \$5.00 is paid for the use of \$100 for 1 year. In the second case \$5.00 is paid for the use of \$95 for 1 year.

SECTION 6 Promissory Notes

When money is borrowed, there is a legal obligation on the part of the borrower to (a) pay interest for the use of the loan and (b) to repay the loan. Frequently the borrower will be required to sign a promissory note similar to the one illustrated in figure 12-6.

Figure 12-6

A Non-Interest Bearing Promissory Note

Form No. DISCOUNT
UNIVERSITY OF GUELPH

BANK OF MONTREAL

(Branch) March 20 19 \$ 500.00

Sixty days after date I promise to pay
to the order of Arthur Jones
the sum of Five Hundred. ⁰⁰/₁₀₀ Dollars,
with interest after maturity at the rate of 7 per cent. per annum,
until paid, at the Bank of Montreal here. Value received.

Jack Bowling

Note: Interest must be paid after the date of maturity if the loan is not paid when due.

A *promissory note* is a written unconditional promise made and signed by one person, called the *maker*, to pay a certain sum of money to the lender, called the *payee*, at a future fixed or determinable date. If interest is to be paid, the rate will be stated. The date on which the money is due is the *maturity date*; the amount of money due is called the *maturity value*.

The *face* of the note is the amount of money mentioned in the note; this may be the same as the maturity value in a non-interest bearing note, as in figure 12-6, or different as in the case of an interest bearing note, as in figure 12-7. The *period* of the note is the length of time for which the money is borrowed. The *date of the note* is the date the note is signed.

In the above note, figure 12-6, \$500 is the face of the note, March 20 is the date, and 60 days is the period. Arthur Jones, to whom the note is payable, is the payee. Jack Bowling, who signed the note, is the maker. \$500 is the maturity value. It should be noted that this non-interest bearing promissory note may also be a discount note. In other words, the proceeds on March 20 will be less than the face of the note by the amount of discount charged and deducted from \$500.00. *Three days of grace* are allowed by Canadian law on promissory notes; so, in order to ascertain the maturity date of the above note, we

Figure 12-7

An Interest-Bearing Promissory Note

BANK OF MONTREAL

Form 107 (1-1-1957)
Unchanged in content

1-2 TORONTO 1, ONT.
MAIN OFFICE - KING & BAY STREETS

Due May 22, 19- DATE OF MATURITY March 20 19 DATE OF PAYMENT

(BRANCH)

Sixty days after date I promise to pay

to the order of Arthur Jones

the sum of Five Hundred..... ⁰⁰/₁₀₀ Dollars

with interest at the rate of 7 per cent per annum, as well after as before

maturity, minimum charge \$ at the Bank of Montreal

here. Value received.

Jack Bowling

The terms of a promissory note are summarized in figure 12-8.

Figure 12-8

The Terms of a Promissory Note

Amount Due (Maturity date)
in figures (face)
INTEREST Everytown (Date of the note)

(Period of the note) after date I.....promise to
pay (the payee) or order
(the lender)

..... (the face of the note) Dollars

with interest at the rate of% per annum
as well after as before maturity until paid.
Value received.

Signed (the Maker)

THE BORROWER

(The maturity value would be the face value plus required
interest based on the face value.)

Example: On June 15, Effie Jones of Kilton borrowed \$150 from her uncle Thomas Jones for a period of three months at 5% per annum interest. Write out the promissory note that Effie would give her uncle.

\$150

Int: \$1.88

Due *Sept. 18, 19-1*Kilton, *June 15, 19-1*Three months after date *I* promise to pay*Thomas Jones* or order*One Hundred and Fifty* dollars

with interest at 5% per annum as well after as before maturity until paid.

Value received.

Effie Jones

When a promissory note is paid, the payee writes "Paid" across the face, followed by the date and signature, and returns the note to the maker.

Example: On September 18, Effie Jones repaid her uncle the \$150 plus the interest on the note. What happens to the note?

The note is held in the possession of the payee Thomas Jones. On September 18, Effie Jones mails a cheque to Thomas Jones paying the note in full. Thomas Jones receipts the note.

\$150

Int: \$1.88

Due *Sept. 18, 19-1*Kilton, *June 15, 19-1*Three months after date *I* promise to pay*Thomas Jones* or order*One Hundred and Fifty* dollars

with interest at 5% per annum as well after as before maturity until paid.

Value received.

Effie Jones

The note is then returned to Effie Jones.

WORKOUT EXERCISE VIII

Draw up promissory notes for the loans made in Workout Exercise II, problems 22 and 23.

Draw up promissory notes for the loans made in Workout Exercise III, problems 22 and 23.

- Notes: 1. Omit the maturity date.
2. A company will use 'we' instead of 'I'.
3. Signature will include the name of the company and a signing officer, e.g.,
- Mattagami Mutual Mule Co.,
per Serge Sedgwick,
General Manager.
4. Keep these promissory notes for the next exercise.

Finding the Maturity Date

The maturity date on a promissory note may be stated in days, months or years. If the period is stated in years, the due date is calculated by counting the years from the date of the note and adding three days of grace. For example, a two-year note dated July 10 is due July 13 two years later.

If the period is stated in months, the due date is calculated by counting the months from the date of the note and adding three days of grace. For example, a two-month note dated February 27 would be due on April 30 in the same year. A one-month note dated January 31 is due March 3 (February 28, or 29 in leap year, plus 3 days.)

If the period is stated in days, the due date is calculated by counting the exact number of days from the date of the note and adding three days of grace to this date. For example, a 90-day note dated April 16 would be calculated as follows:

Number of days remaining in April (30-16)	14
Number of days in May	31
Number of days in June	30
Number of days to be counted in July (90-75)	<u>15</u>
Total number of days	<u>90</u>

The day the note is due is therefore

$July\ 15 + 3\ \text{days of grace} = July\ \underline{\underline{18}}$

Using Tables to Find the Due Date

When a business organization has to compute a great many due dates in the course of a day's work, the foregoing method would be very time-consuming, and, to save time, a table has been drawn up to locate the due date quickly. This is table number 3 at the back of the book which lists days in the order of their occurrence from January 1, number 1, to December 31, number 365.

We can now use this table to find the due date for the 90-day note dated April 16, mentioned above.

First: Locate April 16: Find the month along the top line and go down the left-hand column to the 16th day. Follow this horizontally to the April column; the figure where the two meet is the number of the day in the year, the 106th day.

Second: Find the number of the day required:

April 16 is the	106th day of the year
The period of the note is	90 days
Days of grace	<u>3</u>
Note is due on the	<u>199th</u> day of the year

Third: Find the date of the required day: Look for 199 in the body of the table. The month is read at the top of the columns in which 199 is found, that is, July. The date of the month is read on the same vertical line as 199 at the extreme left or the extreme right, that is, 18. The due date is therefore July 18.

If the year is a leap year and February 29 is contained in the period of the note, subtract one day from the calculation. For example, the due date of a 30-day note dated February 20 in a leap year would be found as follows:

From the table, February 20 is the 51st day.

Add period of note:	30
Add days of grace:	3
Subtract correction for	
leap year:	— <u>1</u>
	<u>83rd</u> day

∴ The due date is the 83rd day = March 24.

Example: Find the maturity date of the following notes by:

(a) calculation, and (b) from Table 3:

30-day note dated February 10,

60-day note dated March 15,

45-day note dated December 27.

	Calculation	Table
30-day note:	February <u>18</u>	February 10 = 41st day
	March 12	Note 30
	Days of grace <u>3</u>	Days of grace <u>3</u>
	Due March <u>15</u>	<u>74th</u> day
		74th day = March 15

60-day note:	March	16	March 15 =	74th day
	April	<u>30</u>	Note	60
	May	14	Days of Grace	<u>3</u>
	Days of grace	<u>3</u>		<u>137th day</u>
	Due May	<u>17</u>	137th day =	May 17
45-day note:	December	4	December 27 =	361st day
	January	<u>31</u>	Note: Old year	4
	February	10	New year	41
	Days of grace	<u>3</u>	Days of grace	<u>3</u>
	Due February	<u>13</u>		<u>44th day</u>
			44th day =	February 13

Note: Interest is charged on the 3 days of grace in addition to the period of the note.

WORKOUT EXERCISE IX

1. Calculate the maturity dates of the five promissory notes drawn up in Workout Exercise VIII by the calculation method. Check your answers by using Table 3.

Questions for Class Discussion:

2. Assume each of the promissory notes in Workout Exercise VIII was paid on the due date. Receipt each note. What happens to the note after being receipted?
3. What is a promissory note? List and explain the terms.
4. What is the legal due date of a promissory note?
5. Explain the difference between an interest and a non-interest bearing promissory note.
6. Trace the ownership of a promissory note from the moment it is signed until the debt is paid.

SECTION 7 Estimating the Answer

It has been suggested several times in previous chapters that you should always estimate your answer roughly before proceeding with the formal solution to the problem.

Here is a method you can use to quickly estimate interest charges: 60 days are approximately $\frac{1}{6}$ of a year; and if the rate of interest is 6%, then the interest for $\frac{1}{6}$ of a year at 6% is:

$$P \times \frac{6}{100} \times \frac{1}{6} = \frac{1}{100} \text{ of } P.$$

To use this method, move the decimal point in the principal two places to the left and then adjust roughly for the change of time and percentage in the problem.

Example: Find the interest on \$500 for 90 days at 5%.

$$\text{Interest on \$500 for 60 days at 6\%} = \underline{\$5.00}$$

$$\text{Interest on \$500 for 90 days at 6\%} = \underline{7.50}$$

As the rate is 1%, or $\frac{1}{6}$ less, take off

$$\frac{1}{6} \text{ of } \$7.50 \text{ for change in rate} = \underline{1.25}$$

$$\text{Rough estimate} = \underline{\underline{\$6.25}}$$

$$\text{Actual: } \frac{500}{1} \times \frac{90}{365} \times \frac{5}{100} = \$6.16$$

WORKOUT EXERCISE X

Estimate the answers by the above method to the problems in Workout Exercise II.

SECTION 8 Figuring for Fun

1. Here are some interest quickies.

(a) In each of the following cases all you do is move the decimal place in the principal two places to the left. Suppose we invest \$375.00 for:

$$4 \text{ months at } 3\% \text{ per annum } \left(375 \times \frac{4}{12} \times \frac{3}{100} \right) = \$3.75$$

$$3 \text{ months at } 4\% \text{ per annum } \left(375 \times \frac{3}{12} \times \frac{4}{100} \right) = \$3.75$$

$$2 \text{ months at } 6\% \text{ per annum } \left(375 \times \frac{2}{12} \times \frac{6}{100} \right) = \$3.75$$

$$1 \text{ month at } 12\% \text{ per annum } \left(375 \times \frac{1}{12} \times \frac{12}{100} \right) = \$3.75$$

From these basic figures we can calculate others:

If you multiply the time and divide the rate, e.g.,

$$8 \text{ months at } 1\frac{1}{2}\% \left(375 \times \frac{8}{12} \times \frac{3}{200} \right) = \$3.75.$$

If you multiply the time and the answer by the same figure, e.g.,

$$9 \text{ months at } 4\% = \$3.75 \times 3 = \$11.25.$$

If you divide the time and the answer by the same figure, e.g.,

$$1 \text{ month at } 3\% = \$3.75 \div 4 = \$0.94.$$

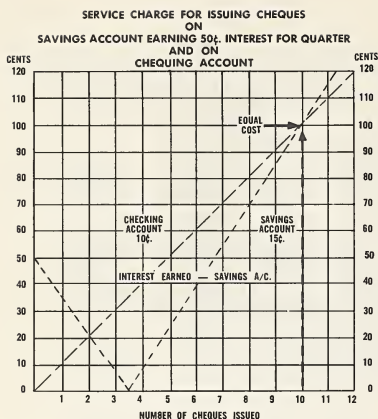
Recalculate Workout Exercise II, questions 3, 4, 6, 9, 12, and Workout Exercise III, questions 3, 7, 15, to see how fast you can do them.

(b) To find interest at 5% per annum on \$425 for

$$\begin{aligned}
 73 \text{ days: } & \left(\frac{73}{365} \times \frac{5}{100} \times \frac{425}{1} \right) = \frac{1}{100} \times \frac{425}{1} = \$4.25 \\
 146 \text{ days: } & \left(\frac{146}{365} \times \frac{5}{100} \times \frac{425}{1} \right) = \frac{2}{100} \times \frac{425}{1} = \$8.50 \\
 219 \text{ days: } & \left(\frac{219}{365} \times \frac{5}{100} \times \frac{425}{1} \right) = \frac{3}{100} \times \frac{425}{1} = \$12.75 \\
 292 \text{ days: } & \left(\frac{292}{365} \times \frac{5}{100} \times \frac{425}{1} \right) = \frac{4}{100} \times \frac{425}{1} = \$17.00 \\
 365 \text{ days: } & \left(\frac{5}{100} \times \frac{425}{1} \right) = \$21.25
 \end{aligned}$$

These basic figures may be used to find others by dividing the time and the answer by the same figure or by adding together any combination. Try Workout Exercise II, questions 7, 8, 11, 13, 16 17, 23, and Workout Exercise III, questions 1, 5, 9, 10, 16, 23.

2. Which is correct: 9 and 9 are 17 or 9 and 9 is 17?



Money and Banking 13

A Business Problem

Sammy Midas owns your local drug store. His cash receipts amount to about \$800 dollars per week, out of which he pays, in cash, a part time assistant \$40 per week and his operating expenses of \$600 per week. He uses 60% of the remainder for his personal living expenses and then hides the balance in an old sock under his bed. He complains he does not trust banks and they are closed anyway by the time he shuts up the store for the night. What would you tell him about banks to persuade him to open an account? What sort of accounts would you advise him to open, and how much should he deposit in them, in all, each week?

The Canadian Chartered Banks

The present Canadian banking system was founded in 1817 when the Bank of Montreal opened as a private company. In 1822, the Bank of Montreal, together with the Bank of Canada in Montreal, the Quebec Bank in Quebec, and the Bank of Upper Canada in York, were granted charters to operate as limited companies. However, the first bank to be granted a charter in Canada was the Bank of New Brunswick in 1820. These early charters followed closely the charter drawn up by the great financier Alexander Hamilton for the Bank of the United States. Hamilton, in turn, had followed closely the practices of the English and Scottish banks.

At the time of Confederation, 37 banks were in operation out of the 62 that had received provincial charters up until that time. These 37 banks had 164 branches in various parts of Canada. In all, 101 banks have started in business, but today only 8 remain. 39 have failed, and 54 have amalgamated. In 1867, the British North America Act gave complete legislative authority in banking matters to the Parliament of Canada. Up to that time they had been the responsibility of the individual provinces.

The first Bank Act was passed in 1871 and set out the regulations and safeguards to be followed by the banks. One provision was that the Act be reviewed every ten years. There have been eight revisions since the original Act; the last revision was in 1954.

The banks now operating in Canada are:

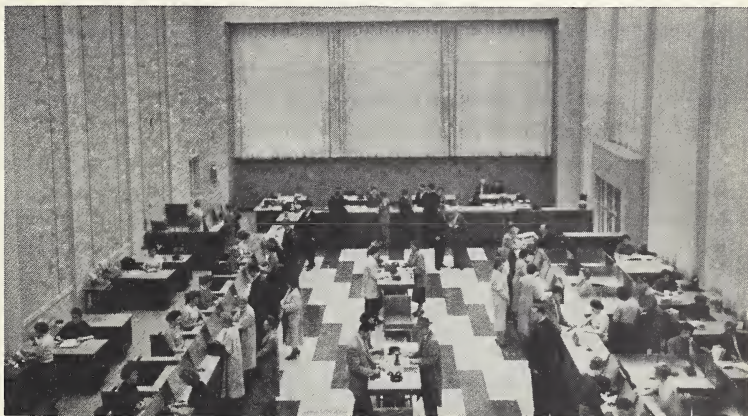
- Bank of Montreal, 1817.
- Bank of Nova Scotia, 1832.
- Provincial Bank of Canada, 1861.
- Royal Bank of Canada, 1864.
- Banque Canadienne Nationale, 1874.
- Mercantile Bank of Canada, 1953.
- Toronto-Dominion Bank, 1955.
- Imperial Bank of Commerce, 1961.

The Toronto-Dominion Bank is the result of the amalgamation, in 1955, of the Bank of Toronto, 1856, and the Dominion Bank, 1871. The Imperial Bank of Commerce is the result of the amalgamation, in 1961, of the Canadian Bank of Commerce, 1867, and the Imperial Bank of Canada, 1875.

Although the number of chartered banks has diminished over the years, the number of branches has increased until there are now approximately 5,000 branches of the eight chartered banks in Canada.

Figure 13-1

A Branch Bank



The Bank of Montreal, City Hall Branch, London, Ontario

This system has come to be known as *branch banking* and facilitates the transfer of funds and services to the place where they are most needed.

The Functions of the Canadian Chartered Banks

The two principle functions of a bank are:

1. To accept for safe-keeping money deposited by individuals, companies, and governments;
2. To use part of the money deposited as loans which will help the production of commodities, general trade, and governmental operation.

The Bank Act safeguards the depositor's money by establishing rules for the types of loans that may be made.

Other important services the bank performs include:

1. Buying and selling securities for bank customers;
 2. Helping customers who trade abroad to pay accounts or make collections in foreign countries;
 3. Providing facilities for the storage of valuables;
 4. Selling travellers' cheques and money orders; arranging letters of credit;
 5. Accepting payment for the bills of the local utility companies;
- And many other services.

Until 1935, all the chartered banks were allowed to issue their own currency. Up to that time, you could receive your pay in, say, Imperial Bank bills or, perhaps, Royal Bank bills, and you might well have bills issued by several of the existing banks in your wallet.

The Bank of Canada — The Banker's Bank

The Bank of Canada is owned by the Government of Canada and was established in 1934 to regulate the issue of currency and the volume of credit.

The Functions of the Bank of Canada

1. The Issuing of Bank Notes.

The Bank of Canada gradually took over the issuing of bank notes until, in January 1945, the chartered banks were forbidden to place additional notes in circulation; in January 1950, the Bank of Canada took over all the currency of the chartered banks then outstanding. The chartered banks now obtain their supply of bank notes from the Bank of Canada.

2. The Control of Credit.

In order to have cash available at any time a depositor may wish to withdraw it, the chartered banks must keep eight per cent of their total deposits in cash. They must also maintain a secondary reserve which can be quickly turned into cash should the need arise. This secondary reserve is usually in the form of 91-day Government Treasury Bills. The Bank of Canada buys these securities from the chartered banks and so releases money to them to lend; alternatively, the Bank of Canada will sell the securities to the chartered banks in order

to curtail the amount of money the chartered banks have available to lend. This is one of the ways in which the Bank of Canada controls the loan activities of the chartered banks.

3. A Clearing House for Cheques.

The Bank of Canada maintains clearing houses in major cities throughout Canada for the settlement of cheques issued on the accounts of depositors of the eight chartered banks.

WORKOUT EXERCISE I

Questions for Class Discussion:

1. What was the name of the first bank to be granted a charter in Canada? When did this occur? Is this bank still in operation?
2. How many of the banks granted charters in 1822 are still operating?
3. How many banks have been granted charters since 1820? How many banks held charters at the time of Confederation? What are the names of the banks still holding charters? What happened to the rest of the banks? Which four banks have most recently amalgamated?
4. Discuss the Bank Act.
5. What are the functions of the Canadian Chartered Banks?
6. What are the functions of the Bank of Canada?
7. Project: Pay a class visit to a large branch of a bank and collect as many different bank forms as possible. Obtain a blank cheque, a cancelled cheque and a bank statement. Write to several of the chartered banks for their literature and arrange a class bulletin board display.
8. If a bank savings department allows 2% interest, calculated (compounded) quarterly, on their deposits, to how much will the following sums amount: \$100 in 3 years; \$15 in $2\frac{1}{2}$ years; \$99.50 in $4\frac{3}{4}$ years; \$7.35 in $1\frac{3}{4}$ years; \$25.60 in $9\frac{1}{4}$ years?

SECTION 1 Opening a Bank Account

A Savings Account

Money deposited in a savings account will earn interest, calculated, or compounded, quarterly, which the bank pays to you in return for the use of your money. This interest is based on the minimum balance, cents ignored, in your account during the three-month period.

If you need the money, you can withdraw it as you require it. Although the legal requirement for withdrawing funds from a savings account is seven days' notice, this right is seldom, if ever, enforced.

Any member of the bank's staff will assist you to open an account. They will require your name, address, occupation, and a specimen of your signature. Your signature is kept on file for comparison with cheques you may issue and sign. A service charge of 15¢ is levied for

each cheque you issue, although some banks do allow a small number of free cheques depending on your minimum quarterly balance; for instance, one free cheque for every hundred dollars you have on deposit. You will then be supplied with a bank passbook marked with your account number and a book of cheques.

Depositing Money

When you deposit money in your account, you will complete a deposit slip similar to figure 13-2.

Figure 13-2

A Savings
Deposit Slip

SAVINGS DEPT. Form 1736 (7-55)
THE ROYAL BANK OF CANADA

ACCOUNT NUMBER
4791

March 20, 19--
(DATE)

CREDIT
Penelope Wyse
(NAME OF ACCOUNT)

CHEQUES	CASH	DOLLARS	CTS.
	x 1		
	1 x 2	2	00
	2 x 5	10	00
	1 x 10	10	00
	x 20		
6	80		
	x		
	SILVER	2	00
7	COPPER		63
	BOND COUPONS		
14	CHEQUES	14	75
TOTAL		39	38
LESS CASH, RECEIPT OF WHICH IS HEREBY ACKNOWLEDGED			
(SIGNATURE) NET TOTAL		39	38
LESS EXCHANGE			
NET DEPOSIT		39	38

TELLER'S INITIALS

P. Wyse
DEPOSITED BY

You should see that your deposit is entered in your Pass Book when made, or you should present your deposit slip in duplicate and request the Teller to receipt and return the duplicate copy.

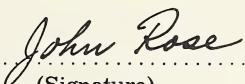
List each cheque separately in the column provided, and extend the total to the right-hand column.

List all currency separately in the right-hand column showing the total of each denomination of coin and bill.

Total the deposit.

Should you wish to receive cash for a cheque or cheques you are depositing, the amount required is entered under the total, with your signature. For instance, you may receive a pay cheque for \$100 and wish to deposit \$90 and receive \$10 in cash. Complete the deposit as

above, enter \$10 in the required space, and sign where indicated as an acknowledgement of receipt of cash.

Example:		Total	\$ 100.00
Less cash, receipt of which is hereby acknowledged			
 (Signature)			10.00
Net Total			90.00
Less exchange (if any)			
Net Deposit		\$	90.00
..... Teller's Initials			

You then hand your deposit, with the deposit slip, to the bank teller. The teller will keep the deposit slip and record the amount of the deposit in your bank passbook. It should be noted, however, that there is no obligation on the part of the bank to pay a portion of the cheque in cash unless the payor also has his account at the same bank.

A Current Account

A current account is used by business firms. Money deposited is payable on demand. No interest is allowed on deposits. Bank passbooks are not issued, but a monthly statement of account is provided. Cheques which have been issued and paid from the account during the month are returned to you with the monthly statement. A nominal monthly charge is made for this service, but no service charge is made for cheques issued on the account.

Figure 13-3

A Current Account Deposit Slip

FORM 400 1C (REV. 9-62)		THE		NAME OR PLACE		AMOUNT	EXCH.		
TORONTO-DOMINION		BANK		Gottawa		100 -	15	53	1 53 -
CREDIT		March 5/50--		Hamilton		182 17		17	2 34 -
Tailspin Assembly		Limited		London		105 -	15	18	5 90 -
						95 -	15	11	10 110 -
								3	20 60 -
									X
									COIN
									13 84
									\$ 360 84
									CHEQUES (FORWARD)
									482 17
									\$ 843 01
									LESS EXCHANGE
									45
									NET DEPOSIT \$
									842 56
ACCOUNT CODE NUMBER		3824		SUB-TOTALS		482 17	45		
						» FORWARD »			

A current account will be opened up in the same way as a savings account. Frequently a business firm will require more than one person to sign a cheque, and, in this case, the signatures of all persons entitled to sign cheques must be on file with the bank.

A Personal Chequing Account

A personal chequing account is similar to a current account and is used by individual depositors. The money is payable on demand and no interest is paid. A service charge of 10¢ is levied for each cheque issued. Bank passbooks are not issued, but most banks send a monthly or bi-monthly statement together with the cancelled cheques issued by the depositor during the time covered by the statement.

For depositors who normally have a low balance and issue a number of cheques, it is often less expensive to pay 10¢ for each cheque issued and receive no interest for money on deposit than it is to pay 15¢ for each cheque issued and receive interest for money on deposit. Figure 13-4 shows a deposit slip for a personal chequing account.

Figure 13-4

A Personal Chequing Account Deposit Slip

NO. 240737 <i>Timothy Thrift</i> NAME OF ACCOUNT DEPOSITED ON THE DATE STAMPED HEREON THE AMOUNT OF \$ 154 75	PERSONAL CHEQUING ACCOUNT	THE TORONTO-DOMINION BANK DEPOSIT <i>December 18, 19</i> DATE CREDIT THE ACCOUNT OF <i>Timothy Thrift</i> NAME <i>18 Toddington Drive</i> ADDRESS	PCA NO. 240737 <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="font-size: small;">CURRENCY</td> <td style="text-align: right;">\$ 41 00</td> </tr> <tr> <td style="font-size: small;">COIN</td> <td style="text-align: right;">3 75</td> </tr> <tr> <td style="font-size: small;">CHEQUES OR COUPONS — List on back if necessary.</td> <td style="text-align: right;">110 00</td> </tr> <tr> <td style="font-size: small;">TOTAL</td> <td style="text-align: right;">154 75</td> </tr> <tr> <td style="font-size: small;">LESS Received in Cash</td> <td></td> </tr> <tr> <td style="font-size: small;">SIGNATURE</td> <td style="text-align: right;">154 75</td> </tr> <tr> <td style="font-size: small;">AMOUNT OF DEPOSIT</td> <td style="text-align: right;">\$</td> </tr> </table>	CURRENCY	\$ 41 00	COIN	3 75	CHEQUES OR COUPONS — List on back if necessary.	110 00	TOTAL	154 75	LESS Received in Cash		SIGNATURE	154 75	AMOUNT OF DEPOSIT	\$
CURRENCY	\$ 41 00																
COIN	3 75																
CHEQUES OR COUPONS — List on back if necessary.	110 00																
TOTAL	154 75																
LESS Received in Cash																	
SIGNATURE	154 75																
AMOUNT OF DEPOSIT	\$																
TELLER'S STAMP		TELLER'S STAMP															

Form 399 (Rev. 3-61)

Example: Frank Truman receives a monthly pay cheque of \$525. He cashes it at his bank on the first of each month and deposits \$400 in his account, taking the remainder in cash. He had \$63 in the bank on September 30 and issued the following cheques:

In October, 18 cheques totalling	\$385
In November, 15 cheques totalling	\$295
In December, 21 cheques totalling	\$391

What is his total expense if he has:

- (a) A savings account paying 2%, compounded quarterly?
- (b) A personal chequing account?

(a) *Savings account:*

Balance, September 30	\$ 63.00	
Deposit, October 1	<u>400.00</u>	
Balance, October 1	463.00	
October cheques	\$385.00	
Service charge, 18 × 15¢	<u>2.70</u>	387.70
Balance, October 31	75.30	= minimum balance
Deposit, November 1	<u>400.00</u>	
Balance, November 1	475.30	
November cheques	\$295.00	
Service charge, 15 × 15¢	<u>2.25</u>	297.25
Balance, November 30	178.05	
Deposit, December 1	<u>400.00</u>	
Balance, December 1	578.05	
December cheques	\$391.00	
Service charge, 21 × 15¢	<u>3.15</u>	394.15
Balance, December 31	<u>\$183.90</u>	
Interest: $\frac{2}{100} \times \frac{75}{1} \times \frac{1}{4}$...	\$.37	
Service charges: 54 × 15¢ ...	\$ 8.10	
Interest earned	<u>.37</u>	
Total expense	<u>\$ 7.73</u>	

(b) *Personal chequing account:*

Total expense:

$$54 \times 10 = \$ \underline{\underline{5.40}}$$

WORKOUT EXERCISE II

1. Draw a chart to show the similarity and the points of difference between a savings account, a chequing account and a current account.
2. Draw a graph similar to the one heading this chapter to show the service charges for issuing cheques on (a) a savings account paying interest at 2% per annum, compounded quarterly, where the minimum quarterly balance is \$250, and (b) a personal chequing account. At what point would the total cost be equal? When would it be more advantageous to use the savings account?
3. Percy Craig maintains a minimum balance of \$480 in his account. If a savings account allows 2¾% per annum, compounded quarterly, and a free cheque for every \$100 of minimum balance, at what point

would it be advantageous for Percy Craig to use a personal chequing account?

4. Mr. U. R. Clerk deposited in the current account of the Coldade Beverages Limited 50 coins of each denomination, 15 bills of each denomination under \$50, and the following cheques: \$25.18, \$78.23, \$18.53, \$31.57 and \$99.87. Make out the deposit slip.

5. Amy Amiable deposited her monthly pay cheque for \$283.10 and a dividend cheque for \$24.85 in her personal chequing account. Make out her deposit slip.

6. Fred Tumbel deposited in his savings account 18 coppers, 23 nickels, 17 dimes, 11 quarters, 7 half-dollars, 19 dollar bills, 13 two-dollar bills, 3 five-dollar bills, and the following amounts in cheques: \$7.95, \$37.49 and \$1.97. Make out his deposit slip.

SECTION 2 Cheques

Issuing Cheques

When you write a cheque, you are ordering your bank to pay out of your account a certain sum of money to the person named on the cheque.

Figure 13-5

A Savings Account Cheque

<i>Toronto, Ont. June 7 19-- N^o 576</i> MAIN OFFICE - KING & BAY STREETS	
<i>To the</i> <i>Bank of Montreal</i>	
<i>Pay to the order of Grayburn Realty Co Ltd \$105⁰⁰</i>	
<i>One Hundred and Five⁰⁰/₁₀₀ Dollars.</i>	
SAVINGS DEPARTMENT ACCOUNT No. 2899 PLEASE INSERT NUMBER OF ACCOUNT	<i>Amy L. Bridgeport</i>

Exchange and Bank Charges

In a city where there are branches of several of the eight chartered banks, any one of these banks will cash a cheque drawn on another bank within the same city without charge because there is usually a local 'clearing house' where cheques are sorted out and forwarded to the correct bank. However, if you receive a cheque from out-of-town and present it for payment at your local bank, the bank will charge exchange on it. The rate is $\frac{1}{8}$ of 1% with a minimum charge of 15¢ for branches of the same bank. $\frac{1}{4}$ of 1%, with a minimum charge of 25¢, may legally be charged if the cheque is drawn on another bank which is out-of-town.

Example: The Soggy Log Wood Products received the following cheques in the mail from out-of-town points: \$310, \$406, \$120, \$57, \$115. To how much would the bank exchange amount?

Exchange: $\frac{1}{8}$ of \$310	=	\$.39
$\frac{1}{8}$ of \$406	=	.51
$\frac{1}{8}$ of \$120	=	.15
Minimum charge on \$57	=	.15
Minimum charge on \$115	=	.15
Total exchange charged	=	<u>\$1.35</u>

When our customers pay us by accepting a draft drawn on them by us, the bank will look after the draft for us and collect the money from our customers. When the draft is due, the bank will deposit the money in our account. For this service there is a bank charge of $\frac{1}{8}\%$ with a minimum charge of 20¢.

Example: The bank collected a draft for us which was drawn on our customer, and accepted by him, for \$460. To how much did the bank charges amount?

Bank charges: $\frac{1}{8}\%$ of \$460	=	\$0.575
	=	\$0.58

Receiving Cheques

When you receive a cheque in payment of a debt or as a gift, you should take it to the bank where you have your account even if it is drawn on one of the other banks. When you present it for payment, you must *endorse* it on the back, signing your name exactly the same way as it is written on the cheque. If the person has misspelled your name, write it this way first and then sign it with the correct spelling underneath.

Current Account Cheques

Current account cheques often come in book form with a number of cheques on each page. The cheques are detached from the right-hand side and a permanent record kept on the *stubs* at the left.

As can be seen in figure 13-6, the cheque stubs have space for recording deposits that have been made, as well as the cheques that are written. Thus a running bank balance can be obtained from the stubs. This type of record in the cheque book serves as an additional control or check of your bank account.

Example: The Soggy Log Wood Products issued the following cheques: On April 11, \$45 to the Federated Gas Company in payment of a heating bill; on April 12, \$61 to Wehavit Supplies for office supplies; on April 13, \$157.30 to Gordon Bros. Ltd. for merchandise.

They had an initial balance of \$500 and deposited \$50 on April 10 and \$121 on April 13. Make out these cheques in a current account cheque book.

See figure 13-6.

Figure 13-6
Current Account Cheques

Toronto, Ont. April 11, 19-1. No. 21 1485 To The Canadian Bank of Commerce Pay to the order of The Federated Gas Company \$45.00 Twenty - four only Charles Gordon - President Ed. Dracer - Treasurer		CHEQUES AND DEPOSITS BALANCE FORWARD \$ 500.00 CHEQUE NO. 21 \$ 45.00 DATE April 11 455.00 TO: Federated Gas Co. ADD for: Heating DEPOSIT April 10 \$ 50.00 DATE April 11 505.00
Toronto, Ont. April 12, 19-1. No. 22 1486 To The Canadian Bank of Commerce Pay to the order of Wehavit Supplies \$61.00 Sixty - one only Charles Gordon - President Ed. Dracer - Treasurer		DEDUCT CHEQUE NO. 22 \$ 61.00 DATE April 12 444.00 TO: Wehavit Supplies ADD for: Office supplies DEPOSIT April 12 \$ 44.00 DATE April 12 444.00
Toronto, Ont. April 13, 19-1. No. 23 1487 To The Canadian Bank of Commerce Pay to the order of Gordon Bros. Ltd. \$157.30 One hundred and fifty-seven and 30/100 Dollars Charles Gordon - President Ed. Dracer - Treasurer		DEDUCT CHEQUE NO. 23 \$ 157.30 DATE April 13 287.00 TO: Gordon Bros. Ltd. ADD for: Merchandise DEPOSIT April 13 \$ 121.00 DATE April 13 408.00 BALANCE CARRIED FORWARD 408.00

WORKOUT EXERCISE III

1. Draw a facsimile of a savings account cheque, a current account cheque with stub, and a personal chequing account cheque with stub, and fill in all the details.
2. In the day's bank deposit of the Crown Cheddar Creamery, the following cheques were from out of town: \$76.80, \$21.30, \$176.19, \$342.20 and \$147.80. An additional cheque for \$782.30 was deposited on which the higher charge was levied. To how much did the bank charges amount?
3. Obtain a pad of current account cheques from a local bank and fill in the following week's transactions for the Yellow Fox Corporation. Assign names to the payees. What is the final balance in the account?
November 1, \$350 balance. Issued: \$25, \$47.50, \$75.35, \$49.80; Deposited: \$180;
November 2, issued: \$85.30, \$98.30, \$38.20; Deposited: \$295;
November 3, issued: \$38.40, \$17.60, \$38.90, \$64.25, \$2.98, \$43.20; Deposited: \$201.18;
November 4, issued: \$157.30, \$19.17, \$198.11, \$2.30; Deposited: \$476.54;
November 5, issued: \$76.21, \$47.83, \$23.97, \$68.76, \$213.58, \$7.95; Deposited: \$316.34.
4. During the month of June the bank collected the following drafts which Hospital Supplies Limited had drawn on their customers: \$780.25, \$1,128.17, \$384.17, \$98.14, \$76.13, \$182.49, \$59.80, \$2,361.98, \$97.18, \$120. What were the total bank charges?
5. There are several methods of endorsing cheques. Consult a business practice textbook to find out what some of these methods are and the reasons for them. For example, how would you endorse a cheque that is payable to you but which is to be carried to the bank and deposited in your account by someone else?

SECTION 3 Bank Statement and Reconciliation

Each depositor should check his account periodically and compare the balance calculated with the balance reported by the bank.

Savings Account Pass Book and Ledger Card

A savings account ledger card is maintained at the bank to record all your transactions with the bank, including the service charges for the cheques you issue and the interest they pay you on the money in your account. The interest is calculated on the minimum quarterly balance and added to your account twice a year on April 1 and November 1. Each time you are in the bank, you should give the teller your pass book and he will bring it up-to-date with the bank ledger card. A page from a pass book and a savings account ledger card are shown in figure 13-7.

2. \$75 for $4\frac{1}{4}$ years at $2\frac{1}{2}\%$ per annum, compounded quarterly.
3. \$1,025 for $10\frac{1}{2}$ years at 7% per annum, compounded semi-annually.
4. \$3,172.50 for $1\frac{3}{4}$ years at 9% per annum, compounded three times a year.
5. \$18.95 for $18\frac{3}{4}$ years at 3% per annum, compounded quarterly.
6. \$346.83 for $3\frac{1}{2}$ years at $4\frac{1}{2}\%$ per annum, compounded semi-annually.
7. \$976.27 for 15 years at $5\frac{1}{2}\%$ per annum, compounded annually.
8. \$84.91 for 11 years at 6% per annum, compounded monthly.
9. \$758.20 for $7\frac{2}{3}$ years at $4\frac{1}{2}\%$ per annum, compounded three times a year.
10. \$1,029,751.10 for $12\frac{1}{2}$ years at 4% per annum, compounded quarterly.
11. Draw up a pass book and a savings account ledger card to record the following: Balance, January 1, \$121. Cheques issued in January: 3rd \$17.50, 5th \$75.00, 7th \$8.00, 9th \$2.95, 13th \$49.10, 21st \$54.30, 26th \$48.63, 29th \$85, \$25, \$10.95. Deposited in January: 10th \$141.20, 25th \$116.16. Cheques issued in February: 11th \$9.50, 13th \$48.21, 17th \$13.95, 23rd \$27.75, 28th \$85, \$25, \$4.20. Deposited in February: 10th \$105.20, 27th \$121.40. Cheques issued in March: 3rd \$31.02, \$4.17, 11th \$17.95, 14th \$3.10, 17th \$18.95, 23rd \$56.30, 26th \$19.55, 30th \$85, \$25, \$15.95. Deposited in March: 2nd \$25.00, 10th \$121.16, 29th \$146.18. Minimum balance October 1 to December 1 was \$28.35. No free cheques are allowed and interest at $2\frac{3}{4}\%$, compounded quarterly, is calculated on the minimum quarterly balance and credited to the account March 31 and September 30. Service charge is made at the end of each quarter. What was the balance in the account on April 1?

Current Account Bank Statement

As part of the service to current account and personal chequing account customers, the bank prepares a monthly statement of account showing the transactions that have occurred in the depositor's account during the month. The bank also returns to the customer all the cancelled cheques; that is, the cheques the depositor has issued and which payees have taken to the bank and cashed during the month. A typical bank statement is shown in figure 13-8.

The bank statement shows:

- (a) The balance at the end of the previous month.
- (b) Deposits made during the month.
- (c) Cheques paid out during the month.
- (d) Bank service charges.
- (e) The daily balance.
- (f) The balance at the end of the month.

A Bank Statement

Figure 13-8

THE SOGGY LOG WOOD PRODUCTS,
1314 ELM LANE,
CEDARVALE.

In Account With **BANK OF MONTREAL**

DEBITS	DEBITS	CREDITS	DATE	NO. OF ENTRIES	BALANCE
	BALANCE BROUGHT FORWARD		MAR. 1		1,376.82 S
132.18-			MAR. 2		1,244.64 S
68.40-	13.95-	1,301.20	MAR. 3		2,463.49 S
141.62-			MAR. 4		2,321.87 S
91.16-	1.50-	432.00	MAR. 5		2,661.21 S
263.24-	54.68-		MAR. 6		2,343.29 S
480.21-	155.75-	312.17	MAR. 8		2,019.50 S
920.42-			MAR.10		1,099.08 S
37.80-		575.25	MAR.11		1,636.53 S
43.28-			MAR.12		1,593.25 S
318.84-	12.95-		MAR.13		1,261.46 S
18.90-	21.40-		MAR.15		1,221.16 S
18.95 DM		756.20	MAR.15		1,958.41 S
48.20-	17.60-		MAR.18		1,892.61 S
178.63-		423.16	MAR.19		2,137.14 S
478.20-			MAR.22		1,658.94 S
510.38-		162.11	MAR.24		1,310.67 S
242.43-			MAR.25		1,068.24 S
43.20-			MAR.26		1,025.04 S
		220.00	MAR.27		1,245.04 S
18.32-	58.11-	261.84	MAR.29		1,430.45 S
4.27 SC			MAR.30		1,426.18 *

Please examine at once and report any errors to the ACCOUNTANT'S DEPARTMENT

EXPLANATION OF CHARACTERS

CC - Certified Item

CM - Credit Memo

O - Loan or Discount

DM - Debit Memo

EC - Error Corrected

IN - Interest

LT - Total of Several Cheques

DO - Overdraft

RT - Returned Item

SC - Service Charge

Kindly notify us promptly of change of address

This statement gives the transactions that the bank has on record. It does not, however, always correspond exactly with your own record of transactions. Suppose that on March 31 you write out and mail three cheques to pay the rent, the telephone bill, and a charge account. You then enter these cheques in your records, and they reduce your bank balance. The cheques will not reach their destination until April 1 or 2. So the recipients will not be able to cash them until April. Consequently, the bank will have no record of these cheques when they make up your statement for March, and the balance they show in your statement will be more than the balance you have in your own records. On the other hand, the bank may charge you a fee for services rendered you during the month. The first notification you have of this is on the bank statement, and so the fee will not have been entered in your own records.

From the "Explanation of Characters" at the foot of the statement, we see that SC opposite the amount of \$4.27 means this is the amount charged for the bank services. The letters DM appearing after the amount of \$18.95 indicate that the bank have debited the account, or deducted from the account, this amount. This would happen, for example, if a customer gave the company a cheque which they deposited and which was later returned N.S.F. (Not Sufficient Funds) from the customer's bank. A "debit memo" is then sent to the depositing company to inform them that this has been done.

Businesses usually make it a rule that all cash and cheques received must be deposited each day. If it is necessary to make a deposit after banking hours, the deposit may be put in the bank's night deposit box. The bank will supply the company with a key to the box for this purpose. However, the bank will not 'receive' the deposit until it opens on the next business day. So if, for example, you make a deposit in the night deposit box on the last day of the month, the bank will not receive it until the first business day of the next month; and so it too will not be shown on the bank statement, even though you have recorded the amount in your own records.

Reconciliation of Bank Statement

As pointed out above, the bank statement and your records may not have exactly the same balances, on the same day, due to some temporary conditions. In order to show your true bank balance at the end of the month, and in business it is important to do this, you prepare a statement to reconcile the two balances and to show the amount of the true bank balance.

Example: The Soggy Log Wood Products receive from the bank the statement shown in figure 13-8, together with the cancelled cheques, for the month of March. The balance in the company's records is \$1,786.85. Mr. Ebony, the accountant, compares the cancelled cheques with his record of cheques issued and finds that the following cheques have not been presented to the bank for payment:

Cheque #51 — \$176.85

Cheque #54 — \$ 59.20

Cheque #55 — \$ 17.73

Cheque #56 — \$121.14

The Soggy Log Wood Products had mailed in a deposit of \$731.32 on March 31 which does not appear on the statement.

A bank charge of \$4.27 had been deducted by the bank.

What is the true bank balance as of March 31st ?

To find this, we draw up a bank reconciliation statement as shown in figure 13-9.

Figure 13-9

THE SOGGY LOG WOOD PRODUCTS Bank Reconciliation Statement March 31, 19-1				
Balance as per: Company's record	\$1,786.85	Balance as per: Bank statement		\$1,426.18
Deduct: Service charge	4.27	Deduct: Outstanding cheques:		
		#51	\$176.85	
		#54	59.20	
		#55	17.73	
		#56	121.14	\$ 374.92
				<u>\$1,051.26</u>
		Add:		
		Late deposit		731.32
True bank balance	<u>\$1,782.58</u>	True bank balance		<u>\$1,782.58</u>

This type of reconciliation statement resembles an account. Like all statements it must have a heading and a date.

The left side starts with the balance recorded in the depositor's own books. In this case the Soggy Log Wood Products show a balance of \$1,786.85 on their books, but the bank charge of \$4.27 has not yet been deducted; so it will be necessary to deduct this charge.

The right side of the bank reconciliation statement starts with the balance reported on the bank statement, in this case \$1,426.18. The statement however does not include four cheques which have been deducted from the Soggy Log records as being paid but which have not as yet been presented to the bank for payment. These are called *outstanding cheques*. This amount, totalling \$374.92, must be deducted from the reported bank balance. In addition a deposit sent to the bank on March 31 did not reach the bank in time to be recorded on the statement; so we add in this amount of \$731.32. When we have adjusted the bank balance to show what it should be if completely up-to-date, that is, all cheques paid out and all deposits entered, we find both sides of our reconciliation statement show the true balance of the money we have in the bank on March 31.

WORKOUT EXERCISE V

1. The bank balance, according to the records of Service Planters Limited, is \$1,328.17 on September 30. When the bank statement is received, it shows a balance of \$1,335.69. The cancelled cheques are checked against the company's record, and it is found that the following cheques have not yet been cashed: No. 17, \$100; No. 21, \$17.50; No. 22, \$84.27; No. 24, \$119.50 and No. 25, \$15.00. A deposit of \$325 was made by Service Planters Limited too late to be included in the bank's statement. The bank's monthly service charge amounts to \$3.75 and has not been recorded in the company's books. Prepare a reconciliation statement to show the true bank balance.

2. Prepare a bank reconciliation statement for Knutty Kennel Krisps Company for the month of August to show their true bank balance. Their books record a balance of \$5,432.57, and the bank statement records a balance of \$5,191.70. Outstanding cheques are: No. 419, \$17.50; No. 427, \$542.20; No. 428, \$59.30; No. 429, \$98.99. A late deposit amounted to \$929.11. The bank recorded a debit memo for \$25.00 for an N.S.F. cheque and service charges of \$4.75.
3. You have been asked to help the bookkeeper of the Bovary Bovine Feeds reconcile their bank statement for the month of June. You are given the following information: Bank balance on the company's books: \$1,678.37; bank balance on the bank statement: \$1,328.70. Outstanding cheques not yet presented for payment: No. 813, \$157.20; No. 897, \$13.20; No. 898, \$76.13 and No. 899, \$50.00. A deposit of \$625.45 did not reach the bank in time to be included in the statement. A bank service charge of \$2.75 had not been recorded in the company's books. You also discover that cheque No. 805 was issued for \$157.18, but in the bank statement this has been recorded and deducted as \$175.18. Prepare the required bank reconciliation statement.
4. Prepare the bank reconciliation statement for the Far-Away Travel Agency for the month of December. The company's books show a balance of \$11,428.33. The bank statement shows a balance of \$10,737.94. Outstanding cheques are: No. 73, 75, 76, 79 for \$528.50, \$1,029.40, \$764.64, and \$329.40, respectively. An outstanding deposit amounted to \$3,420.18. Cheque No. 55 was recorded in the bank statement as \$727.27 but was incorrectly recorded in the Far-Away Travel Agency books as \$277.72. The bank recorded a service charge of \$5.50 for the collection of a draft for the company, and the deposit of the proceeds of the draft of \$628.35 in their account (a credit memo), and the deduction of \$95.45 from their account for an N.S.F. cheque (a debit memo), none of which the company had recorded.
5. The bank statement for Morgan Miffelin Mufflers showed an overdraft of \$97.80 on November 30. Outstanding cheques not cashed by the bank were \$78.60, \$15.45, \$320.18, \$59.80, and \$38.40. A late deposit of \$945.17 had not been recorded on the bank statement. If the bank statement recorded service charges of \$2.50, interest charges of \$7.50 and a debit memo for \$47.50, none of which had been recorded by the company, how much did the company's records show the bank balance to be? What was the true bank balance? Prepare a reconciliation statement.
6. Majorie Noble, the owner of the Noble Poodle Boutique, has asked you to prepare a bank reconciliation statement for her business for the month of October. The balance according to her records is \$1,420.76. The following cheques which she has issued have not yet been cashed by the bank: No. 156, \$18.75; No. 158, \$73.80; No. 165, \$17.95; No. 168, \$33.18 and No. 169, \$110.50. A late deposit of \$540.27 was not recorded

on the bank statement. The bank had recorded a service charge of \$3.50 and had cashed a cheque on the account issued by Noble Products for \$79.65. What balance did the bank statement show? Prepare a reconciliation statement to show the true bank balance.

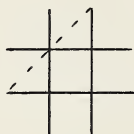
7. C. P. Martin Manufacturing show a cash balance on their books of \$3,220.18. The bank statement received for the month of January reveals the following items:

- (a) Outstanding cheques not presented: Nos. 198, 236, 237, 240 and 242 for \$629.80, \$17.65, \$84.50, \$328.13 and \$247.20, respectively.
- (b) Deposit of \$384.42 not recorded.
- (c) Service charge, \$2.50, and interest charge, \$15.00.
- (d) A cheque for \$78.28 for G. P. Martin has been charged to the account.
- (e) A cheque for \$313.21 has been entered on the statement as \$331.12.
- (f) The bank have collected and deposited in the account the proceeds of a draft amounting to \$1,174.32 (credit memo).

What is the amount of the balance on the bank statement? Prepare a reconciliation statement showing the true bank balance.

SECTION 4 Figuring for Fun

1. Draw 4 lines to form a double cross.



Form an 8-pointed star joining the points of the double cross as indicated in the diagram.



Aim of the game:

To place seven separate coins on seven of the eight points of the star according to the following rules:

Rules for play:

- (1) Take only one coin at a time.
 - (2) Choose a point of the star and place coin on it.
 - (3) Move the coin along either of the straight lines leading from the point of the star to an adjacent point.
 - (4) You may only move the coin *once*.
 - (5) The point the coin occupies after it has been moved is then out of play, and another point must be chosen for the next coin.
2. A French proverb: Money doesn't bring happiness, but it calms the nerves.

APPENDIX A

ROUNDING OFF

Whole Numbers

In many mathematical calculations it is more meaningful to give the answer in terms of a large unit. For instance, if the census-taker reports the population of a country as 21,321,971, this figure could be reported as 21,000,000, rounded off to the nearest million.

<i>Examples:</i> Round off 8,321 to the nearest hundred	= 8,300
Round off 72,916 to the nearest thousand	= 73,000
Round off 41,351 to the nearest hundred	= 41,400
Round off 445 to the nearest ten	= 440
Round off 4,349 to the nearest hundred	= 4,300
Round off 4,351 to the nearest hundred	= 4,400
Round off 3,750 to the nearest hundred	= 3,800
Round off 7,450 to the nearest hundred	= 7,400
Round off 7,451 to the nearest hundred	= 7,500

Rule: To round off a number to a specified place, replace all the digits to the right of that place by zeros.

Rule: To adjust the digit immediately to the left of the part dropped:

(a) If the part dropped begins with 1, 2, 3, or 4, the digit to the left remains unchanged.

(b) If the part dropped begins with 6, 7, 8, or 9, the digit to the left is increased by 1.

(c) If the part dropped begins with 5 and is followed only by zeros, the preceding digit is increased if it is an odd number but left unchanged if it is an even number. (This is the rule followed in this text, although some authorities always increase the retained digit.) See examples 7 and 8 above.

(d) If the part dropped begins with 5 and is followed by a digit other than zero, the preceding digit is increased by one. See last example above.

Decimal Fractions

The above method can be extended to include decimal fractions.

Rule: To round off a number to a specified place, drop all the digits to the right of that place.

Rule: To adjust the digit immediately to the left of the part dropped, proceed as above for whole numbers.

Examples:

	Tenths Hundredths Thousandths Ten Thousandths			Units Tenths Hundredths
(1)	3 . 1 7 6	to the nearest tenth	=	3 . 2
(2)	6 . 0 3 5 2	to the nearest hundredth	=	6 . 0 4
(3)	4 . 5 9 7	to the nearest hundredth	=	4 . 6 0
(4)	18 . 9 7	to the nearest tenth	=	19 . 0
(5)	79 . 9 9	to the nearest tenth	=	80 . 0
(6)	5 . 8 1 6	to the nearest unit	=	6 .
(7)	616 . 3 4 9	to the nearest ten	=	620 .

Note: The zeros must be included in the result:

- In (3): The zeros in the tenths and hundredths places must be included; otherwise 4.6 would appear to be given to the nearest tenth instead of to the nearest hundredth as requested.
- In (4): The zero in the tenths place must be retained; otherwise 19 would appear to be given to the nearest unit instead of to the nearest tenth as requested.
- In (5): The zero for the unit digit must be retained as a place holder for the 8 tens, and the zero in the tenths place retained to indicate the answer is correct to the nearest tenth.
- In (7): The zero must be retained in the units place as a place holder.

APPENDIX B

MULTIPLICATION AND DIVISION – SOME SHORT CUTS

Some Short Cuts in Multiplication

To multiply by:	Procedure	Answer
5	Add one zero	$139 \times 5 = 1,390 \div 2 = 695$ $3,162 \times 50 = 316,200 \div 2 = 1,581$ $623 \times 500 = 623,000 \div 2 = 311,500$
50	Add two zeros	
500	Add three zeros	
11	Separate the two digits and insert their sum between: $11 \times 31 = 3 (3+1) 1 =$ If the sum of the two digits exceeds 9, carry 1 to the left hand digit: $11 \times 37 = 3 (3+7) 7 = 3 (10) 7 =$ $11 \times 49 = 4 (4+9) 9 = 4 (13) 9 =$	341 407 539

To multiply by:	Procedure	Answer
	If there are more than two digits, separate the end digits; then, starting with the two right hand digits, add two at a time from right to left: $11 \times 312 = 3 (3+1) (1+2) 2 =$ $11 \times 371 = 3 (3+7) (7+1) 1 = 3 (10) (8) 1 =$ $11 \times 6,721 = 6 (6+7) (7+2) (2+1) 1 =$ $6 (13) (9) (3) 1 =$ $11 \times 103,762 = 1 (1+0) (0+3) (3+7) (7+6) (6+2) 2 =$ $1 (1) (3) (10) (13) (8) 2 =$	3,432 4,081 73,931 1,141,382
12 to 19	(a) Multiply the multiplicand by the <i>last</i> digit of the multiplier. (b) Write down the product one place to the <i>right</i> of the multiplicand and add: $4,962 \times 13$ $\begin{array}{r} 4962 \\ 14886 \\ \hline 64506 \end{array}$	64,506
21, 31, 41, etc.	(a) Multiply the multiplicand by the <i>first</i> digit of the multiplier. (b) Write down the product one place to the <i>left</i> of the multiplicand and add: $4,962 \times 31$ $\begin{array}{r} 4962 \\ 14886 \\ \hline 153822 \end{array}$	153,822
	Note: The last two methods, i.e., $\times 13$ and $\times 31$, use similar methods. To remember the difference: $\times 13$ — the digit 3 is to the right; place the product to the right of the multiplicand, $\times 31$ — the digit 3 is to the left; place the product to the left of the multiplicand.	
15	(a) Add one zero to the number. (b) Divide by two and add: 75×15 $\begin{array}{r} 750 \\ 375 \\ \hline 1125 \end{array}$	1,125
25	(a) Add two zeros. (b) Divide by four for the answer: 136×25 $\begin{array}{r} 13600 \\ 3400 \\ \hline 3400 \end{array}$	3,400
19, 29, 39, etc.	(a) Multiply multiplicand by next highest figures, e.g., 30, and place product <i>above</i> multiplicand. (b) Subtract multiplicand from product obtained in (a): 419×29 $\begin{array}{r} 12570 \\ 419 \\ \hline 12151 \end{array}$ $\begin{array}{r} (\times 30) \\ (\times 1) \\ (\times 29) \end{array}$	12,151

To multiply by:	Procedure	Answer
21, 31, 41, etc.	(a) Multiply multiplicand by next lowest figures, e.g., 30, and place product above multiplicand (b) Add: $\begin{array}{r} 419 \times 31 \\ \quad 419 \\ 12570 \\ \hline 12989 \end{array}$	12,989
A number close to 100	(a) Add two zeros. (b) Multiply multiplicand by amount over or under one hundred. (c) If over a hundred, add: $\begin{array}{r} 571 \times 103 \\ \quad (a) \ 57100 \\ \quad (b) \ 1713 \\ \hline 58813 \end{array}$ (d) If under a hundred, subtract: $\begin{array}{r} 571 \times 98 \\ \quad (a) \ 57100 \\ \quad (b) \ 1142 \\ \hline 55958 \end{array}$	58,813 55,958
1½, 2½, 3½, etc.	Divide multiplicand by 2. Multiply multiplier by 2. $8,472 \times 3\frac{1}{2} = 4,236 \times 7 =$ $7,428 \times 7\frac{1}{2} = 3,714 \times 15 =$	29,652 55,710

Some Short Cuts in Division

See Chapter 2, Figuring for Fun Section for Austrian division which is a very fast and efficient method of long division.

To divide by:	Procedure	Answer
	Move decimal point:	
5	1 place to the left	85.0
50	2 places to the left	156.8
500	3 places to the left	196.850
12½	2 places to the left and multiply by 8	34.32
25	2 places to the left and multiply by 4	3,372.84
	The above method can be used with all aliquot parts of 10, 100, 1,000, etc.	
10	Move decimal point 1 place to the left, $78,943 \div 10 =$	7,894.3
100	Move decimal point 2 places to the left, $78,943 \div 100 =$	789.43
1,000	Move decimal point 3 places to the left, $78,943 \div 1,000 =$	78.943
	The above method can be used with all multiples of 10.	

To divide by:	Procedure	Answer
$1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, etc.	Multiply divisor by 2. Multiply dividend by 2. $47 \div 3\frac{1}{2} = 94 \div 7 =$ $79\frac{1}{2} \div 7\frac{1}{2} = 159 \div 15 =$	$13\frac{3}{4}$ $10\frac{3}{5}$

Tests For Divisibility

Can be divided by:	If:
2	the final digit is divisible by 2: Example: 494. The final digit 4 is divisible by 2; therefore 492 is divisible by 2.
3	the sum of the digits is divisible by 3: Example: 136,821. Sum of the digits = 21. 21 is divisible by 3; therefore 136,821 is divisible by 3.
4	the two final digits are zeros or are divisible by 4: Example: 3,197,264. Final two digits = 64. 64 is divisible by 4; therefore 3,197,264 is divisible by 4.
5	the final digit is 0 or 5.
6	the last digit is an even number, and the sum of the digits is divisible by 3: Example: 3,297,264. Last digit is even. The sum of the digits = 33. 33 is divisible by 3; therefore 3,297,264 is divisible by 6.
8	the three final digits are divisible by 8: Example: 42,936. 936 is divisible by 8; therefore 42,936 is divisible by 8.
9	the sum of its digits is divisible by 9: Example: 362,349. Sum of the digits = 27. 27 is divisible by 9; therefore 362,349 is divisible by 9.
10	the final digit is 0.
11	starting from the right of the number: the sum of the digits in the odd places subtracted from the sum of the digits in the even places is zero, or divisible by 11: Example: 3,826,031. Sum of the odd-placed digits: $1 + 0 + 2 + 3 = 6$ Sum of the even-placed digits: $3 + 6 + 8 = 17$ $17 - 6 = 11$; 11 is divisible by 11; therefore 3,826,031 is divisible by 11.
12	the last two digits are divisible by 4, and the sum of the digits is divisible by 3: Example: 782,354,616. The last two digits, 16, are divisible by 4. The sum of the digits = 42; 42 is divisible by 3; therefore 782,354,616 is divisible by 12.

TABLE 1
Multiplication Table

Products you must know												Products you should know								Products worth knowing				
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625

TABLE 2

Daily Table of Interest
Showing Simple Interest on \$1,000
Based on 365 Days

No. of Days	1%	2%	3%	4%	5%	6%
1	0.027	0.055	0.082	0.110	0.137	0.164
2	0.055	0.110	0.164	0.219	0.274	0.329
3	0.082	0.164	0.247	0.329	0.411	0.493
4	0.101	0.201	0.329	0.438	0.548	0.658
5	0.137	0.274	0.411	0.548	0.685	0.822
6	0.165	0.331	0.493	0.658	0.822	0.986
7	0.192	0.394	0.575	0.767	0.959	1.151
8	0.219	0.438	0.658	0.877	1.096	1.315
9	0.247	0.493	0.740	0.986	1.233	1.479
10	0.274	0.548	0.822	1.096	1.370	1.644
20	0.548	1.096	1.644	2.192	2.740	3.288
30	0.822	1.644	2.466	3.288	4.110	4.932
40	1.096	2.192	3.288	4.384	5.479	6.575
50	1.370	2.740	4.110	5.479	6.849	8.219
60	1.644	3.288	4.932	6.575	8.219	9.863
70	1.918	3.836	5.753	7.671	9.589	11.507
80	2.192	4.384	6.575	8.767	10.959	13.151
90	2.466	4.932	7.397	9.863	12.329	14.795
100	2.740	5.479	8.219	10.959	13.699	16.438
110	3.014	6.027	9.041	12.055	15.068	18.082
120	3.288	6.575	9.864	13.150	16.438	19.726
130	3.562	7.123	10.685	14.247	17.808	21.369
140	3.836	7.671	11.506	15.342	19.178	23.014
150	4.110	8.219	12.330	16.437	20.547	24.657
160	4.384	8.767	13.150	17.534	21.918	26.302
170	4.658	9.315	13.972	18.630	23.288	27.945
180	4.932	9.863	14.794	19.726	24.658	29.588
190	5.206	10.411	15.616	20.822	26.027	31.232
200	5.479	10.959	16.438	21.918	27.397	32.877
210	5.753	11.507	17.259	23.013	28.767	34.521
220	6.027	12.055	18.082	24.110	30.137	36.164
230	6.301	12.603	18.904	25.205	31.507	37.807
240	6.575	13.151	19.725	26.301	32.877	39.453
250	6.849	13.699	20.550	27.395	34.245	41.095
260	7.123	14.247	21.370	28.493	35.616	42.739
270	7.397	14.795	22.191	29.589	36.987	44.385
280	7.671	15.343	23.012	30.684	38.356	46.028
290	7.945	15.890	23.835	31.781	39.726	47.671
300	8.219	16.438	24.658	32.877	41.096	49.315
310	8.493	16.986	25.479	33.973	42.466	50.958
320	8.767	17.534	26.300	35.068	43.836	52.604
330	9.041	18.082	27.123	36.164	45.205	54.245
340	9.315	18.630	27.945	37.260	46.575	55.889
350	9.589	19.178	28.767	38.356	47.945	57.533
360	9.863	19.726	29.589	39.452	49.315	59.177
365	10.000	20.000	30.000	40.000	50.000	60.000

TABLE 3
THE NUMBER OF EACH DAY OF THE YEAR

DAY OF MONTH	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	DAY OF MONTH
1	1	32	60	91	121	152	182	213	244	274	305	335	1
2	2	33	61	92	122	153	183	214	245	275	306	336	2
3	3	34	62	93	123	154	184	215	246	276	307	337	3
4	4	35	63	94	124	155	185	216	247	277	308	338	4
5	5	36	64	95	125	156	186	217	248	278	309	339	5
6	6	37	65	96	126	157	187	218	249	279	310	340	6
7	7	38	66	97	127	158	188	219	250	280	311	341	7
8	8	39	67	98	128	159	189	220	251	281	312	342	8
9	9	40	68	99	129	160	190	221	252	282	313	343	9
10	10	41	69	100	130	161	191	222	253	283	314	344	10
11	11	42	70	101	131	162	192	223	254	284	315	345	11
12	12	43	71	102	132	163	193	224	255	285	316	346	12
13	13	44	72	103	133	164	194	225	256	286	317	347	13
14	14	45	73	104	134	165	195	226	257	287	318	348	14
15	15	46	74	105	135	166	196	227	258	288	319	349	15
16	16	47	75	106	136	167	197	228	259	289	320	350	16
17	17	48	76	107	137	168	198	229	260	290	321	351	17
18	18	49	77	108	138	169	199	230	261	291	322	352	18
19	19	50	78	109	139	170	200	231	262	292	323	353	19
20	20	51	79	110	140	171	201	232	263	293	324	354	20
21	21	52	80	111	141	172	202	233	264	294	325	355	21
22	22	53	81	112	142	173	203	234	265	295	326	356	22
23	23	54	82	113	143	174	204	235	266	296	327	357	23
24	24	55	83	114	144	175	205	236	267	297	328	358	24
25	25	56	84	115	145	176	206	237	268	298	329	359	25
26	26	57	85	116	146	177	207	238	269	299	330	360	26
27	27	58	86	117	147	178	208	239	270	300	331	361	27
28	28	59	87	118	148	179	209	240	271	301	332	362	28
29	29		88	119	149	180	210	241	272	302	333	363	29
30	30		89	120	150	181	211	242	273	303	334	364	30
31	31		90		151		212	243		304		365	31

TABLE 4

Table of Time for Mensuration

60 seconds (sec.)	= 1 minute (min.)	52 weeks	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	12 months	= 1 year
24 hours	= 1 day (da.)	365 days	= 1 year*
7 days	= 1 week (wk.)	100 years	= 1 century (C.)
30 days	= 1 month (mo.)*		

* Do you recall your elementary school rhyme: "30 days have September, April, June, and November. All the rest have 31, except February which has 28; and leap year, coming once in four, gives February one day more."

Leap year, of course, has 366 days.

TABLE 5
Equivalent Discount Factor Table

Rate Per Cent	2	4	5	7 1/2	10	15	20	25
2 1/2	0.9555	0.9360	0.9262	0.9019	0.8775	0.8288	0.7800	0.7312
5	0.9310	0.9120	0.9025	0.8788	0.8550	0.8075	0.7600	0.7125
5-2 1/2	0.9077	0.8892	0.8799	0.8568	0.8336	0.7873	0.7410	0.6947
5-5	0.8844	0.8664	0.8574	0.8348	0.8122	0.7671	0.7220	0.6769
5-7 1/2	0.8612	0.8436	0.8348	0.8128	0.7909	0.7469	0.7030	0.6591
5-10	0.8379	0.8208	0.8122	0.7909	0.7695	0.7268	0.6840	0.6412
10	0.8820	0.8640	0.8550	0.8325	0.8100	0.7650	0.7200	0.6750
10-7 1/2	0.8158	0.7992	0.7909	0.7701	0.7492	0.7072	0.6660	0.6244
10-10	0.7938	0.7776	0.7695	0.7492	0.7290	0.6885	0.6480	0.6075
15	0.7080	0.6936	0.6864	0.6683	0.6502	0.6141	0.5780	0.5419
15-5	0.7914	0.7752	0.7671	0.7469	0.7268	0.6864	0.6460	0.6056
15-10	0.7497	0.7344	0.7268	0.7076	0.6885	0.6502	0.6120	0.5738
20	0.7840	0.7680	0.7600	0.7400	0.7200	0.6800	0.6400	0.6000
20-5	0.7448	0.7296	0.7220	0.7030	0.6840	0.6460	0.6080	0.5700
20-10	0.7056	0.6912	0.6840	0.6660	0.6480	0.6120	0.5760	0.5400
20-20	0.6272	0.6144	0.6080	0.5920	0.5760	0.5440	0.5120	0.4800
25	0.7350	0.7200	0.7125	0.6938	0.6750	0.6375	0.6000	0.5625
25-5	0.6982	0.6840	0.6769	0.6591	0.6412	0.6056	0.5700	0.5344
25-10	0.6615	0.6480	0.6412	0.6244	0.6075	0.5738	0.5400	0.5062
25-15	0.6248	0.6120	0.6056	0.5897	0.5738	0.5419	0.5100	0.4781
25-20	0.5880	0.5760	0.5700	0.5550	0.5400	0.5100	0.4800	0.4500
25-25	0.5512	0.5400	0.5344	0.5203	0.5062	0.4781	0.4500	0.4219

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